4: Significance Testing

Machine Learning and Real-world Data

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Lent 2019

Last session: Zipf's Law and Heaps' Law

- Zipf's Law: small number of very high-frequency words; large number of low-frequency words ("long tail").
- Heaps' Law: as more text is gathered, there will be diminishing returns in terms of discovery of new word types in the tail.
  - We will systematically always encounter new unseen words in new texts.
- Smoothing works by
  - lowering the MLE estimate for seen types
  - redistributing this probability to unseen types (e.g. for words in long tail we might encounter during our experiment).

Observed system improvement

- This produced a better system.
- Or at least, you observed higher accuracies.
- Today: we use a statistical test to gather evidence that one system is really better than another system.

Variation in the data

- Documents are different (writing style, length, type of words used, . . .)
- Some documents will make it easier for your system to score well, some will make it easier for the other system.
- Maybe you were just lucky and *all* documents in the test set are in your favour?
  - This could be the case if you don't have enough data.
  - This could be the case if the difference in accuracy is small.
- Maybe both systems perform equally well in reality?

Statistical Significance Testing

- Null Hypothesis: two result sets come from the same distribution
  - System 1 is (really) equally good as System 2.
- First, choose a significance level (α), e.g., $\alpha = 0.01$ or 0.05.
- We then try to reject the null hypothesis with confidence $1 - \alpha$ (99% or 95% in this case)
- Rejecting the null hypothesis means showing that the observed result is very unlikely to have occurred by chance.

Reporting significance

- If we successfully pass the significance test, and only then, we can report:

  > *"The difference between System 1 and System 2 is statistically significant at* $\alpha = 0.01$*."*

- Any other statements based on raw accuracy differences alone are strictly speaking meaningless.

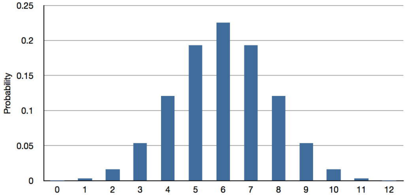
Sign Test (non-parametric, paired)

- The sign test uses a binary event model.
- Here, events correspond to documents.
- Events have binary outcomes:
  - Positive: System 1 beats System 2 on this document.
  - Negative: System 2 beats System 1 on this document.
  - (Tie: System 1 and System 2 do equally well on this document / have identical results – more on this later).
- Binary distribution allows us to calculate the probability that, say, (at least) 1,247 out of 2,000 such binary events are positive.
- Or otherwise the probability that (at most) 753 out of 2,000 are negative.

Binomial Distribution $B(N, q)$

- Call the probability of a negative outcome q (here $q = 0.5$)
- Probability of observing $X = k$ negative events out of N:

$$P_q(X = k|N) = \binom{N}{k} q^k (1-q)^{N-k}$$



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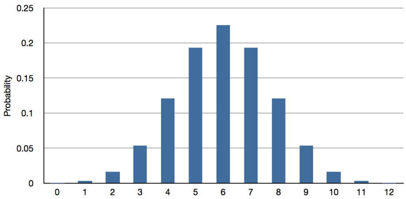
$$P_q(X = k|N) = \binom{N}{k} q^k (1-q)^{N-k}$$

- At most k negative events:

$$P_q(X \leq k|N) = \sum_{i=0}^{k} \binom{N}{i} q^i (1-q)^{N-i}$$



Binary Event Model and Statistical Tests

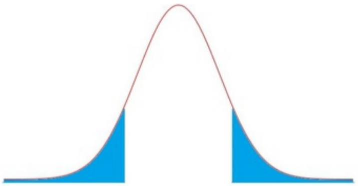
- If the probability of observing our events under the Null Hypothesis is very small (smaller than our pre-selected significance level α, e.g., 0.01), we can safely reject the Null hypothesis.
- The $P(X \leq k)$ we just calculated directly gives us the probability we are interested in.
- If $P(X \leq k) \leq 0.01$, this means there is less than a 1% chance that System 1 does not actually beat System 2.

Two-Tailed vs. One-Tailed Tests

A more conservative, rigorous test would be a non-directional one (though some debate on this!)

- Testing for statistically significant difference regardless of direction: **a two-tailed test**
- We are now interested in the value of k at which 0.01 of the probability exists in the two tails.

- $B(N, 0.5)$ is symmetric so we are now interested in $2P(X \leq k)$
- For the two-tailed test, if $2P(X \leq k) \leq 0.01$, then there is less than a 1% chance that System 1 does not actually beat System 2.
- We'll be using the two-tailed test for this practical.



Treatment of Ties

- When comparing two systems in classification tasks, it is common for a large number of ties to occur.
- Disregarding ties will tend to affect a study's statistical power.
- Here, we will treat ties by adding 0.5 events to the positive and 0.5 events to the negative side (and round up at the end).

Today's Tasks

- Implement the above-introduced test for statistical significance, so that you can compare two systems.
- Implementation details on moodle (including helper code as before)

Today's Tasks

- Create more (potentially better) systems to use the significance test on.
- Modify the simple lexicon-based classifier by weighting terms with stronger sentiment more.
- The pretester will accept a system where strong indicators have weight 2.
  - You can also empirically find out the optimal weight.
  - We call this process parameter tuning.
  - Use the training corpus to set your parameters, then test on the 200 documents as before.

Starred Tick — Parameter tuning for NB Smoothing

- Formula for smoothing with a constant ω:

$$\hat{P}(w_i|c) = \frac{count(w_i, c) + \omega}{(\sum_{w \in V} count(w, c)) + \omega|V|}$$

- We used add-one smoothing in Task 2 (ω = 1).
- Using the training corpus, we can optimise the smoothing parameter ω.

Literature

- Siegel and Castellan (1988). *Non-parametric statistics for the behavioral sciences*, McGraw-Hill, 2nd. Edition.
  - Chapter 2: The use of statistical tests in research
  - Sign test: p. 80–87