Announcements

- Lab today (Feb 5):
  - **Compulsory presence** at 14:00
  - Brief tutorial on how to operate real Turtlebot.
  - Sign-up (3 students per Turtlebot).
- Turtlebot sharing:
  - 3 students per robot; each robot in a dedicated locker in Intel lab
  - Key to be stored at the **reception** - note weekend closure!
- **Don’t break the robots.**

Mobile Robot Systems

Lecture 6: Navigation & Path Planning

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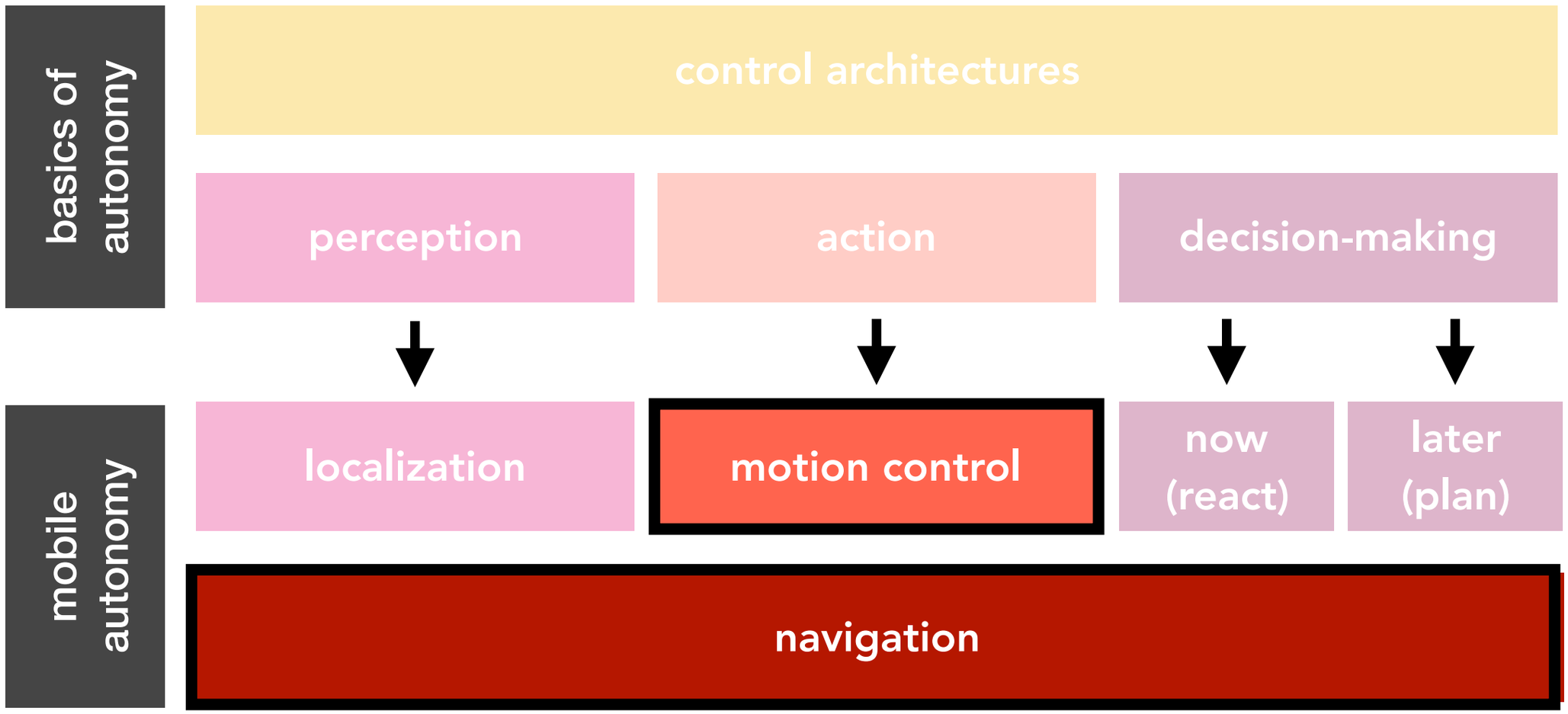
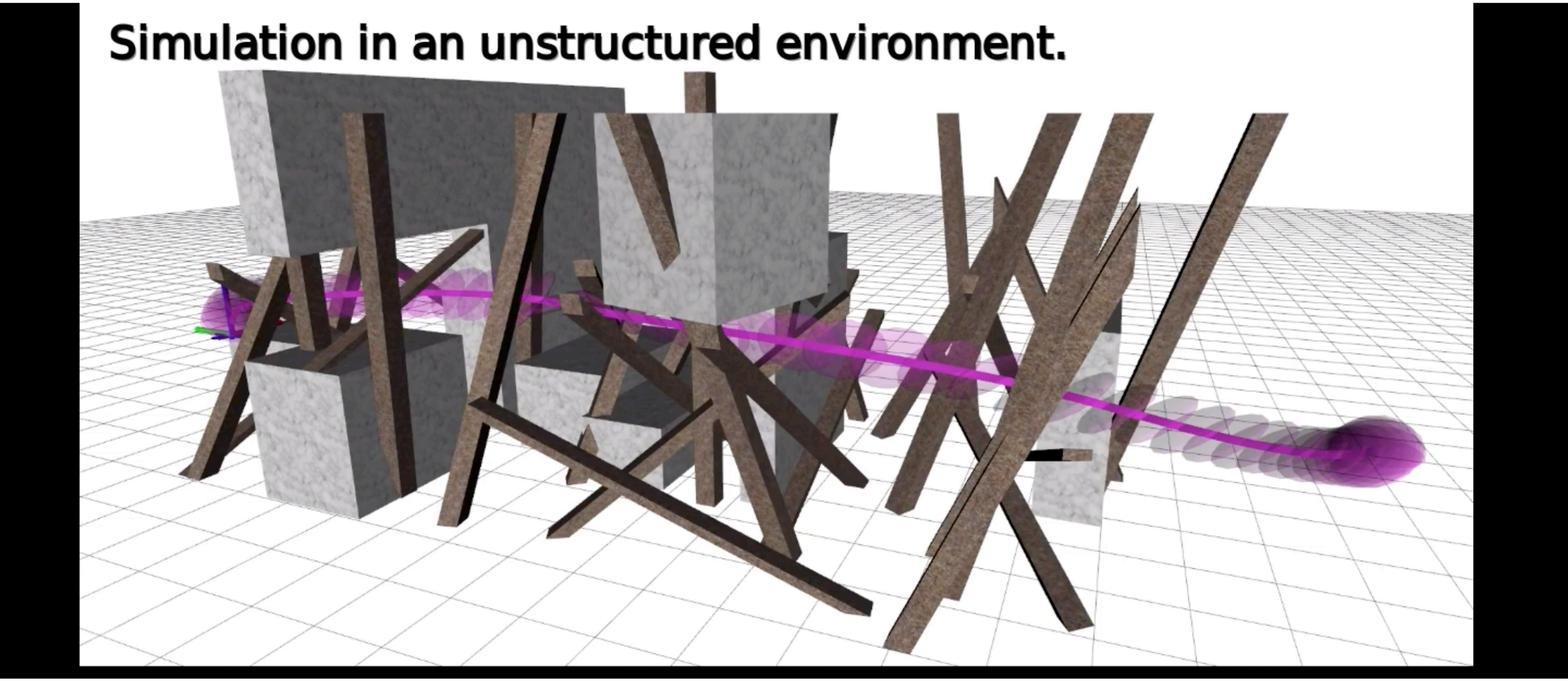
In this Lecture

- Navigation and path planning
- Configuration space
- 3 general method classes
  - Combinatorial
  - Sampling-based
  - Potential fields

Architecture of an Autonomous Mobile Robot



Motion Planning



Configuration Space

The ‘world’ has two entities: **robots and obstacles**. Both considered as closed subset of the world (or workspace): $\mathcal{O} \subset \mathcal{W}$, $\mathcal{R} \subset \mathcal{W}$

The ‘space’ for motion planning is the set of possible transformations that could be applied to the robot (considered as a rigid body).

We refer to this as the **configuration space**.

Important **abstraction** that allows to use the same motion planning algorithms to problems that differ in geometry and kinematics.

[Two common views of the configuration space:
metric space, or topological manifold]

Configuration Space

The robot is mapped to a **single point** in C-space $\mathscr{C}$

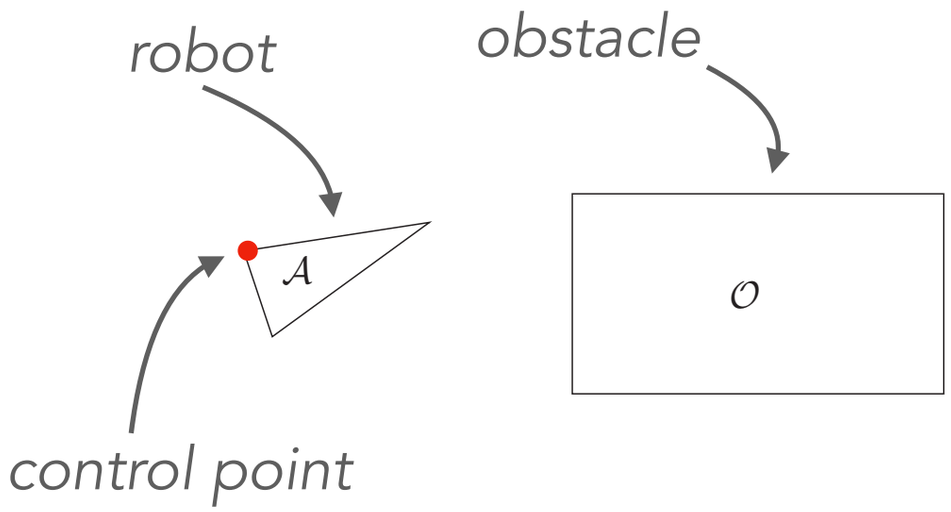
Complete specification of robot configuration: $\mathbf{q} \in \mathscr{C}$

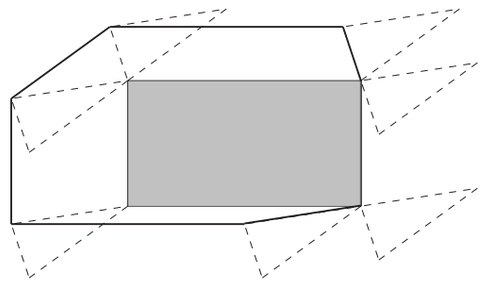
Robot's workspace: $\mathscr{W} = \mathbb{R}^N$ and points occupied by robot: $A(\mathbf{q}) \subset \mathscr{W}$

Obstacle region: $\mathcal{O} \subset \mathscr{W}$

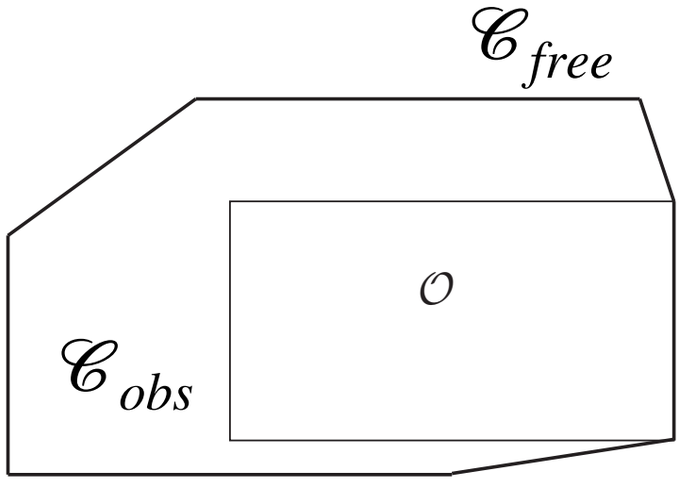
Set of configurations that avoid collision: $\mathscr{C}_{free} = \mathscr{C} \backslash \mathscr{C}_{obs}$

where we have: $\mathscr{C}_{obs} = \{\mathbf{q} \in \mathscr{C} \,|\, A(\mathbf{q}) \cap \mathcal{O} \neq \emptyset\}$



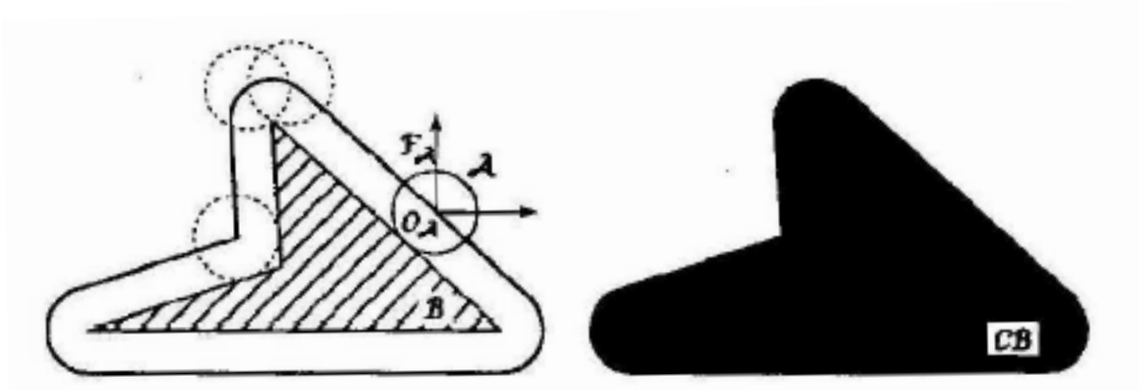


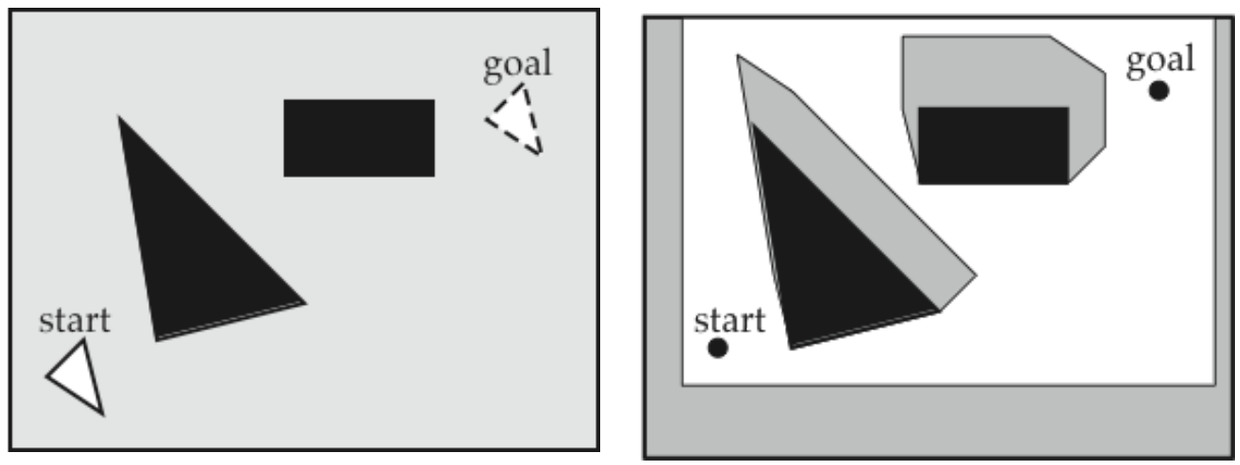
planar translation



Configuration Space

How to compute $\mathcal{C}_{obs}$ and $\mathcal{C}_{free}$?



