Topics in Concurrency

Lecture 2

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The Calculus of Communicating Systems

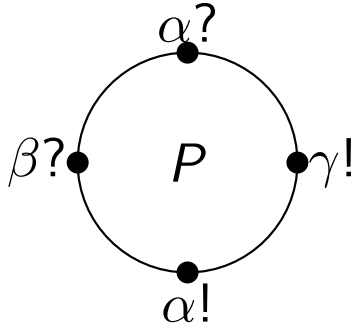
- Introduced by Robin Milner in 1980
- First process calculus developed with its operational semantics
- Supports algebraic reasoning about equivalence
- Simplifies Dijkstra's Guarded Command Language by removing the store (store locations can be encoded as processes)

Action and communication

- Processes communicate values (numbers) on channels.
- Communication is synchronous and between two processes.
- a is an arithmetic expression. Evaluation is written $a \rightarrow n$
- Input: $\alpha?x$
- Output: $\alpha!a$
- Silent action: τ. Internal to the process.
- We will use λ to range over all the kinds of action.

Interface diagrams

- *Interface diagrams* describe the channels used by processes for input and output.
- The use of a channel by a process is called a *port*.
- Example: process P inputs on α, β and outputs on α, γ.



- Later examples: links between processes to represent the possibility of communication

Syntax of CCS

- Expressions: Arithmetic a and Boolean b
- Processes:

p	::=	**nil**	nil process
	$\mid$	$(\tau \to p)$	silent/internal action
	$\mid$	$(\alpha!a \to p)$	output
	$\mid$	$(\alpha?x \to p)$	input
	$\mid$	$(b \to p)$	Boolean guard
	$\mid$	$p_0 + p_1$	non-deterministic choice
	$\mid$	$p_0 \parallel p_1$	parallel composition
	$\mid$	$p \backslash L$	restriction (L a set of channel identifiers)
	$\mid$	$p[f]$	relabelling (f a function on channel identifiers)
	$\mid$	$P(a_1, \cdots, a_k)$	process identifier

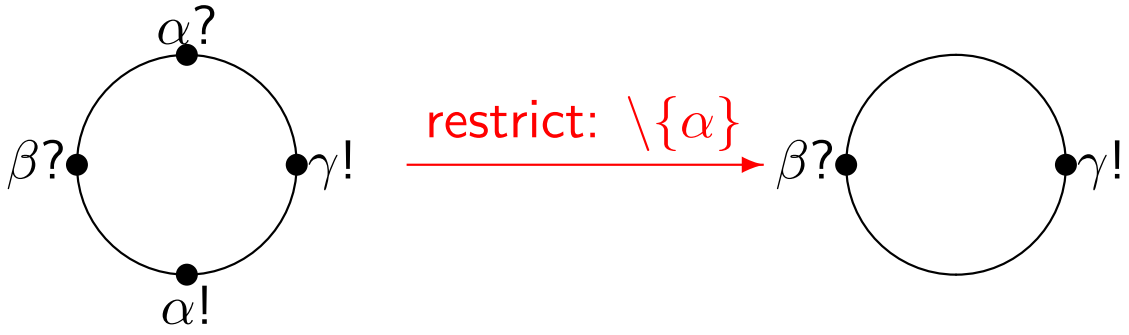
- Process definitions:

$$P(x_1, \cdots, x_k) \stackrel{\text{def}}{=} p$$

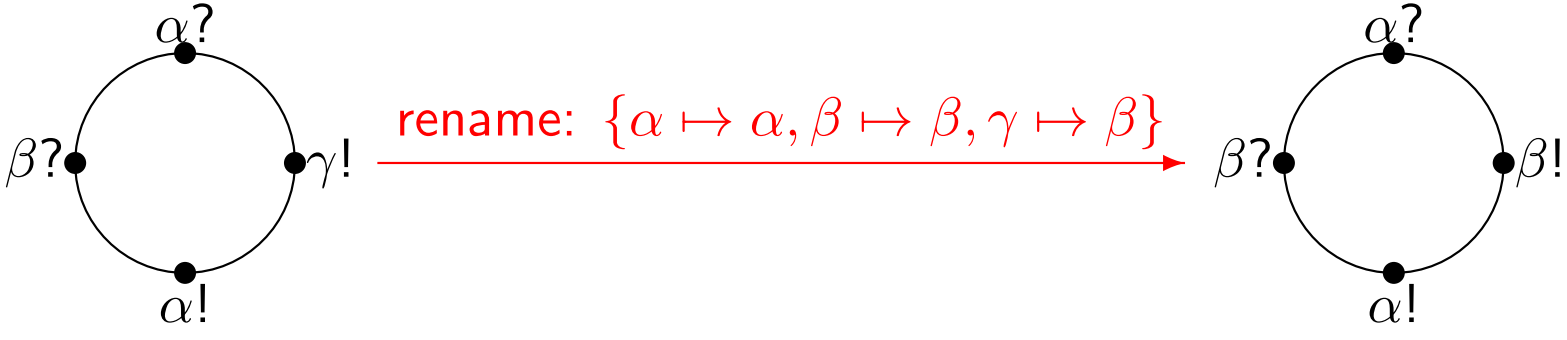
(free variables of $p \subseteq \{x_1, \cdots, x_k\}$)

