Exercise 1: Find a short MATLAB expression to build the matrix

\[ B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 7 & 5 & 3 & 1 & -1 & -3 \\ 4 & 8 & 16 & 32 & 64 & 128 & 256 \end{pmatrix} \]

Answer: \( b = [1:7; 9:-2:-3; 2.^(2:8)] \)

Exercise 2: Give a MATLAB expression that uses only a single matrix multiplication with \( B \) to obtain

(a) the sum of columns 5 and 7 of \( B \)

Answer: \( b * [0 0 0 0 1 0 1]' \)

(b) the last row of \( B \)

Answer: \([0 0 1] * b\)

(c) a version of \( B \) with rows 2 and 3 swapped

Answer: \([1 0 0; 0 0 1; 0 1 0] * b\)

Exercise 3: Give a MATLAB expression that multiplies two vectors to obtain

(a) the matrix

\[ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \]

Answer: \([1 1 1]' * (1:5)\)

(b) the matrix

\[ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{pmatrix} \]

Answer: \((0:4)' * [1 1 1]\)
**Exercise 4:** Modify slide 30 to produce tones of falling frequency instead.

*Answer:* Replace

\[ f = \text{fmin} \times (\text{fmax}/\text{fmin}) \cdot l; \]

with

\[ f = \text{fmax} \times (\text{fmin}/\text{fmax}) \cdot l; \]

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**Exercise 5:**

(a) Write down the function \( g(t) \) that has the shape of a sine wave that increases linearly in frequency from 0 Hz at \( t = 0 \) s to 5 Hz at \( t = 10 \) s.

*Answer:* The instantaneous frequency of function \( g(t) \) at time \( t \) is

\[ f(t) = t \cdot \frac{5 \text{ Hz}}{10 \text{ s}} = \frac{t}{2 \text{ s}^2} \]

and since the phase of a sine wave is \( 2\pi \) times the integrated frequency so far, we get

\[ g(t) = \sin \left( 2\pi \int_0^t f(t') \, dt' \right) = \sin \left( 2\pi \frac{t^2}{4 \text{ s}^2} \right) = \sin \left( \pi \frac{t^2}{2 \text{ s}^2} \right) \]

(b) Plot the graph of this function using MATLAB’s `plot` command.

(c) Add to the same figure (this can be achieved using the `hold` command) in a different colour a graph of the same function sampled at 5 Hz, using the `stem` command.

*Answer:* for (b) and (c)
\( t = 0:0.01:10; \)
\( f = \sin(\pi t^2/2); \)
\( \text{plot}(t,f); \)
\( \text{hold}; \)
\( t2 = 0:1/5:10; \)
\( \text{stem}(t2, \sin(\pi t2^2/2), 'r'); \)

(d) [Extra credit] Plot the graph from (c) separately. Can you explain its symmetry? [Hints: sampling theorem, aliasing].

Answer: A sine wave with a frequency \( f \) larger than half the sampling frequency \( f_s \) cannot be distinguished based on the sample values from a sine wave of frequency \( f_s - f \). In other words, the sample values would have looked the same had we replaced the instantaneous frequency \( f(t) \) with \( f_s/2 - |f_s/2 - f(t)| \), and the latter is symmetric around \( f_s/2 \), which is in this graph 2.5 Hz and occurs at \( t = 5 \) s.

[The above is of course just a hand-waving argument, but shall be sufficient for this exercise. There are actually a few more conditions fulfilled here that lead to the exact symmetry of the plot. Firstly, since we started sampling at \( t = 0 \) s with \( f_s = 5 \) Hz, the positions of the sample values end up being symmetric around \( t = 5 \) s. Secondly, at the symmetry point \( t = 5 \) s, the sine wave was at a symmetric peak from where increasing or decreasing the phase has the same result.]

Exercise 6: Use MATLAB to write an audio waveform (8 kHz sampling frequency) that contains a sequence of nine tones with frequencies 659, 622, 659, 622, 659, 494, 587, 523, and 440 Hz. Append to this waveform a copy of itself in which every other sample has been multiplied by \(-1\). Play the waveform, write it to a WAV file, and use the \texttt{spectrogram} command to plot its spectrogram with correctly labelled time and frequency axis.

Answer:
\[ f = [659 \ 622 \ 659 \ 622 \ 659 \ 494 \ 587 \ 523 \ 440]; \]
fs = 8000; \% sampling frequency
d = 0.5; \% duration per tone
t = 0:1/fs:d-1/fs;
w = \sin(2 \times \pi \times f' \times t)/2;
w = w'; w = w(:)';
w = [w, w \times (\text{mod}((1:length(w)), 2) \times 2 - 1)];
audiowrite('matlab_answer-2.wav', w, fs);
spectrogram(w, 1024, [], [], fs, 'yaxis');