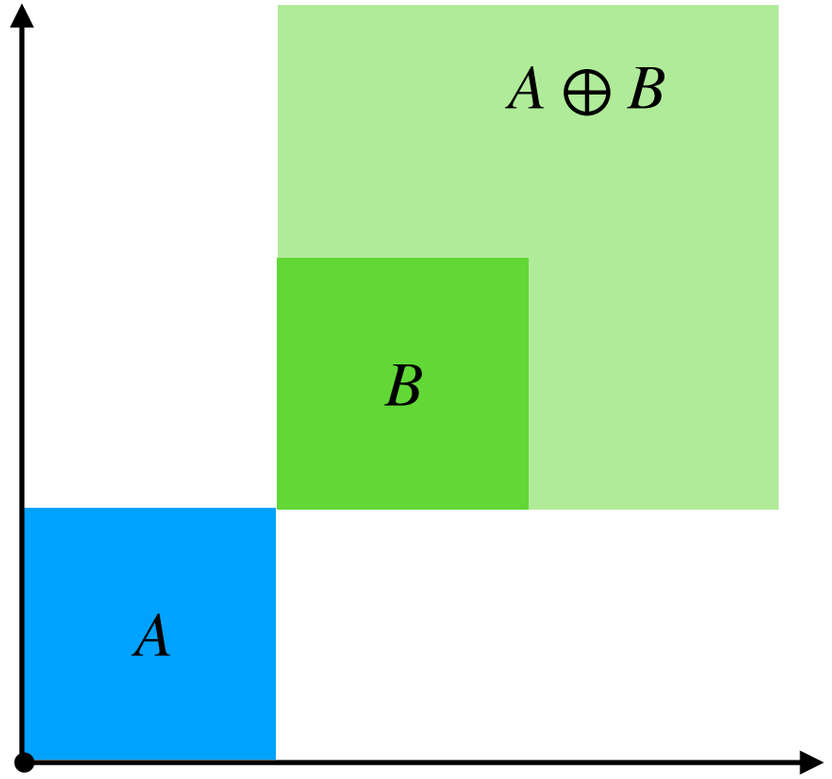


Various methods, e.g. reflect points, Minkowski sum, convex Hull.

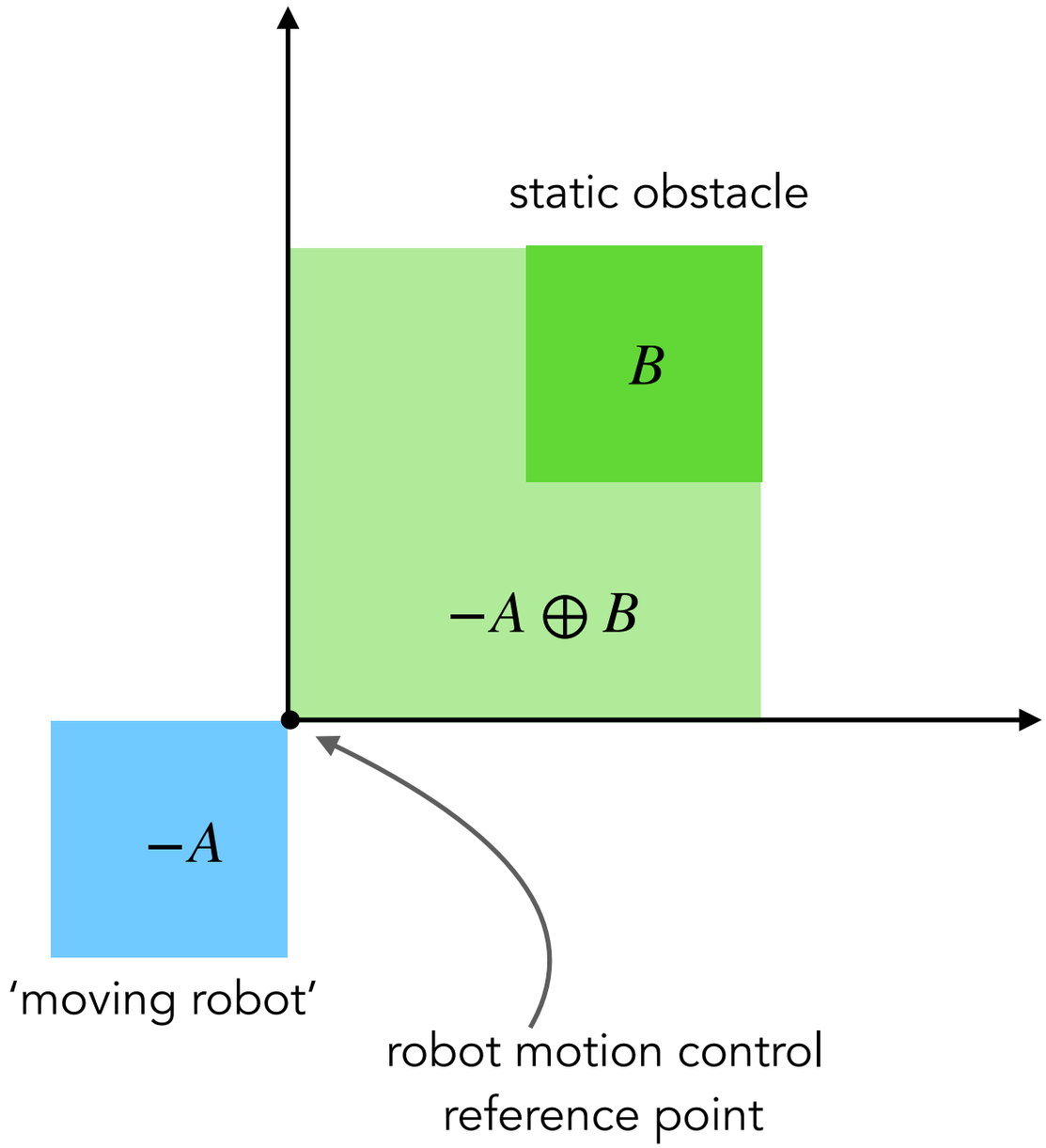
Minkowski Sum

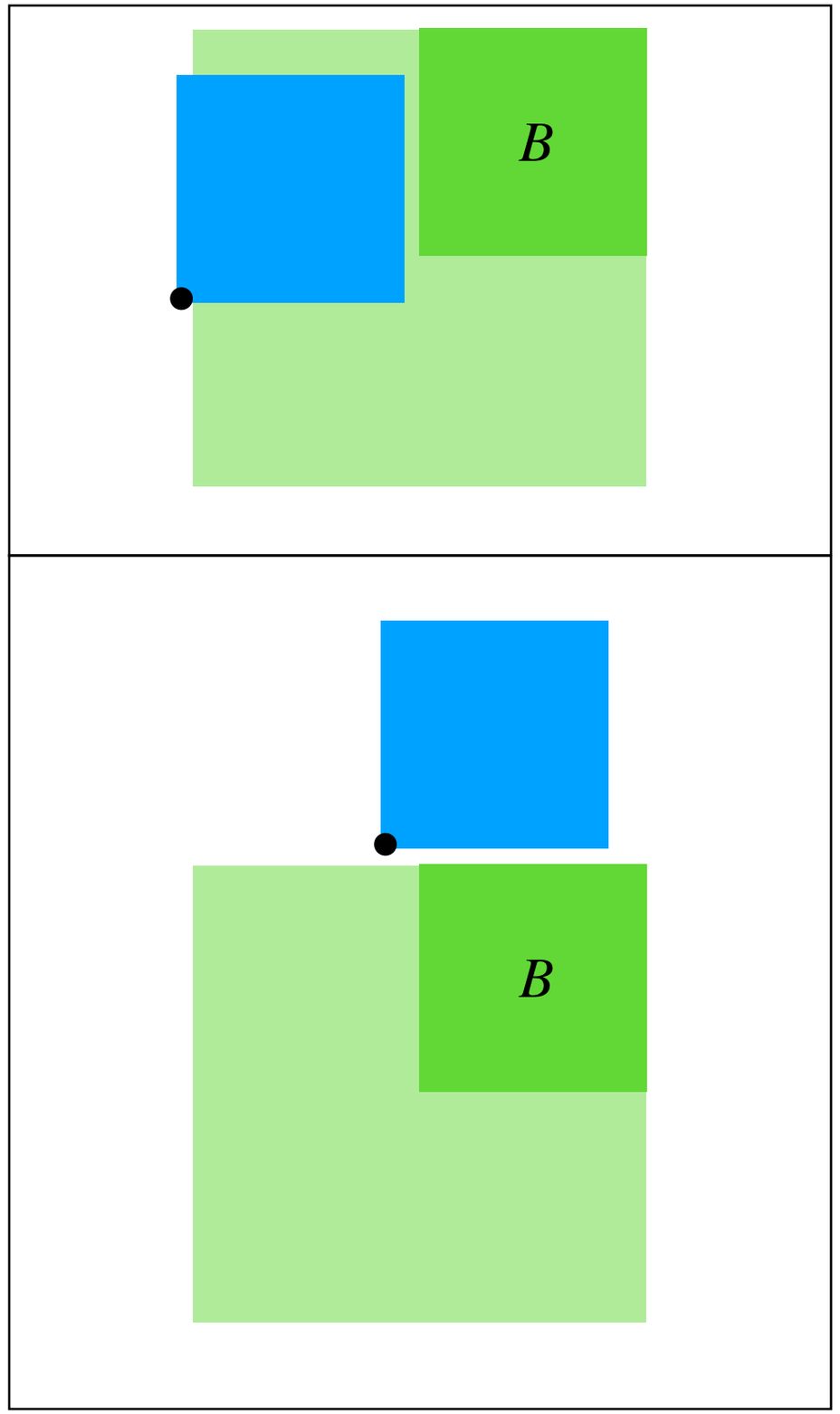
- In geometry, the Minkowski sum (also known as dilation) of two sets of position vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ in Euclidean space is formed by adding each vector in $\boldsymbol{A}$ to each vector in $\boldsymbol{B}$, i.e., the set:

$$A \oplus B = \{\mathbf{a} + \mathbf{b} \,|\, \mathbf{a} \in A, \mathbf{b} \in B\}$$



Minkowski Sum





As long as reference point stays outside dilated area, there will be no collisions.