Restriction and relabelling: interface diagrams

- $p \setminus L$: Disallow external interaction on channels in L



- $p[f]$: Rename external interface to channels by f



Operational semantics of CCS

- **Guarded processes**

  Silent action

$$(\tau \to p) \xrightarrow{\tau} p$$

  Output

$$\frac{a \longrightarrow n}{(\alpha!a \to p) \xrightarrow{\alpha!n} p}$$

  Input

$$(\alpha?x \to p) \xrightarrow{\alpha?n} p[n/x]$$

  Boolean

$$\frac{b \to \mathsf{true} \qquad p \xrightarrow{\lambda} p'}{(b \to p) \xrightarrow{\lambda} p'}$$

- **Sum**

$$\frac{p_0 \xrightarrow{\lambda} p_0'}{p_0 + p_1 \xrightarrow{\lambda} p_0'} \qquad \frac{p_1 \xrightarrow{\lambda} p_1'}{p_0 + p_1 \xrightarrow{\lambda} p_1'}$$

- **Parallel composition**

$$\frac{p_0 \xrightarrow{\lambda} p_0'}{p_0 \parallel p_1 \xrightarrow{\lambda} p_0' \parallel p_1} \qquad \frac{p_0 \xrightarrow{\alpha?n} p_0' \qquad p_1 \xrightarrow{\alpha!n} p_1'}{p_0 \parallel p_1 \xrightarrow{\tau} p_0' \parallel p_1'}$$

$$\frac{p_1 \xrightarrow{\lambda} p_1'}{p_0 \parallel p_1 \xrightarrow{\lambda} p_0 \parallel p_1'} \qquad \frac{p_0 \xrightarrow{\alpha!n} p_0' \qquad p_1 \xrightarrow{\alpha?n} p_1'}{p_0 \parallel p_1 \xrightarrow{\tau} p_0' \parallel p_1'}$$

- **Restriction**

$$\frac{p \xrightarrow{\lambda} p'}{p \setminus L \xrightarrow{\lambda} p' \setminus L} \quad \text{where if } \lambda \equiv \alpha?n \text{ or } \lambda \equiv \alpha!n \text{ then } \alpha \notin L$$

- **Relabelling**

$$\frac{p \xrightarrow{\lambda} p'}{p[f] \xrightarrow{f(\lambda)} p'[f]}$$

where f is extended to labels as $f(\tau) = \tau$ and $f(\alpha?n) = f(\alpha)?n$ and $f(\alpha!n) = f(\alpha)!n$

- **Identifiers**

$$\frac{p[a_1/x_1, \cdots, a_n/x_n] \xrightarrow{\lambda} p'}{P(a_1, \cdots, a_n) \xrightarrow{\lambda} p'}$$

- **Nil process** no rules

A derivation from the operational semantics

$$\dfrac{\dfrac{(\alpha!3 \to \mathbf{nil}) \xrightarrow{\alpha!3} \mathbf{nil}}{(\alpha!3 \to \mathbf{nil}) + P \xrightarrow{\alpha!3} \mathbf{nil}}}{((\alpha!3 \to \mathbf{nil}) + P) \parallel (\tau \to \mathbf{nil}) \xrightarrow{\alpha!3} \mathbf{nil} \parallel (\tau \to \mathbf{nil})}$$

