14: Clique Finding

Machine Learning and Real-world Data (MLRD)

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(based on slides created by Simone Teufel)

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Last session: betweenness centrality

- You implemented betweenness centrality.
- This let you find “gatekeeper” nodes in the Facebook network.
- We will now turn to the task of finding clusters in networks.
- You will test this on a small network derived from one Facebook user.

Clustering in networks

- **clustering**: automatically grouping data according to some notion of closeness or similarity.
- **agglomerative clustering** works bottom-up.
- **divisive clustering** works top-down, by splitting.
- Newman-Girvan method — a form of divisive clustering.
- Criterion for breaking links is edge betweenness centrality.
- When to stop?
  - Prespecified (today's tick): use prior knowledge to decide when to stop, based on number of clusters.
  - Inherent 'goodness of clustering' metric: today's starred tick uses **modularity** (Newman 2004).

Step 1: Code for determining connected components

- Today's graph is disconnected: there are five **connected components**.
- Finding connected components: depth-first search, start at an arbitrary node and mark the other nodes you reach.
- Repeat with unvisited nodes, until all are visited.
- Implementation hint: depth-first, so use recursion (the program stack stores the search state).

Step 2: Edge betweenness centrality

- Previously: $\sigma(s,t|v)$ — the number of shortest paths between s and t going through node v.
- Now: $\sigma(s,t|e)$ — the number of shortest paths between s and t going through edge e.
- Algorithm only changes in the bottom-up (accumulation) phase: $\delta(v)$ much as before, but $c_B[(v,w)]$

Brandes (2008) pseudocode

▼ **accumulation** // — back-propagation of dependencies
  for $v \in V$ **do** $\delta[v] \leftarrow 0$
  while S *not empty* **do**
    pop $w \leftarrow S$
    for $v \in Pred[w]$ **do** $\delta[v] \leftarrow \delta[v] + \frac{\sigma[v]}{\sigma[w]} \cdot (1 + \delta[w])$
    if $w \neq s$ **then** $c_B[w] \leftarrow c_B[w] + \delta[w]$

Edge betweenness

output: betweenness $c_B[q]$ for $q \in V \cup E$ (initialized to 0)

▼ **accumulation**
  for $v \in V$ **do** $\delta[v] \leftarrow 0$
  while S *not empty* **do**
    pop $w \leftarrow S$
    for $v \in Pred[w]$ **do**
      $c \leftarrow \frac{\sigma[v]}{\sigma[w]} \cdot (1 + \delta[w])$
      $c_B[(v, w)] \leftarrow c_B[(v, w)] + c$
      $\delta[v] \leftarrow \delta[v] + c$
    if $w \neq s$ **then** $c_B[w] \leftarrow c_B[w] + \delta[w]$

ignore last line

Step 3: Newman-Girvan method

while number of connected subgraphs $<$ specified number of clusters (and there are still edges):

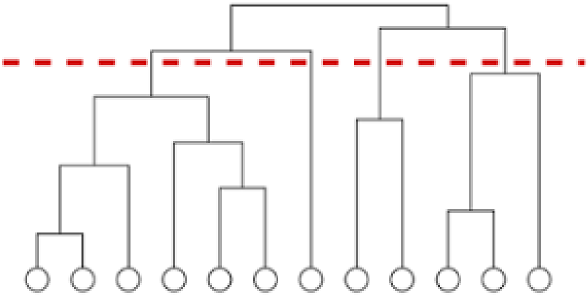
1. calculate edge betweenness for every edge in the graph
2. remove edge(s) with highest betweenness
3. recalculate number of connected components

Note:

- Treatment of tied edges: either remove all (today) or choose one randomly.

Visualization as dendrogram

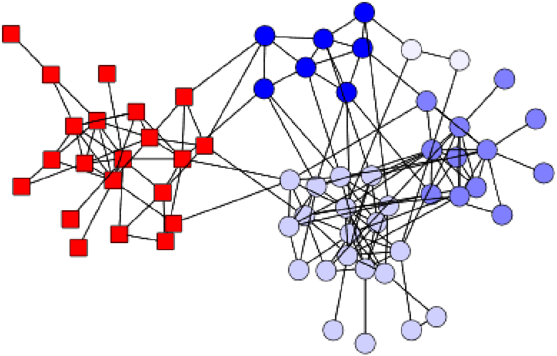
- Either: stop at prespecified level (tick).
- Or: complete process and choose best level by ‘modularity’ (starred tick).



Newman and Girvan (2004)

Dolphin data: different clustering layers

- squares vs circles: first split
- different colours: further splits



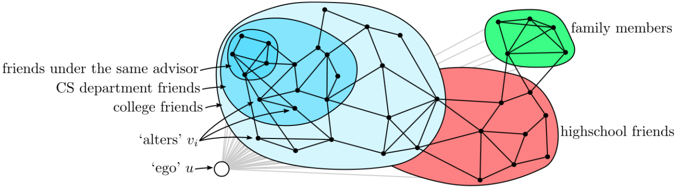
Newman and Girvan (2004)

Facebook circles dataset: McAuley and Leskovec (2012)

- Designed to allow experimentation with automatic discovery of circles: Facebook friends in a particular social group.
- Profile and network data from 10 Facebook ego-networks (networks emanating from one person: referred to as an **ego**).
- Gold-standard circles, manually identified by the egos themselves.
- Average: 19 circles per ego, each circle with average of 22 **alters**.
- Complete network consists of 4,039 nodes in 193 circles.

Facebook circles

Requires more sophisticated methods than Newman-Girvan:
a) nodes may be in multiple circles, b) not just network data.



25% of circles are contained completely within another circle
50% overlap with another circle
25% have no members in common with any other circle

Evaluating simple clustering

- Assume data sets with gold standard or ground truth clusters.
- But: unlike classification, we don't have labels for clusters, number of clusters found may not equal true classes.
- **purity**: assign label corresponding to majority class found in each cluster, then count correct assignments, divide by total elements (cf accuracy).
  `http://nlp.stanford.edu/IR-book/html/htmledition/evaluation-of-clustering-1.html`
- But best evaluation (if possible) is **extrinsic**: use the system to do a task and evaluate that.

Clustering and classification

- Classification (e.g., sentiment classification): assigning data items to predefined classes.
- Clustering: groupings can emerge from data, **unsupervised**.
- Clustering for documents, images etc: anything where there's a notion of similarity between items.

  https://www.theguardian.com/politics/ng-interactive/2019/feb/15/

  how-brexit-revealed-four-new-political-factions
- Most famous technique for hard clustering is **k-means**: very general (also variant for graphs).
- Also soft clustering: clusters have graded membership

Schedule

Task 12:

- Implement the Newman-Girvan method.
- Discover clusters in the network provided.