The Path Planning Problem

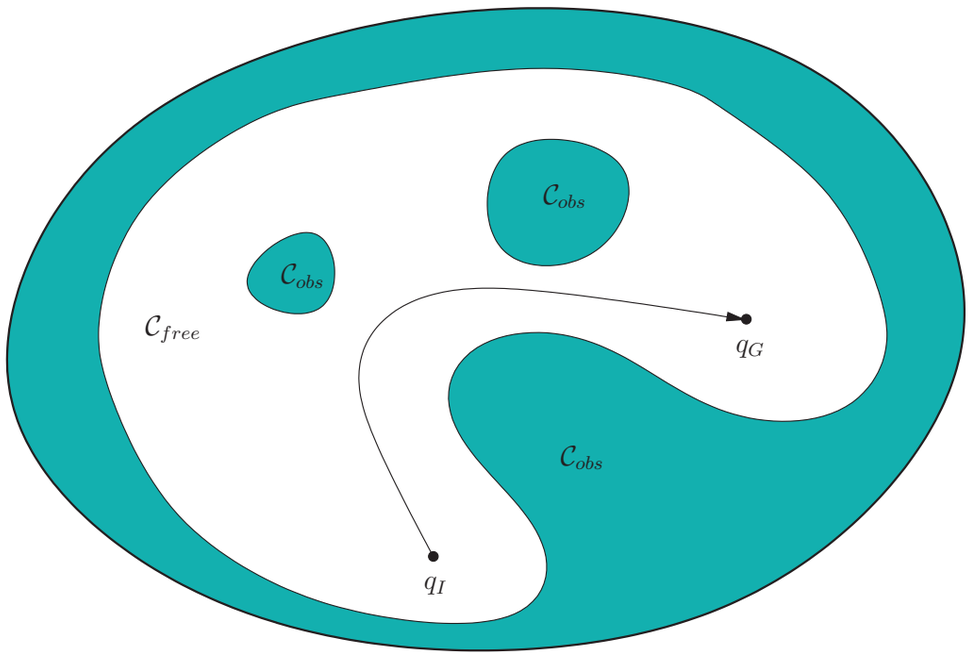
Assume a workspace, obstacle region, and configuration space, with definitions of free and occupied C-space $\mathscr{C}_{free}$ and $\mathscr{C}_{obs}$

Given a *query* $(\mathbf{q}_I, \mathbf{q}_G)$ with

an initial configuration $\mathbf{q}_I \in \mathscr{C}_{free}$

a desired goal configuration $\mathbf{q}_G \in \mathscr{C}_{free}$

compute a path $\tau : [0,1] \mapsto \mathscr{C}_{free}$ such that $\tau(0) = \mathbf{q}_I$ and $\tau(1) = \mathbf{q}_G$



* image credit: Pallotino

Approaches and Guarantees

1. **Combinatorial** planning (exact)
2. **Sampling-based** planning (probabilistic)
3. **Potential-field** methods

These methods use the configuration space abstraction (hence, require preliminary computation of C_{free} and C_{obs})

Guarantees:

- **Complete**: if a solution exists, it finds one, otherwise returns failure
- **Semi-complete**: if a solution exists, if finds one; otherwise may run forever
- **Resolution complete**: If a solution exists, it finds one; otherwise, it terminates and reports that no solution within a specified resolution exists.
- **Probabilistically complete**: If a solution exists, the probability that it will be found tends to one as the number of iterations tends to infinity

Overview of Methods

- **Combinatorial methods**
  - Exact and *complete*
  - Underlying principle: Requires a-priori computation of C-space and generation of a roadmap (e.g., a visibility graph). Then, use a discrete graph-search method on the roadmap.
- **Sampling-based methods**
  - Approximate and not complete; but can be *resolution complete*, or *probabilistically complete*
  - Underlying principle: Avoid explicit construction of roadmaps in C-space; instead, sample a path segment and check for collisions
- **Potential-field methods**
  - Approximate and not complete (suffer from local minima)
  - Underlying principle: Integrate over a vector field to obtain a trajectory (pose as a function of time). Can also be done in reactive mode.

Combinatorial Methods

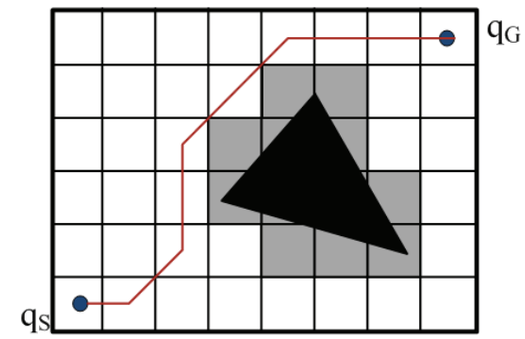
- Recipe:

  1. Compute **C-space** (C_{free} and C_{obs})

  2. Generate a **roadmap** (i.e., a **graph**) in C_{free}

     - Cell decomposition methods: visibility graphs / Voronoi cells / occupancy grid maps
     - Or: maximum clearance roadmaps (direct construction)

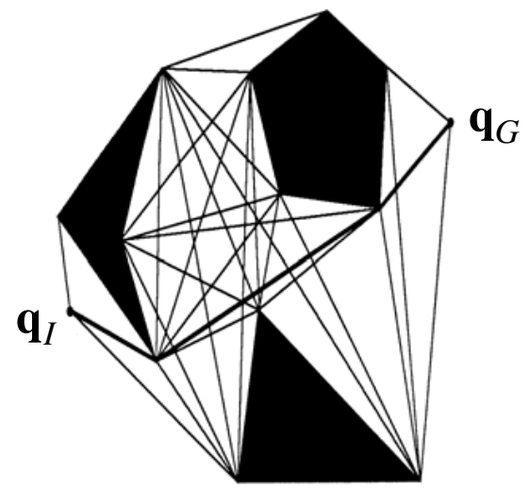
*A valid **roadmap** guarantees accessibility and is connectivity preserving w.r.t. C_{free}*



3. Compute the **minimum-cost path** from initial to goal configuration (cast as a graph search algorithm)

4. Result:

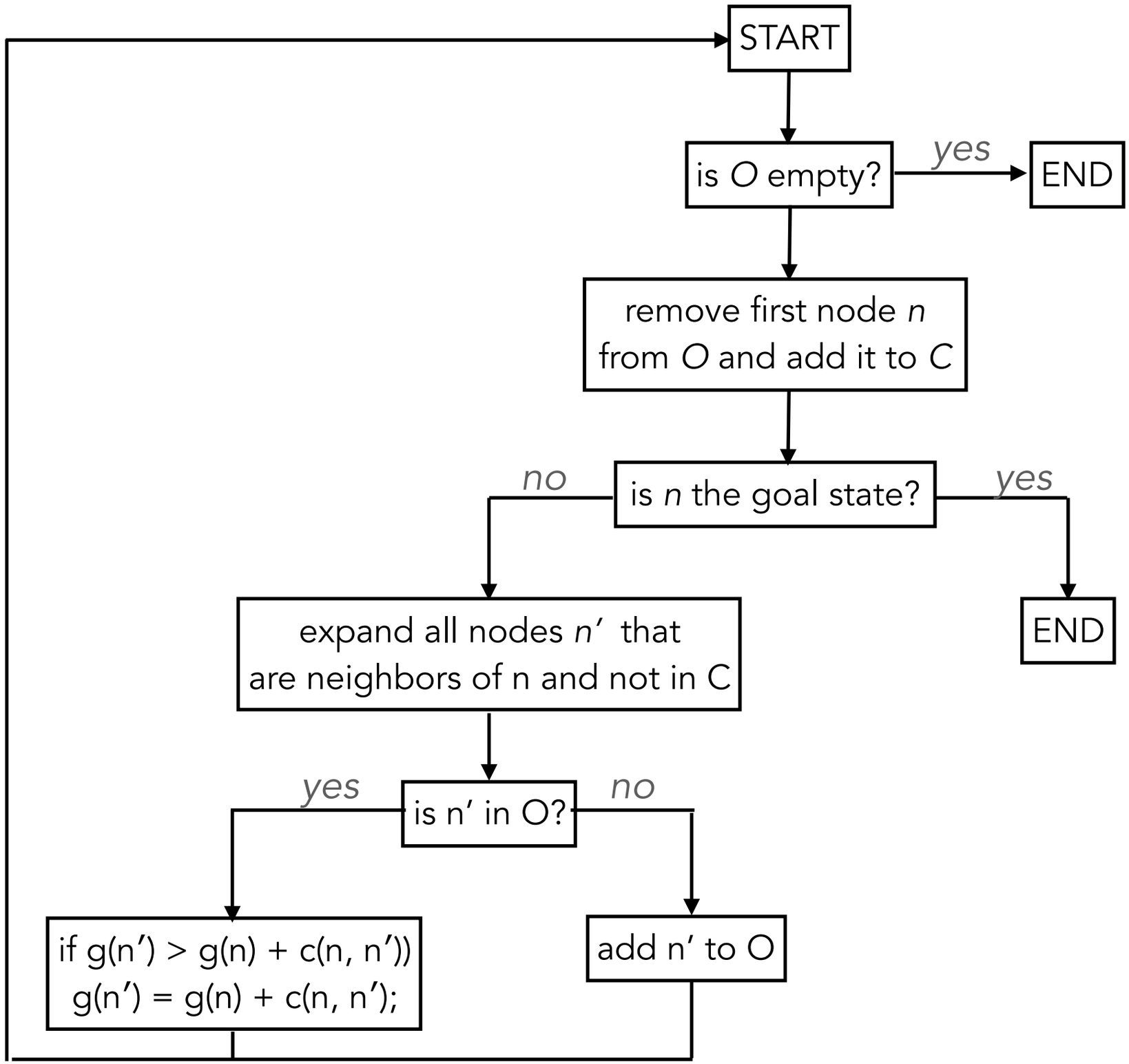
$\tau : [0,1] \mapsto \mathscr{C}_{free}$ such that $\tau(0) = \mathbf{q}_I$ and $\tau(1) = \mathbf{q}_G$



Discrete Search Methods

- How to search roadmap for minimum-cost path?
- One well-known example: **A*** algorithm
- Extension of Dijkstra' search algorithm, to reduce number of states explored (exploiting an informed search using a heuristic)
- Forward search, applied to path planning:
  - Evaluation function: $f(n) = g(n) + h(n)$
  - Operating cost function $g(n)$; cost of path already traversed.
  - Heuristic function $h(n)$; information used to find promising nodes to traverse; heuristic must be **admissible** (i.e., must **underestimate** true cost)!

A*



O: ordered set / open list (priority queue)
C: closed set

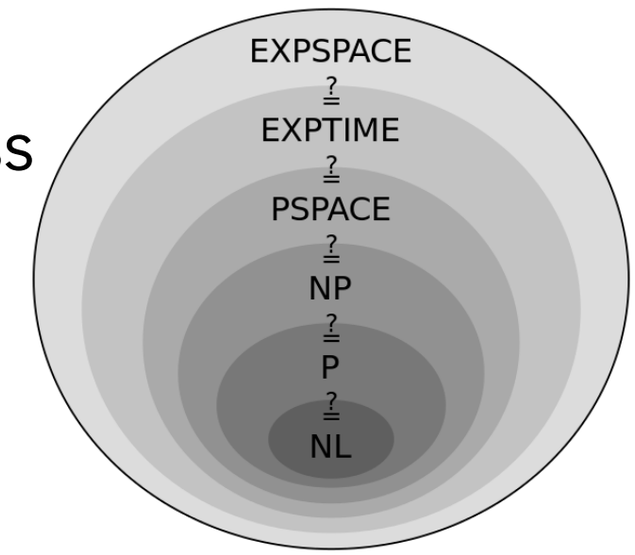
A*

- Requirements
  - Preprocessing to generate roadmap (connected graph) that represents C_{free}
- Pros
  - Optimal path cost and complete
- Cons
  - Memory inefficient (see IDA*)
  - Curse of dimensionality (exponential growth of search space w.r.t. length of solution)

Variants:
replanning algorithms (e.g. D)*
anytime algorithms (e.g. ARA),*
anytime re-planning (e.g. AD)*

Complexity of Path Planning

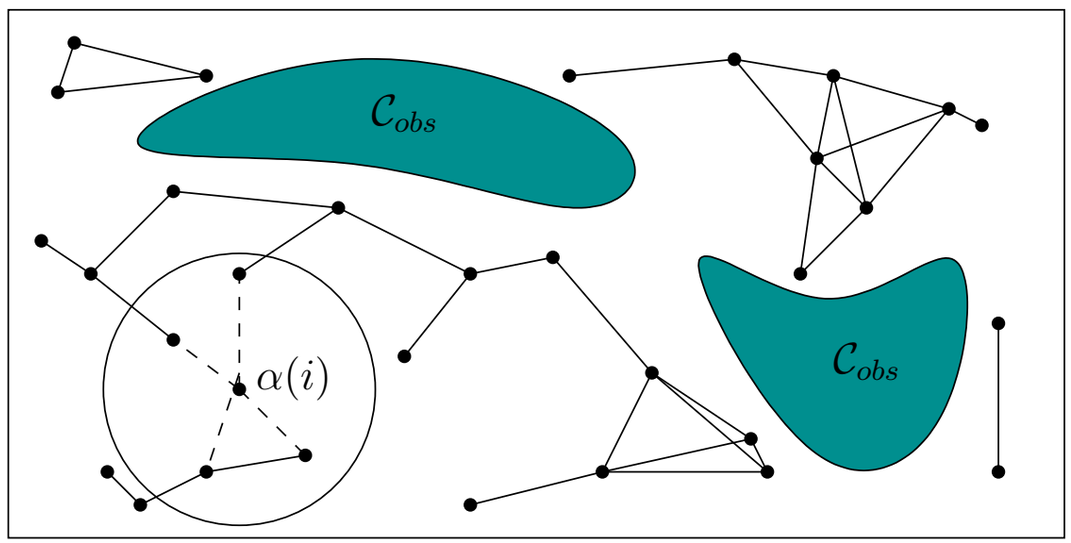
- The general motion planning problem is **PSPACE-hard**
  [Reif 1979; Hopcroft et al. 1984]
- NP ⊆ PSPACE (polynomial amount of memory)
- Challenges:
  - C-space has high dimensionality: E.g.: A rigid body in 3D space has a C-space with 6 dimensions ($\mathscr{C} = \mathbb{R}^3 \times SO(3)$)
  - Simple obstacles have complex C-obstacles; impractical to compute explicit representation of free space for high DOF robots
- Attention has turned to approximation and **randomized** algorithms which trade full completeness of the planner for a major gain in efficiency
  → sampling-based approaches