A derivation from the operational semantics

$$\dfrac{\dfrac{\dfrac{(\alpha!3 \to \mathbf{nil}) \xrightarrow{\alpha!3} \mathbf{nil}}{(\alpha!3 \to \mathbf{nil}) + P \xrightarrow{\alpha!3} \mathbf{nil}}}{((\alpha!3 \to \mathbf{nil}) + P) \parallel (\tau \to \mathbf{nil}) \xrightarrow{\alpha!3} \mathbf{nil} \parallel (\tau \to \mathbf{nil})} \qquad (\alpha?x \to \mathbf{nil}) \xrightarrow{\alpha?3} \mathbf{nil}}{(((\alpha!3 \to \mathbf{nil}) + P) \parallel (\tau \to \mathbf{nil})) \parallel (\alpha?x \to \mathbf{nil}) \xrightarrow{\tau} (\mathbf{nil} \parallel (\tau \to \mathbf{nil})) \parallel \mathbf{nil}}$$

A derivation from the operational semantics

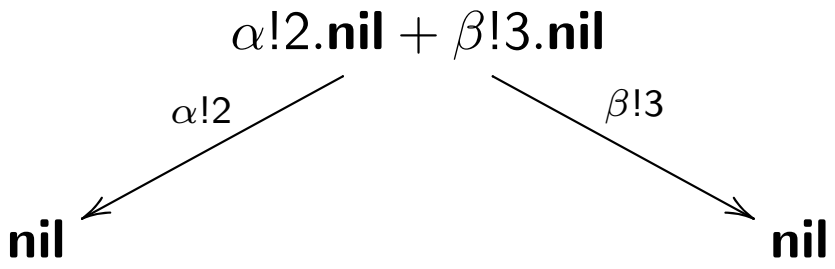
$$\dfrac{\dfrac{\dfrac{(\alpha!3 \to \mathbf{nil}) \xrightarrow{\alpha!3} \mathbf{nil}}{(\alpha!3 \to \mathbf{nil}) + P \xrightarrow{\alpha!3} \mathbf{nil}}}{((\alpha!3 \to \mathbf{nil}) + P) \parallel (\tau \to \mathbf{nil}) \xrightarrow{\alpha!3} \mathbf{nil} \parallel (\tau \to \mathbf{nil})} \qquad (\alpha?x \to \mathbf{nil}) \xrightarrow{\alpha?3} \mathbf{nil}}{(((\alpha!3 \to \mathbf{nil}) + P) \parallel (\tau \to \mathbf{nil})) \parallel (\alpha?x \to \mathbf{nil}) \xrightarrow{\tau} (\mathbf{nil} \parallel (\tau \to \mathbf{nil})) \parallel \mathbf{nil}}$$

$$\overline{(((\alpha!3 \to \mathbf{nil} + P) \parallel \tau \to \mathbf{nil}) \parallel \alpha?x \to \mathbf{nil}) \setminus \{\alpha\} \xrightarrow{\tau} ((\mathbf{nil} \parallel \tau \to \mathbf{nil}) \parallel \mathbf{nil}) \setminus \{\alpha\}}$$

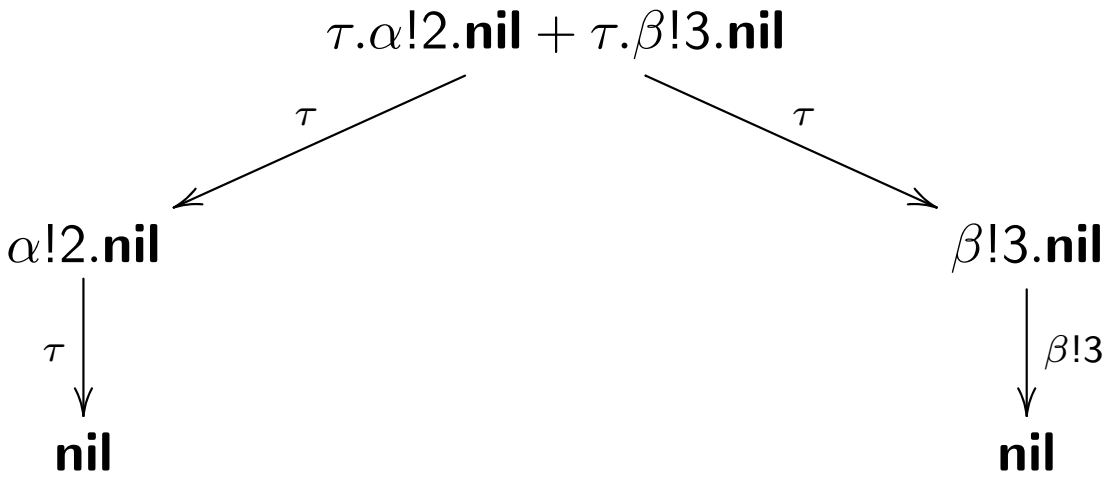
Further examples

(Write . for $\rightarrow$)
Each step justified by a derivation:

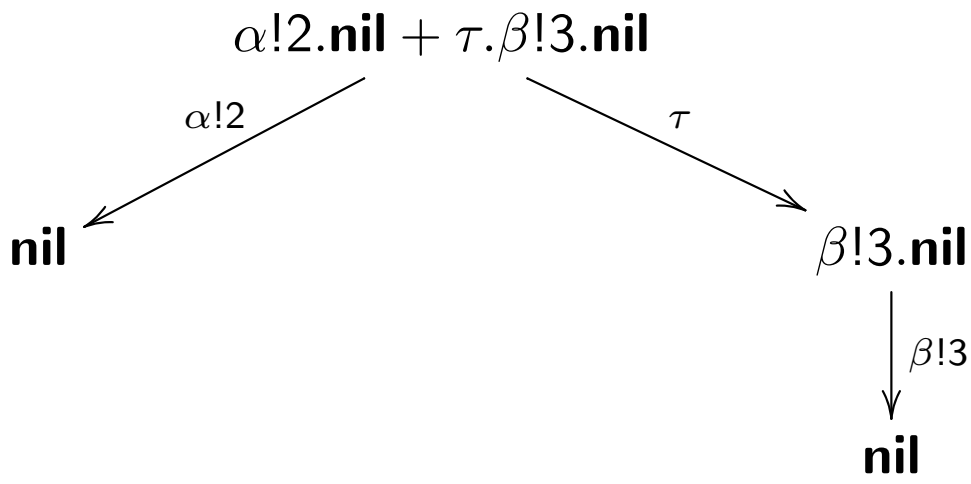
- External choice



- Internal choice



- Mixed choice



- Exercise:

$$\alpha!3.\mathbf{nil} \parallel \alpha?x.\beta!x.\mathbf{nil}$$

- Exercise:

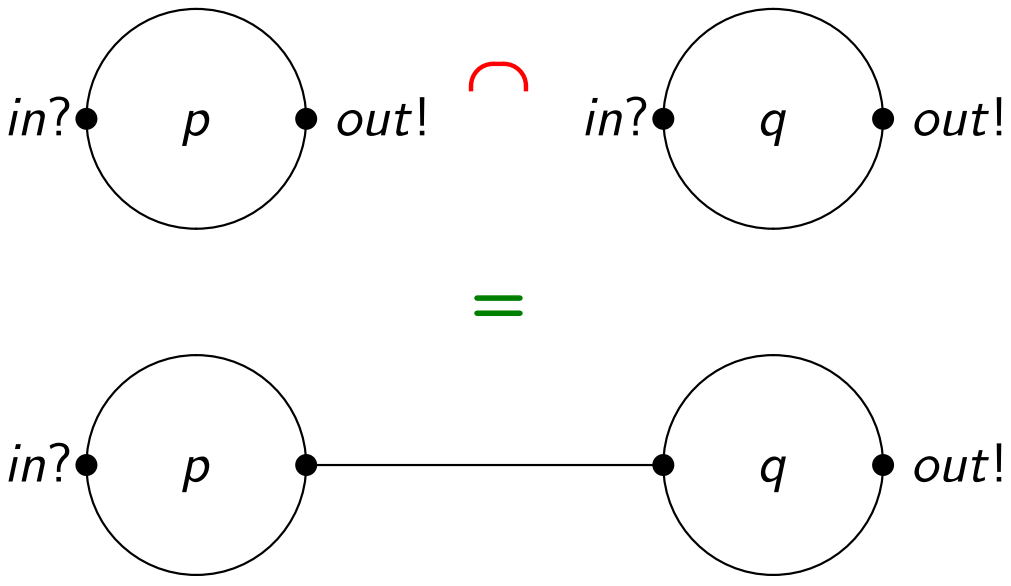
$$(\alpha!3.\mathbf{nil} \parallel \alpha?x.\beta!x.\mathbf{nil}) \setminus \{\alpha\}$$

