Event structures

of the simplest kind, “prime event structures with binary conflict,” as originally introduced

From nets ...
Unfolding a (safe) Petri net:
An event structure
Event structures

An event structure comprises \((E, \leq, \#)\), consisting of

- a set \(E\), of events
- partially ordered by \(\leq\), the causal dependency relation, and
- a binary, irreflexive, symmetric relation \(#\) on \(E\), the conflict relation,

which satisfy

\[
\{e' \mid e' \leq e\} \text{ is finite for all } e \in E,
\]

if \(e \geq e_0 \# e'_0 \leq e'\), then \(e \# e'\)

Say \(e, e'\) are concurrent if \(\neg(e \# e') \& e \not\leq e' \& e' \not\leq e\).
States of an event structure

The *configurations*, $\mathcal{C}(E)$, of an event structure $E$ consist of those subsets $x \subseteq E$ which are

*Consistent*: $\forall e, e' \in x. \lnot(e \neq e')$ and

*Down-closed*: $\forall e, e'. e' \leq e \in x \Rightarrow e' \in x$.

For an event $e$ the set $[e] = \text{def} \{ e' \in E \mid e' \leq e \}$ is a configuration describing the whole causal history of the event $e$.

$x \subseteq x'$, i.e. $x$ is a sub-configuration of $x'$, means that $x$ is a sub-history of $x'$.

$(\mathcal{C}(E), \subseteq)$ is a domain.
Example: Streams as event structures

--- conflict (inconsistency)  \[\rightarrow\] immediate causal dependency
Simple parallel composition

000 \sim 001 010 \sim 011 110 \sim 111

00 \sim 01 : \sim 11

0 \sim 1

aaa \sim aab aba \sim abb bba \sim bbb

aa \sim ab : \sim bb

a \sim b
Another example
Maps of event structures

A map of event structures $f : E \rightarrow E'$ is a partial function $f : E \rightarrow E'$ such that for all $x \in \mathcal{C}(E)$

$$fx \in \mathcal{C}(E') \text{ and } e_1, e_2 \in x \& f(e_1) = f(e_2) \Rightarrow e_1 = e_2.$$ 

Note that when $f$ is total it restricts to a bijection $x \cong fx$, for any $x \in \mathcal{C}(E)$.

Maps preserve concurrency, and locally reflect causal dependency:

$$e_1, e_2 \in x \& f(e_1) \leq f(e_2) \text{ (both defined)} \Rightarrow e_1 \leq e_2.$$ 

A total map is rigid when it preserves causal dependency.
CCS operations on event structures?

\( a \cdot b \cdot \text{nil} \parallel \overline{a} \cdot \text{nil} ? \)