Sampling-Based Methods

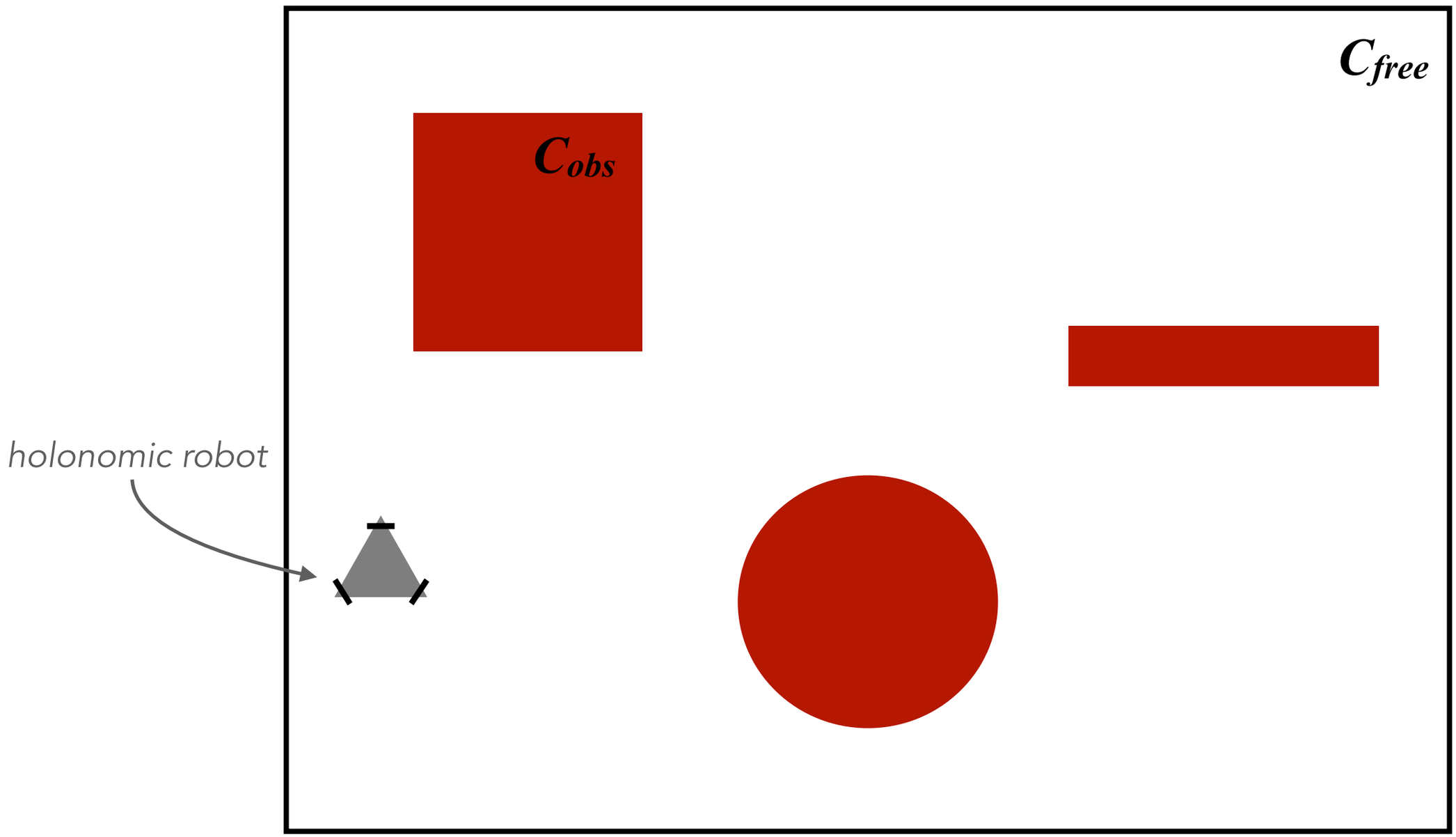
- **Probabilistic Roadmaps** (PRMs) (Kavraki, LaValle, et al.)

1. Initialize an empty graph G.
   Vertices will correspond to configurations, edges to collision-free paths.
2. **Sample** configurations $a(i)$ in C_{free}
3. Use a metric defined in C-space to compute **neighborhood** set of $a(i)$, of vertices $\boldsymbol{q}$ already in G.
4. **Local planner**: check if $a(i)$ can be connected to points $\boldsymbol{q}$ in neighborhood set. Add to edge $(a(i), \boldsymbol{q})$ to edge set if no collision detected.
5. Terminate when N edges added to roadmap.



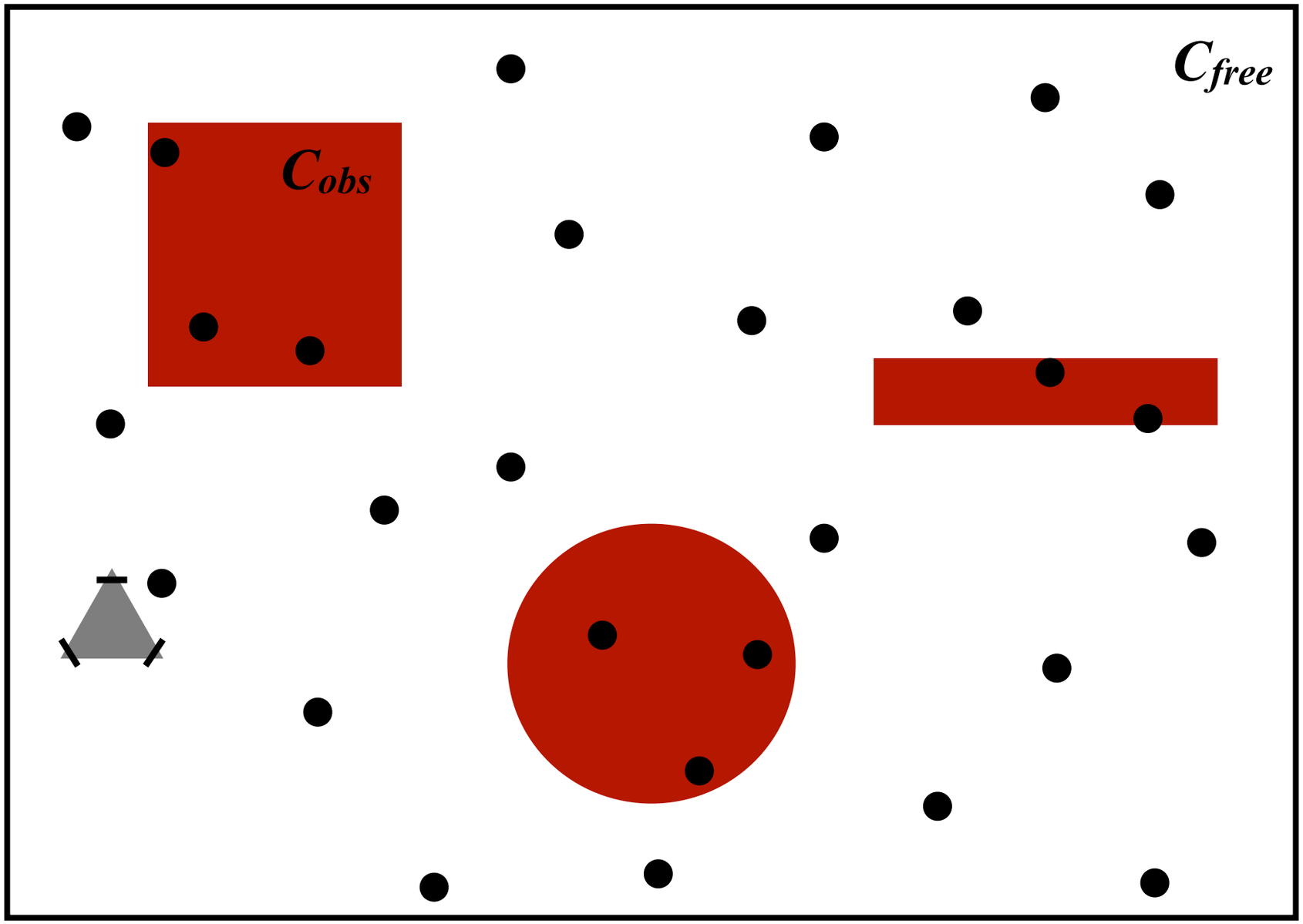
* image credit: Kavraki, 2008

Probabilistic Roadmaps (PRMs)



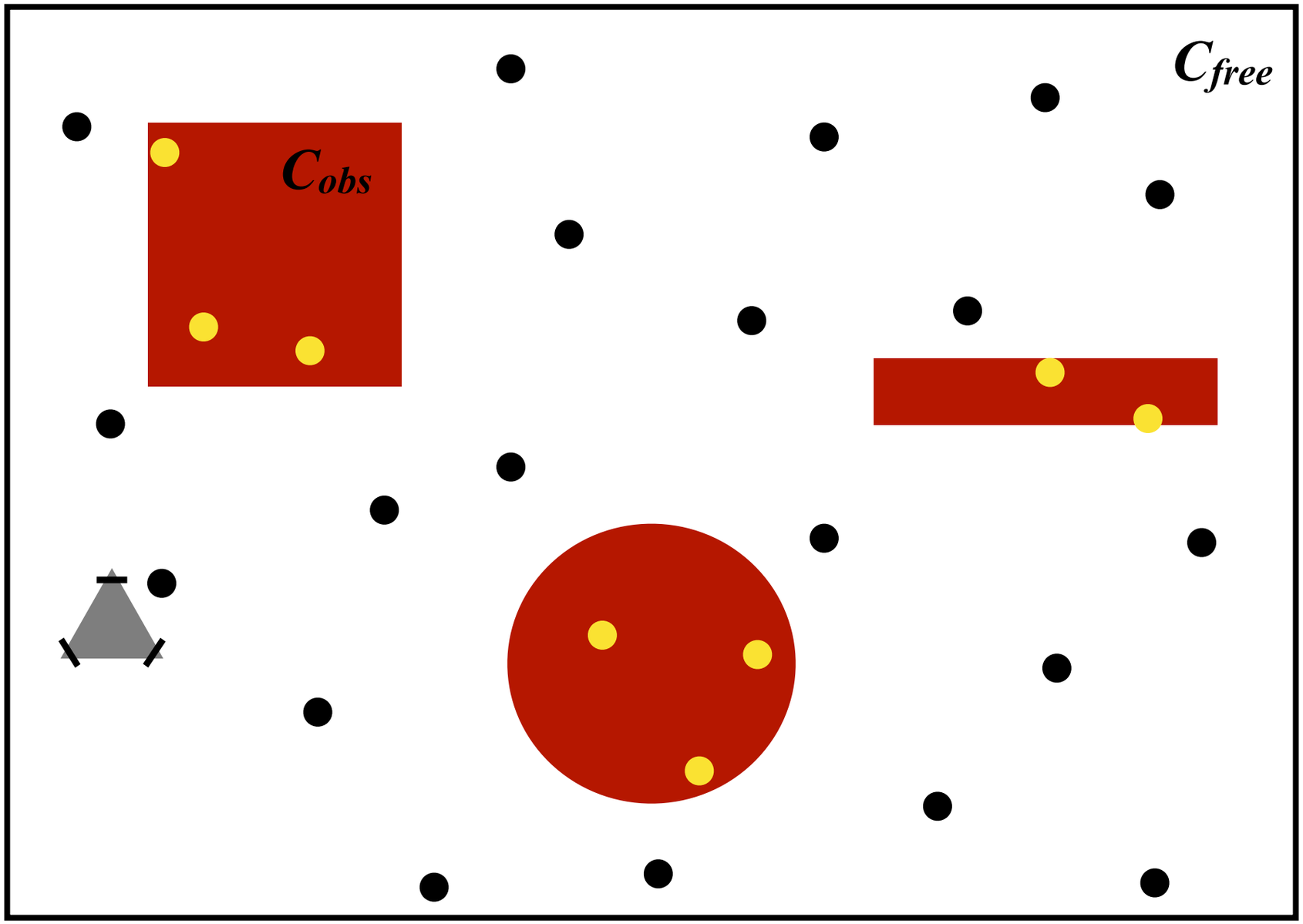
initialize empty graph and C-space

Probabilistic Roadmaps (PRMs)