- Exercise:

$$(\alpha?x.\mathbf{nil} \parallel \beta!4)[\alpha \mapsto \beta, \beta \mapsto \beta]$$

Linking processes

Connect p's output port to q's input port:



Definition:

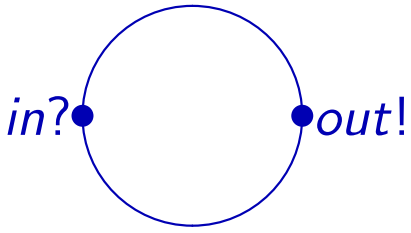
$$p \frown q = (p[c/out] \parallel q[c/in]) \setminus c$$

where c is a fresh channel name

Buffers

- **Definition:**

$$B \stackrel{\text{def}}{=} in?x \rightarrow (out!x \rightarrow B)$$



- n-ary buffer

$$\underbrace{B \frown B \frown \ldots \frown B}_{n \text{ times}}$$

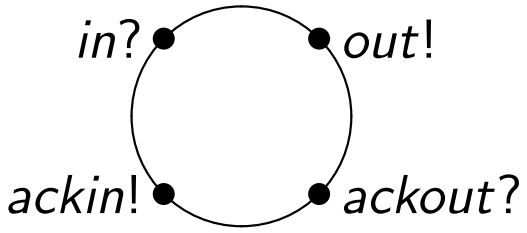
- Exercise: Draw the transition system for $B \frown B$

Remember: $p \frown q = (p[c/out] \parallel q[c/in]) \setminus c$

Buffer with acknowledgements

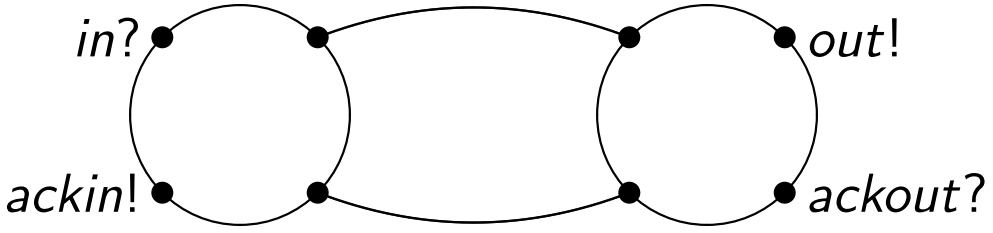
- **Definition:**

$$D \stackrel{\text{def}}{=} in?x \rightarrow out!x \rightarrow ackout? \rightarrow ackin! \rightarrow D$$



- Chaining now establishes two links:

$D \Cap D$

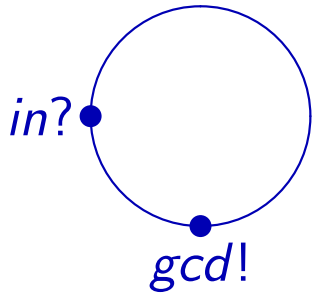


- How would this differ with the following process?

$$D' \stackrel{\text{def}}{=} in?x \rightarrow ackin! \rightarrow out!x \rightarrow ackout? \rightarrow D'$$

Euclid's algorithm in CCS

Interface:



Implementation:

$$
\begin{aligned}
E(x,y) \quad \stackrel{\text{def}}{=} \quad & x = y \to gcd!x \to \mathbf{nil} \\
+ \quad & x < y \to E(x, y-x) \\
+ \quad & y < x \to E(x-y, x)
\end{aligned}
$$

$$
Euclid \stackrel{\text{def}}{=} in?x \to in?y \to E(x,y)
$$

Euclid's algorithm in CCS (without parameterized processes)

$$
\begin{aligned}
\mathit{Step} \stackrel{\text{def}}{=}\ & \mathit{in}?x \rightarrow \\
& \mathit{in}?y \rightarrow \\
& \quad (x = y \rightarrow \mathit{gcd}!x \rightarrow \mathbf{nil}) \\
& + \\
& \quad (x < y \rightarrow \mathit{out}!x \rightarrow \mathit{out}!(y - x) \rightarrow \mathbf{nil}) \\
& + \\
& \quad (y < x \rightarrow \mathit{out}!(x - y) \rightarrow \mathit{out}!y \rightarrow \mathbf{nil})
\end{aligned}
$$

$$
\mathit{Euclid} \stackrel{\text{def}}{=} \mathit{Step}^{\frown}\, \mathit{Euclid}
$$