

# Probability and Computation: examinable material

## 1 Probability and Markov Chains

- Probability spaces & Random variables [Lecture 1]
- The Union Bound [Lecture 1]
- Max Cut Problem (random guessing algorithm) [Lecture 1]
- Balls into Bins [Lecture 1]
- The Markov and Chebychev Inequalities (First and Second moment methods) [Lecture 1]
- Markov Chain Basics: Transition matrices, Aperiodicity, Irreducibility [Lecture 2]
- Stationary distributions: Existence/Uniqueness & Convergence [Lecture 2/3]
- Total Variation/ $\ell^1$ -mixing time (i.e. mixing with respect to total variation) [Lecture 3/4]

## 2 Concentration and Martingales

- Chernoff Bounds [Lecture 5/6]
- Recipe for deriving Chernoff Bounds [Lecture 5/6, Q2 PS3, Lecture 15]
- Basic inequalities [e.g. Lecture 6 page 10, Q1 PS3]
- Conditional Expectation [Lecture 6, Q4,5,6 PS3]
- Definition of Martingale [Lecture 7, Q1 PS4]
- Lipschitz functions and applications of the Bounded Differences inequalities [Lecture 7]

## 3 Spectral techniques for Markov Chains and Algorithms

- Basics of linear algebra for Markov chains [Lecture 9]
- Reversible Markov chains: definitions, relation with random walks on weighted undirected graphs, stationary distribution [Lecture 9, Q4 PS4]
- $\ell_2$ -mixing [Lecture 10]
- Relation between mixing and eigenvalues of reversible Markov chains [Lecture 10]
- Graph clustering and conductance [Lecture 11]
- Cheeger's inequality and spectral partitioning algorithm [Lecture 11]
- Relation between conductance and mixing time of random walks [Lecture 11]
- Spectral clustering algorithm [Lecture 12]

## 4 Applications to Randomised Algorithms

- Sublinear Algorithms [Lecture 13/14]
- Dimensionality Reduction and Random Projection [Lecture 15]