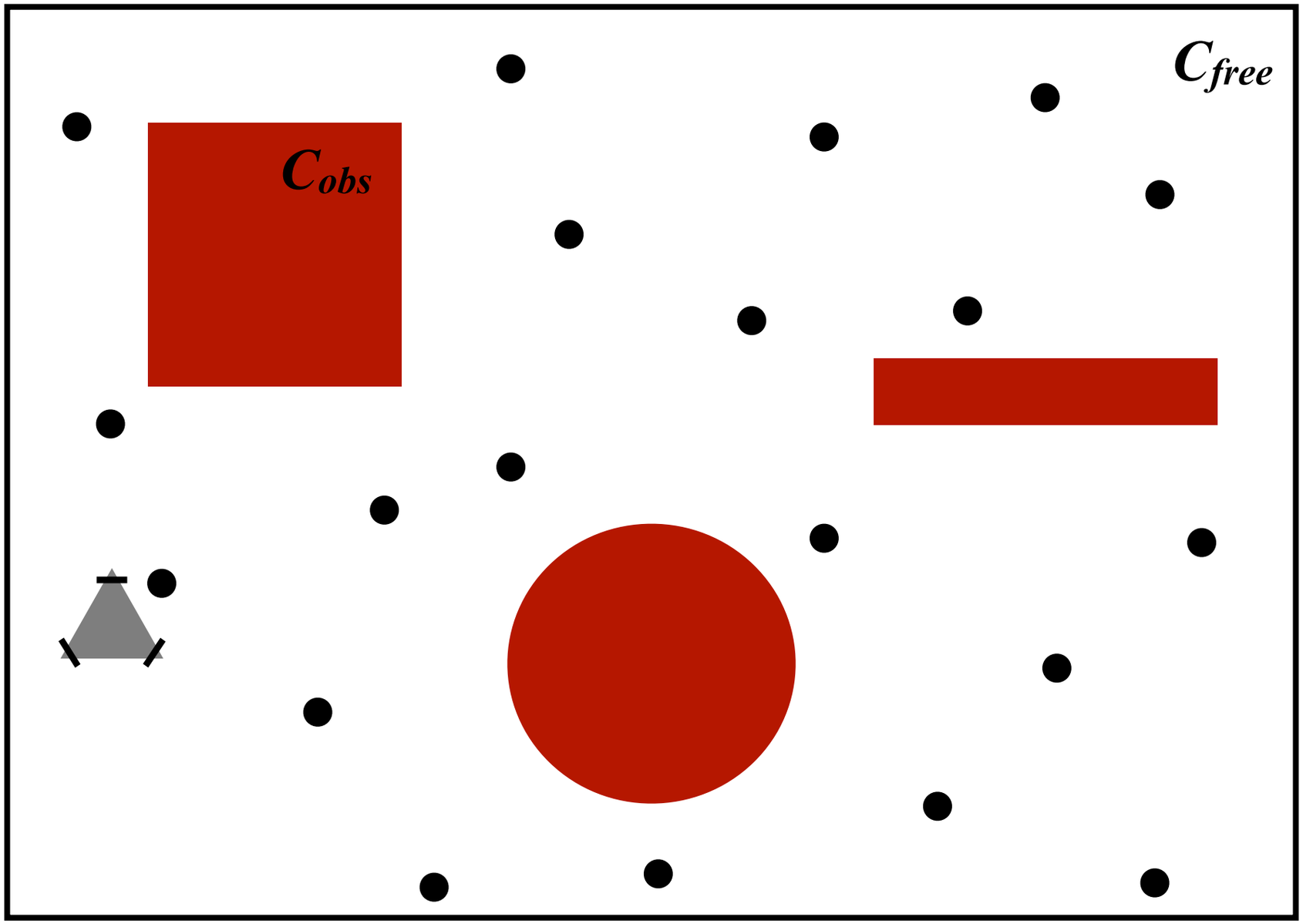
sample random configurations in C-space

Probabilistic Roadmaps (PRMs)



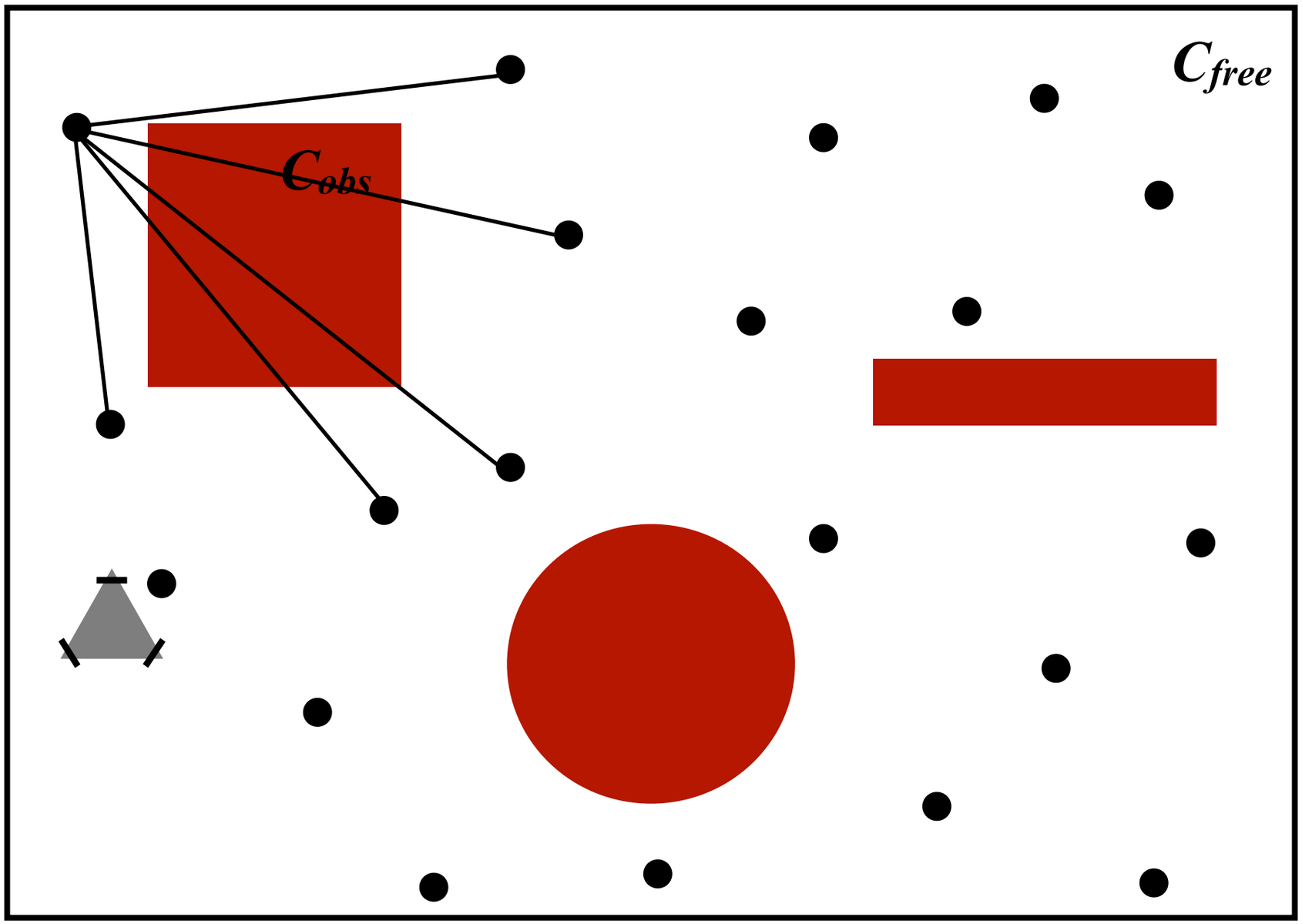
test for collisions

Probabilistic Roadmaps (PRMs)



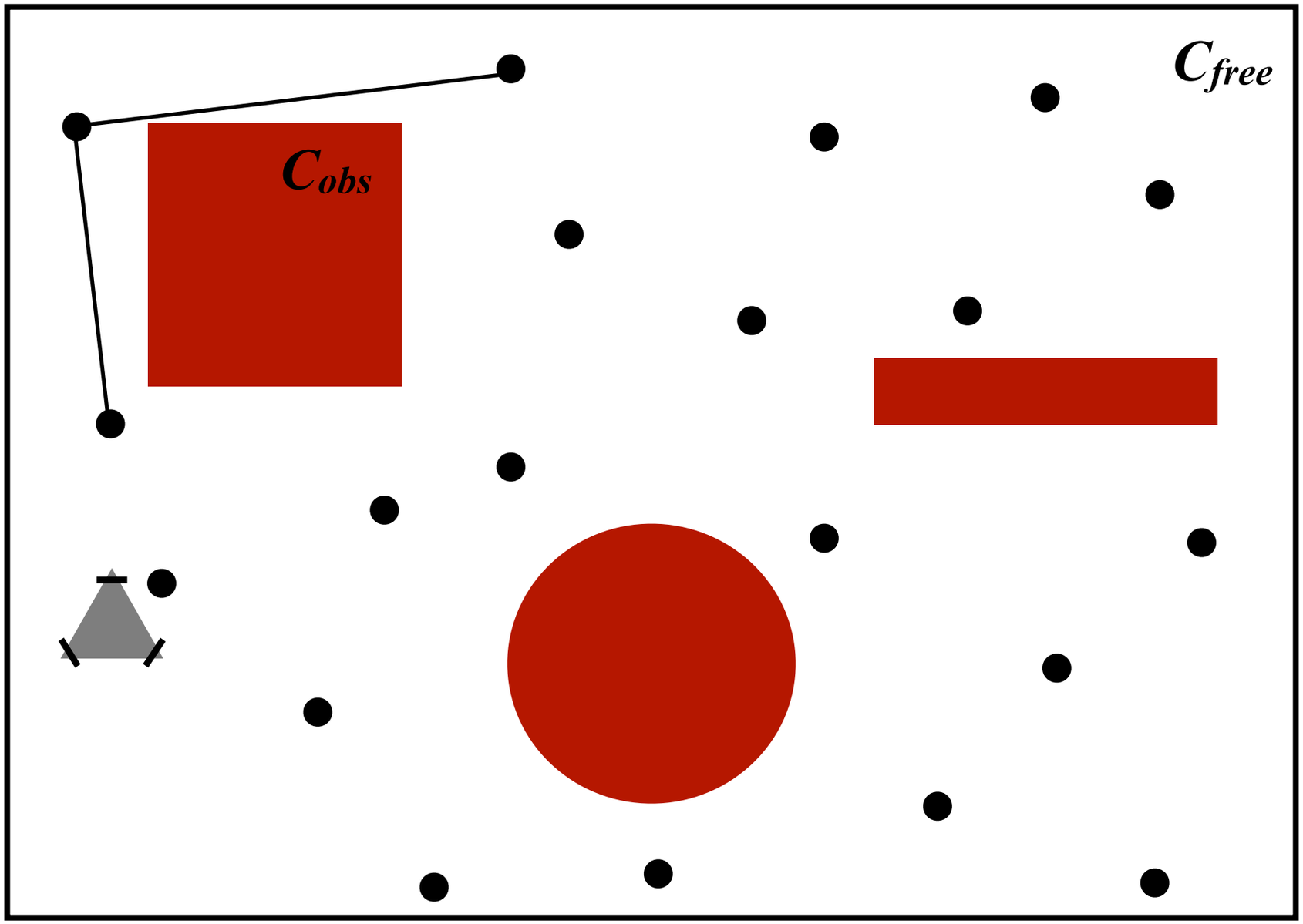
retain collision-free configurations

Probabilistic Roadmaps (PRMs)



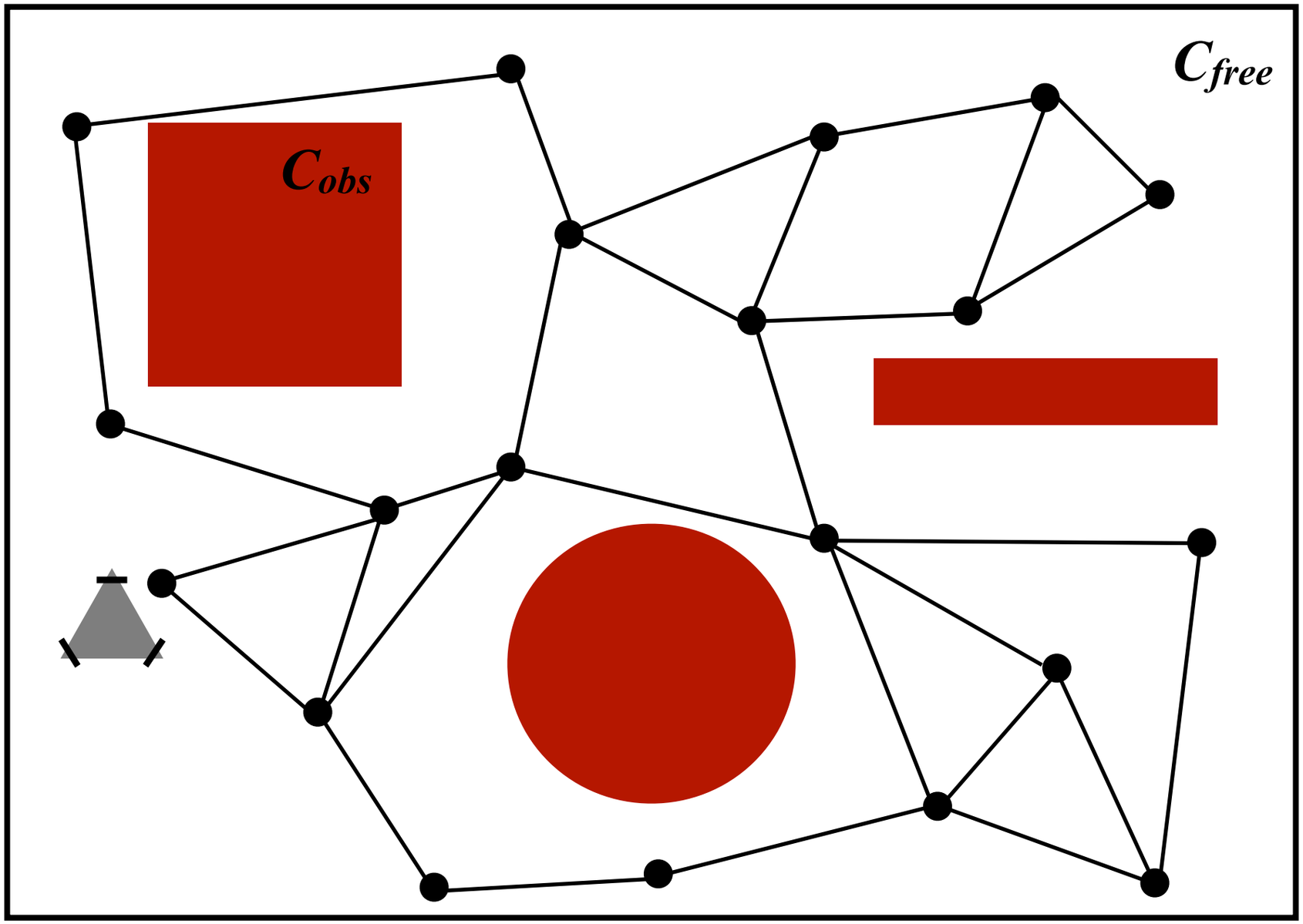
create edges to nearest neighbors

Probabilistic Roadmaps (PRMs)



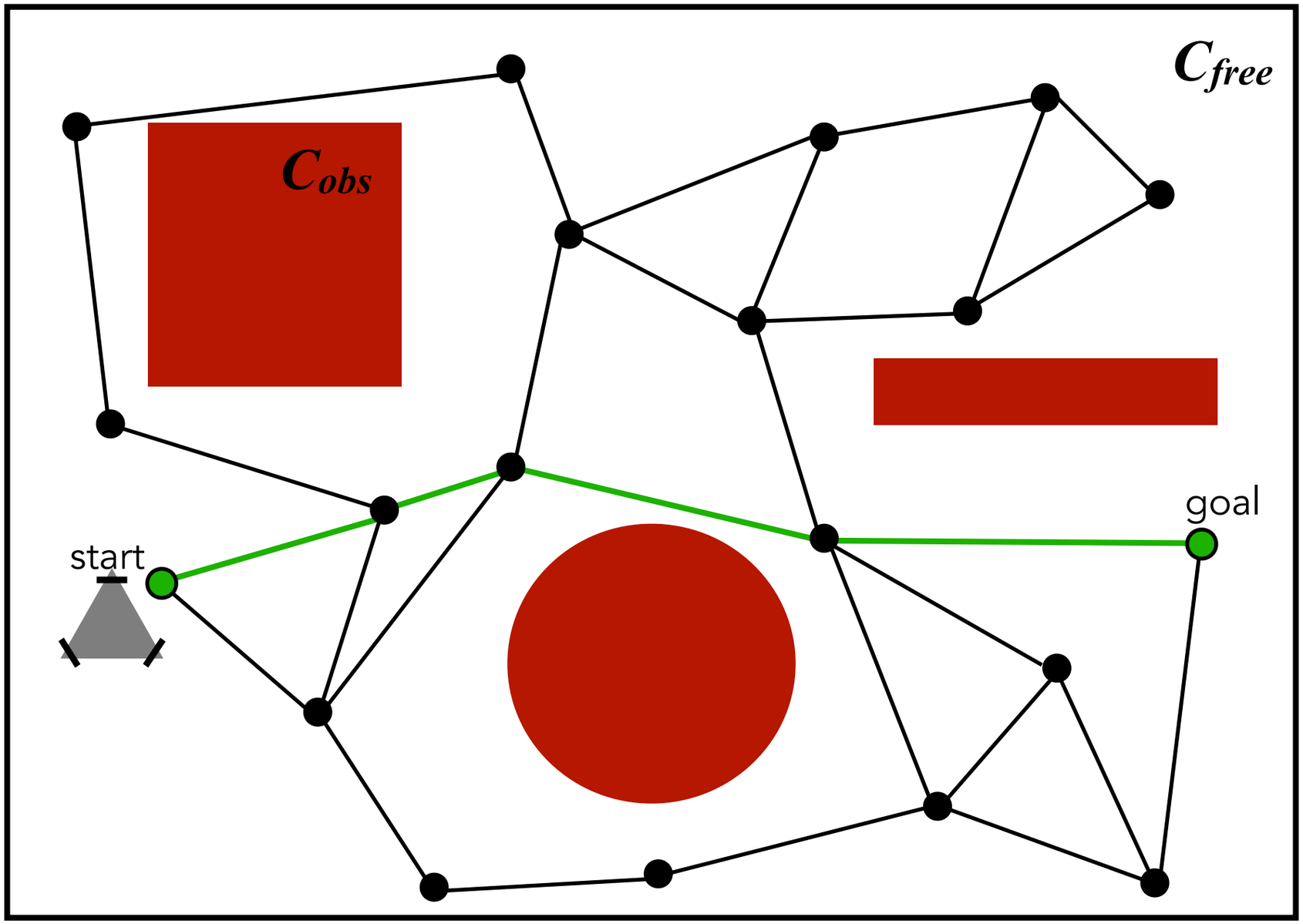
create edges to nearest neighbors

Probabilistic Roadmaps (PRMs)



retain collision-free local paths to generate PRM

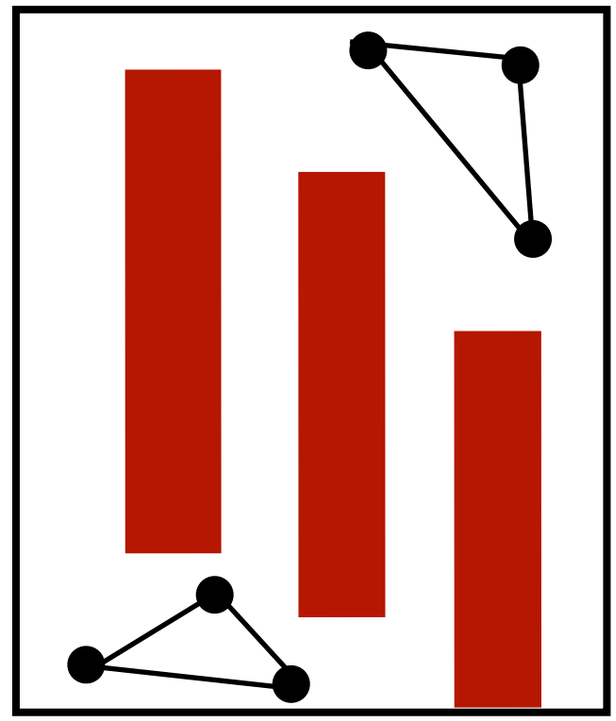
Probabilistic Roadmaps (PRMs)



search PRM for shortest path to goal configuration

Probabilistic Roadmaps (PRMs)

- Primitives required:
  - Method to sample configurations in free C-space
  - Method to check for collisions in C-space
- Pros:
  - Probabilistically complete
  - Apply easily to high-dimensional C-spaces
  - Fast query processing (start, goal)
- Cons:
  - Don’t work well for narrow passages
  - Hard to connect vertices for differential motion constraints (generally requires solution to the Boundary Value Problem)
  - Hard to sample uniformly in configuration space.



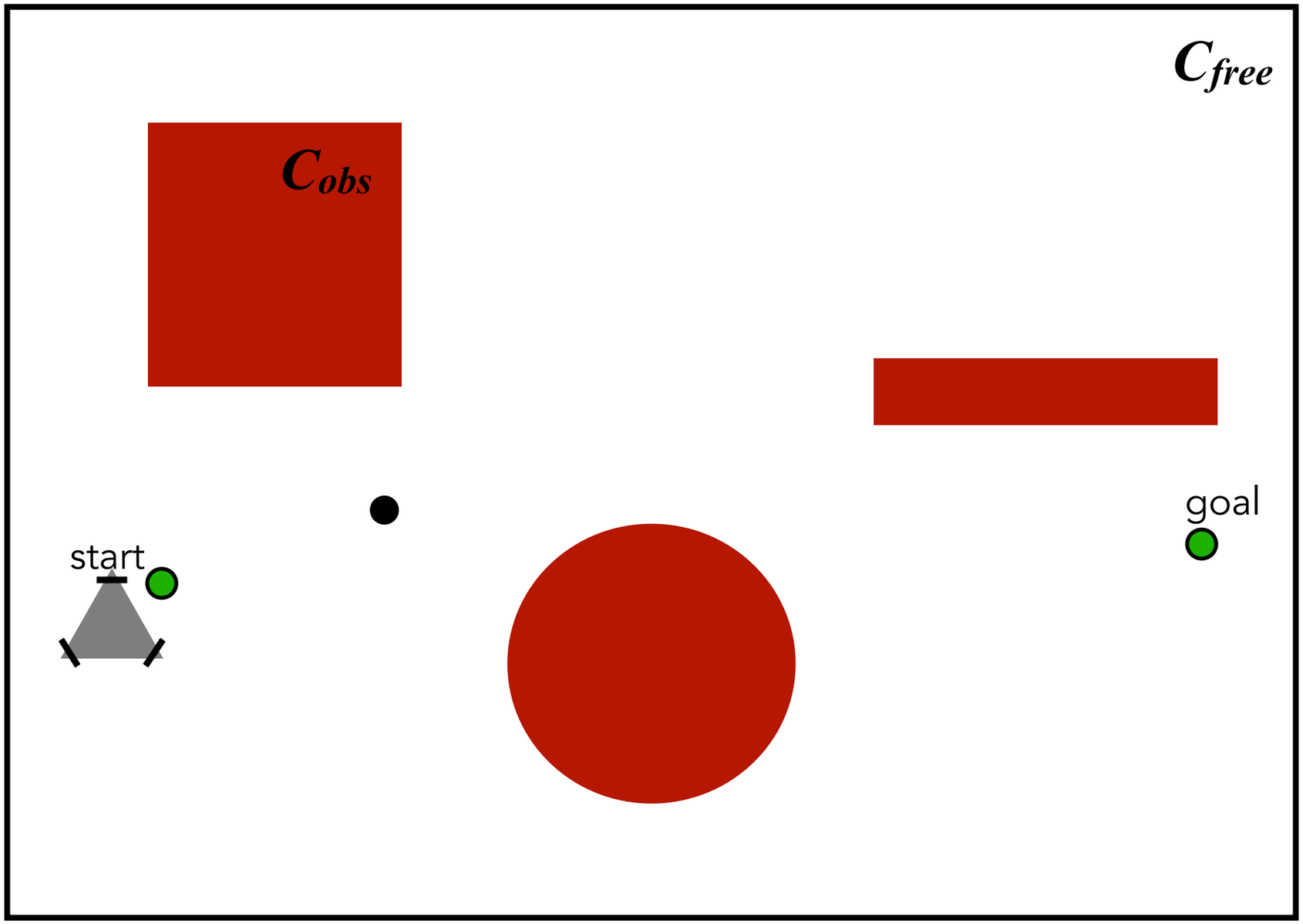
* image credit: Kavraki, 2008

Rapidly Exploring Random Trees (RRTs)

- LaValle and Kuffner, 2000.
- Similar to PRMs, but for single-query problems, a tree is enough.
- Basic idea: Build a tree by generating 'next states' in the tree, by executing random control

1. Initialize a graph G with a vertex corresponding to $\boldsymbol{q_I}$.
   Vertices correspond to configurations, edges to collision-free paths.
2. Sample a random state in C-space $\boldsymbol{q_{rand}}$ (with small probability, sample $\boldsymbol{q_G}$)
3. Find its nearest vertex $\boldsymbol{q_{near}}$ in G.
4. Local planner: For non-holonomic robots, find a new vertex $\boldsymbol{q_{new}}$ in C_{free} close to $\boldsymbol{q_{rand}}$ and check that edge $(\boldsymbol{q_{near}}, \boldsymbol{q_{new}})$ is collision-free. If so, add new edge and new vertex to G *(and discard* $\boldsymbol{q_{rand}}$*)*.
5. Check if G encodes a solution (path from $\boldsymbol{q_I}$ to $\boldsymbol{q_G}$), return G.
6. Return to step 2 or terminate with failure at time-out.

Rapidly Exploring Random Trees (RRTs)



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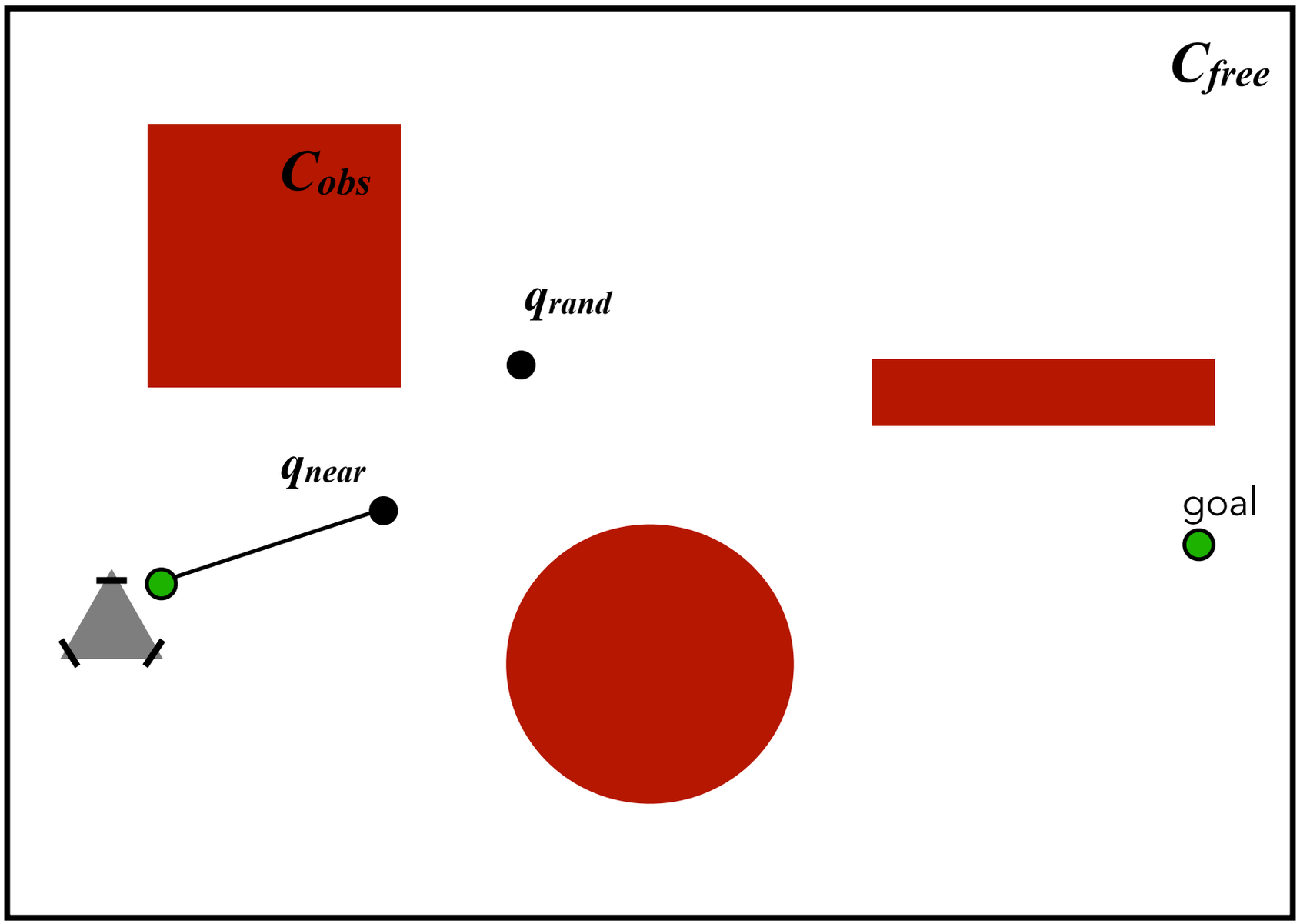

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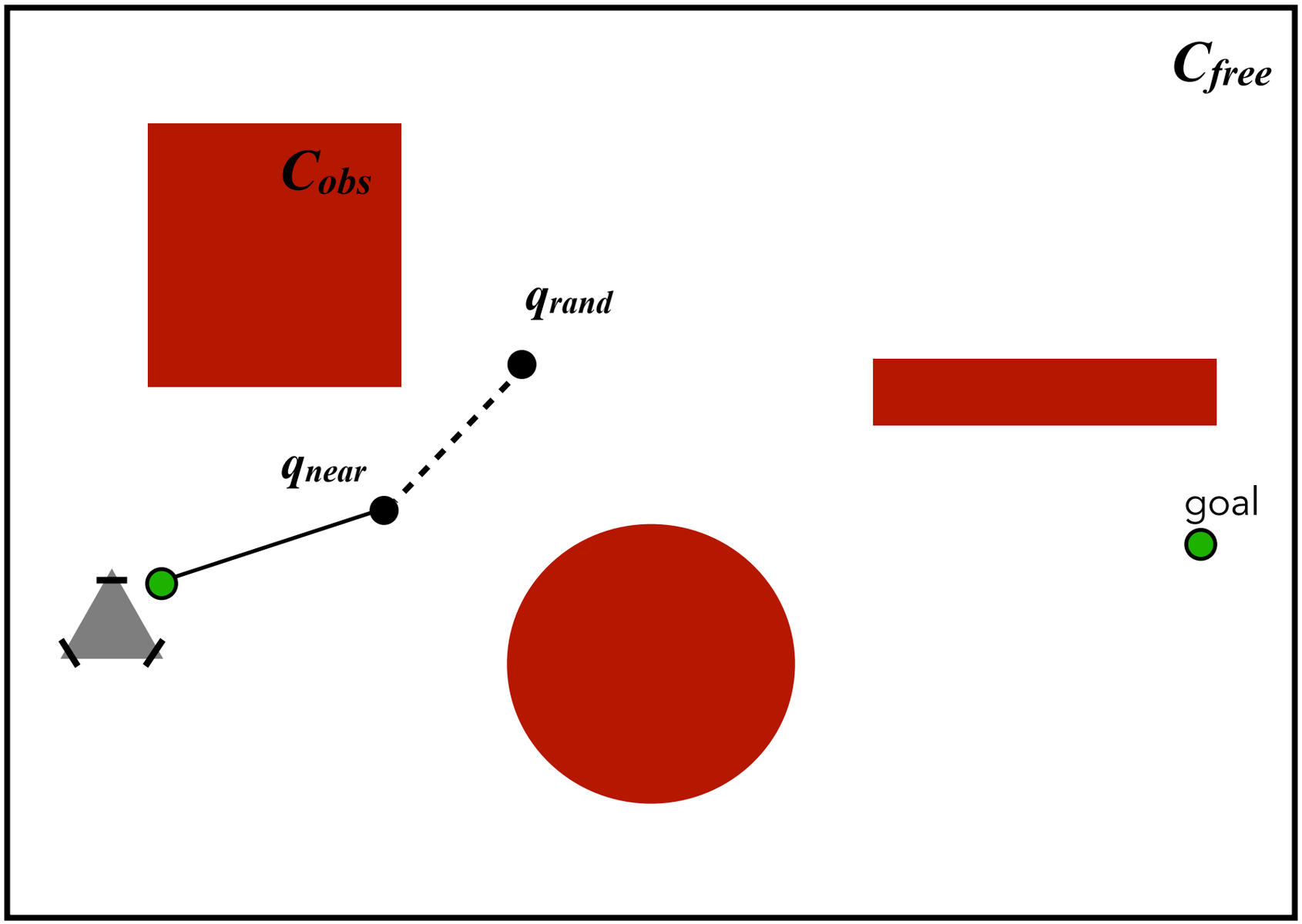

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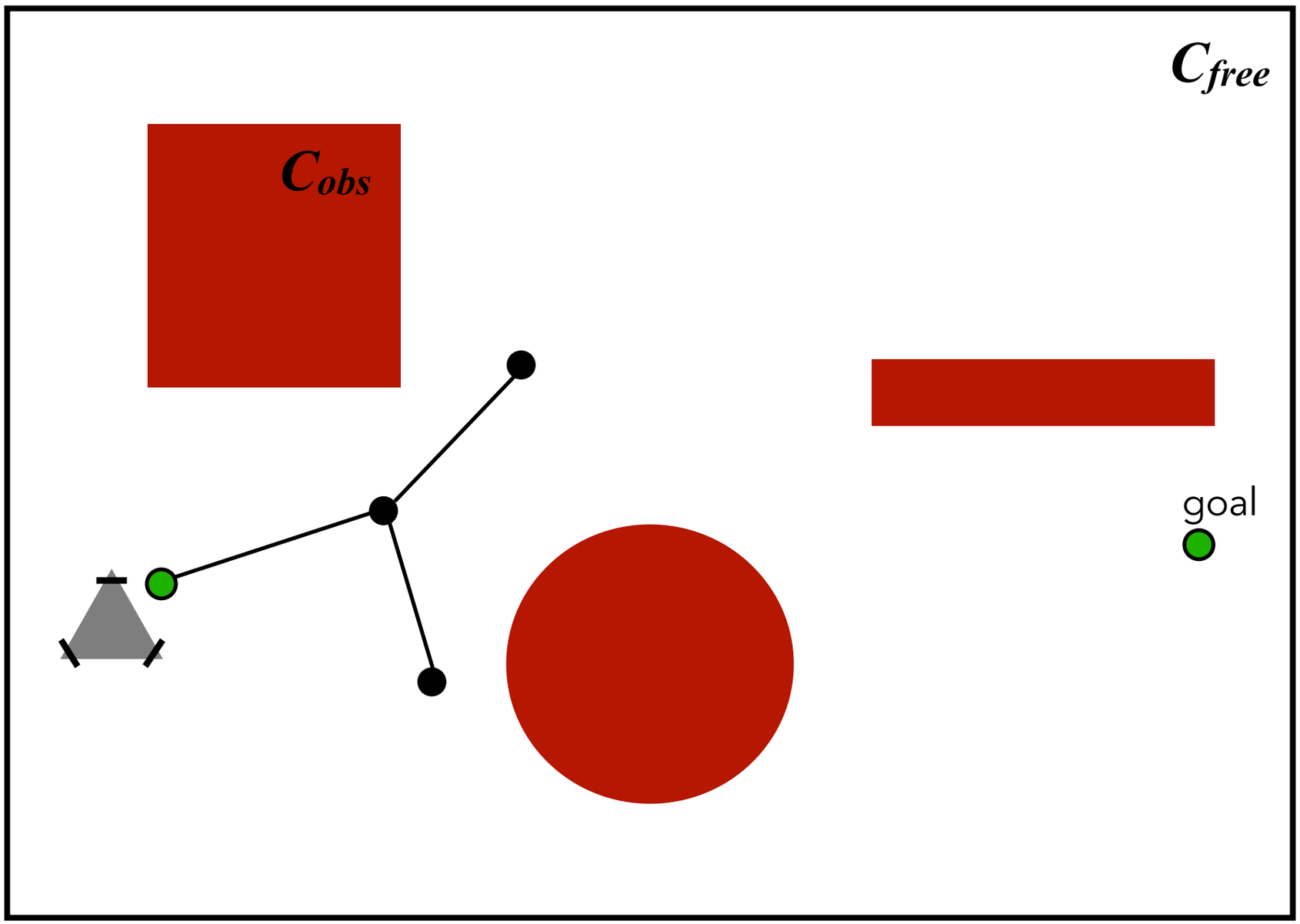

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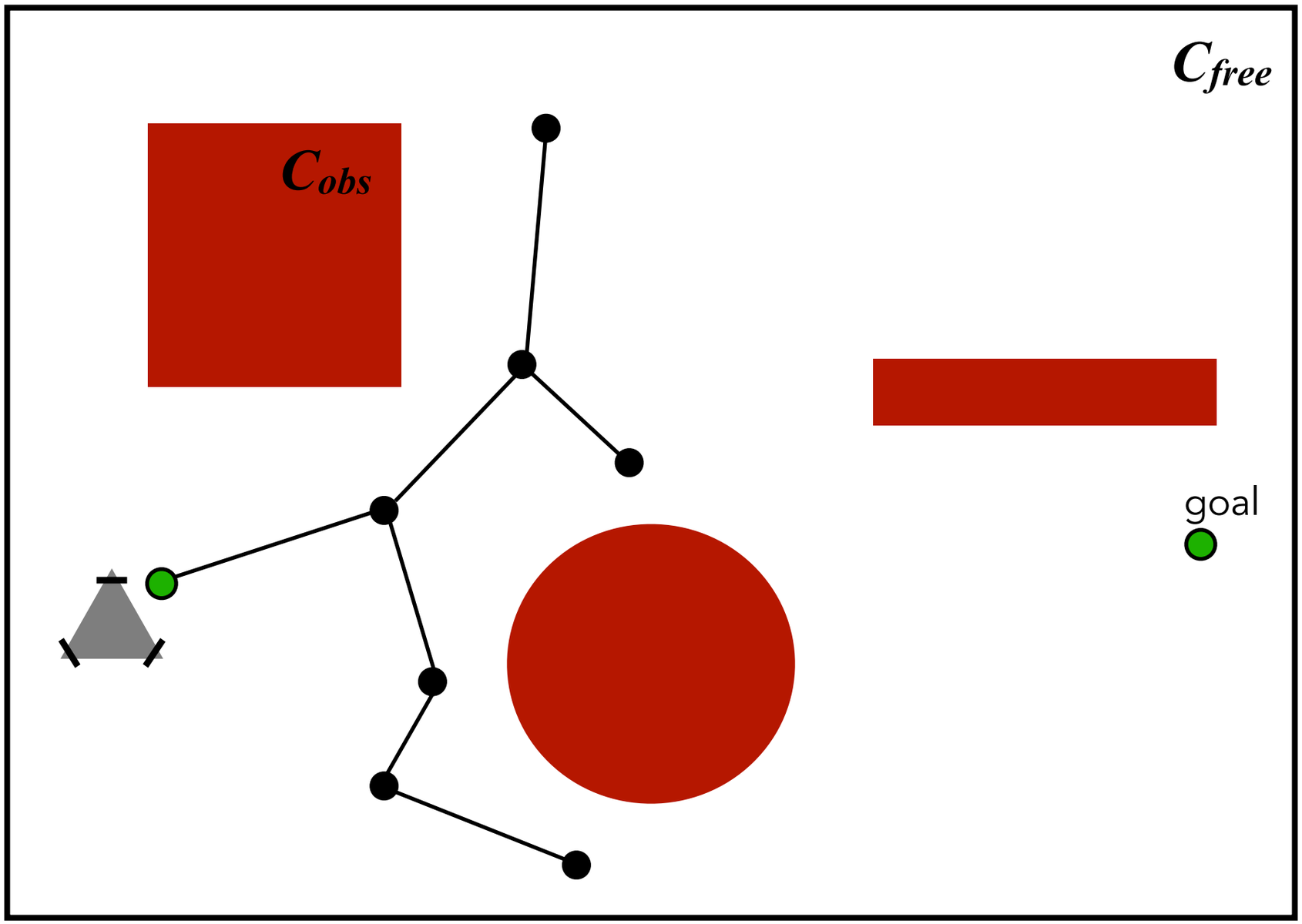

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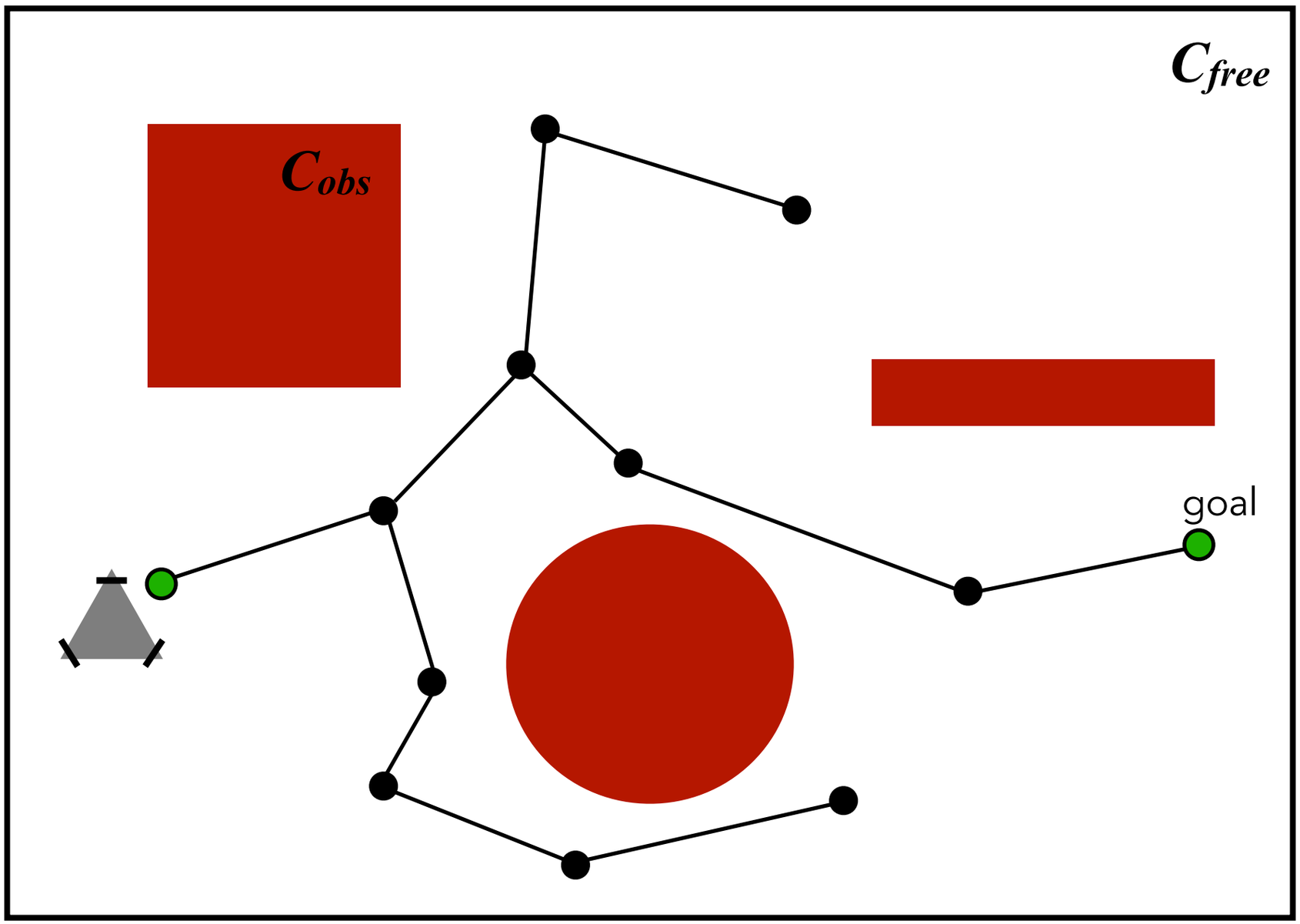


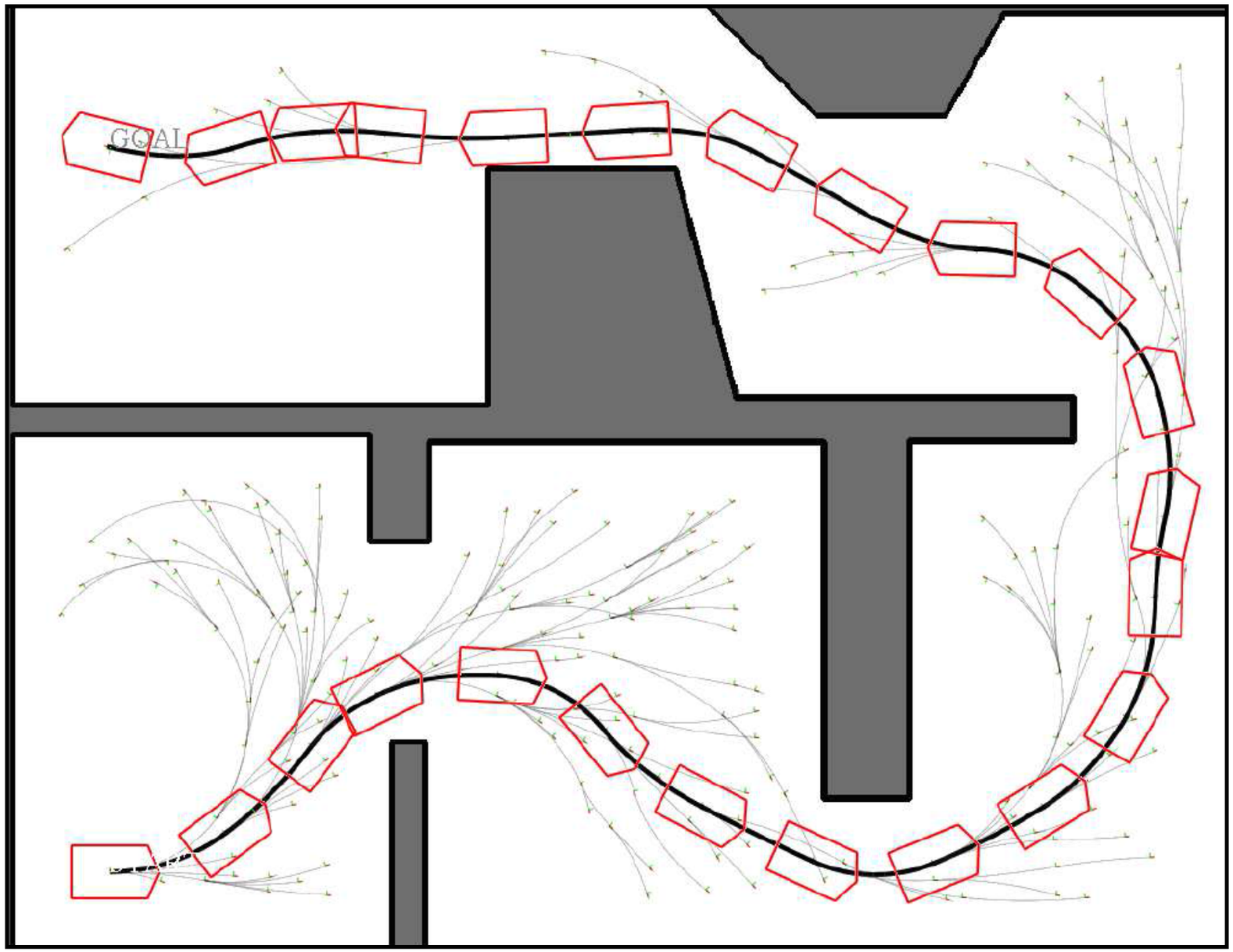
Rapidly Exploring Random Trees (RRTs)

- Primitives required:
  - Method to sample configurations in free C-space
  - Method to check for collisions in C-space
- Pros:
  - Probabilistically complete
  - Exponential rate of decay for the probability of failure
  - Asymptotically optimal (RRT*)
- Cons:
  - For non-holonomic robots, generating edges to sampled configurations requires solution to the *two-point boundary value problem*. (Alternate trick: use **motion primitives**).

* image credit: Kavraki, 2008

Rapidly Exploring Random Trees (RRTs)

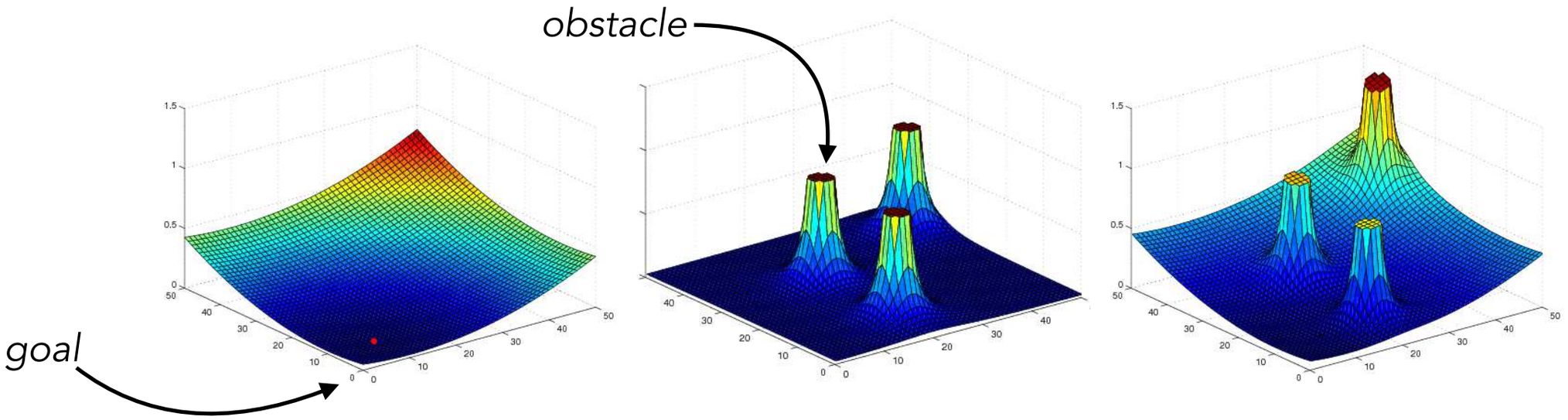
Example: Integration of motion primitives with RRT-based search.



Potential Field Method

- No explicit roadmap; instead, construct a real-valued potential function: $U : \mathbb{R}^m \mapsto \mathbb{R}$

  Attractive potential: $U_a(\mathbf{q})$ Repulsive potential: $U_r(\mathbf{q})$



Differentiate potential $\nabla U(\mathbf{q}) = \left[\frac{\partial U}{\partial \mathbf{q}_1}, \ldots, \frac{\partial U}{\partial \mathbf{q}_m}\right]^\mathsf{T}$

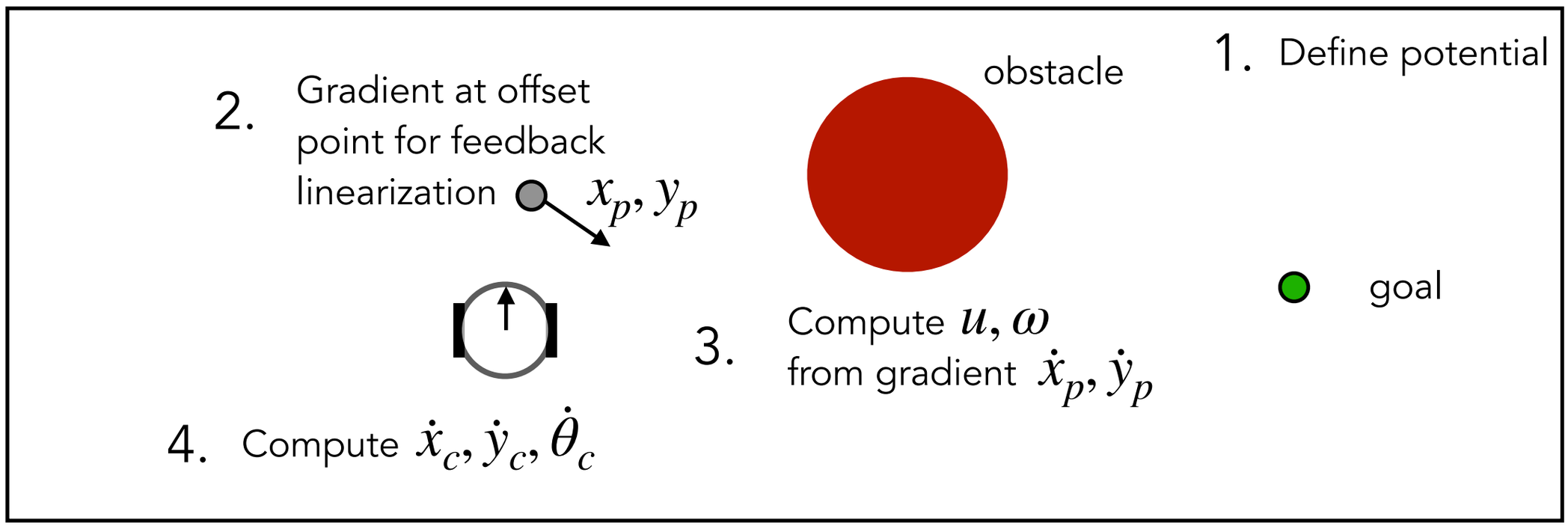
and compute a path via gradient descent (*gradient motion*).

* image credit: Kavraki, 2008

Potential Field Method

- Creating the potential field in C-space is hard.
- One possible strategy:
  1. Create potential field in robot work-space
  2. Compute gradient
  3. Use feedback linearization to get control inputs

4. (Compute gradient in C-space from control inputs)



Vector Fields

- Requirements
  - Representation of work-space / C-space
- Pros
  - Computationally efficient (once C-space is constructed)
  - More than just 1 path: motion strategy (feedback control) is readily computed from any point in field.
- Cons
  - Local minima (require methods to resolve them)
  - Representation of C-space is hard in high dimensions

Further Reading

Books that cover fundamental concepts:

- Steven M. LaValle. "Planning Algorithms". 2006. Cambridge University Press.
- http://planning.cs.uiuc.edu/

Seminal papers:

- S. M. LaValle and J. J. Kuffner; Rapidly-exploring random trees; 2001.
- S. Karaman, E. Frazzoli; Sampling-based Algorithms for Optimal Motion Planning; 2011
- J. Barraquand and J-C. Latombe. Robot motion planning: A distributed representation approach;1991.