UNIVERSITY OF CAMBRIDGE

Hoare Logic and Model Checking

Model Checking Lecture 3 Supplement

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CST Part II – 2018/19

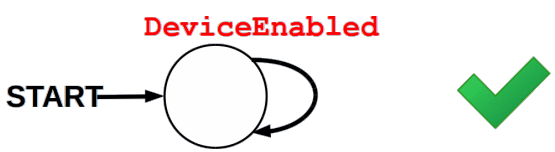
LTL formulas and models

- Examples from slide 75.
- Based on board-work during lecture 3.
- Example models, indicating whether the formula holds.

- Exercise: for failing models, give a counter-example path/trace.
- Reminder: LTL formulas are implicitly “for all paths”.

LTL formulas and models (1)

- “`DeviceEnabled` holds infinitely often along every path”
- **G** (**F** `DeviceEnabled`)



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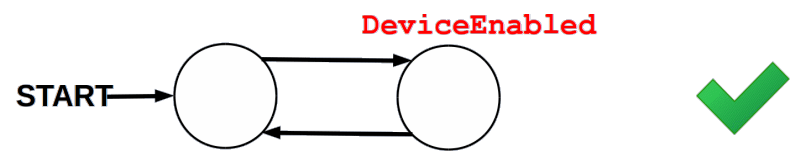

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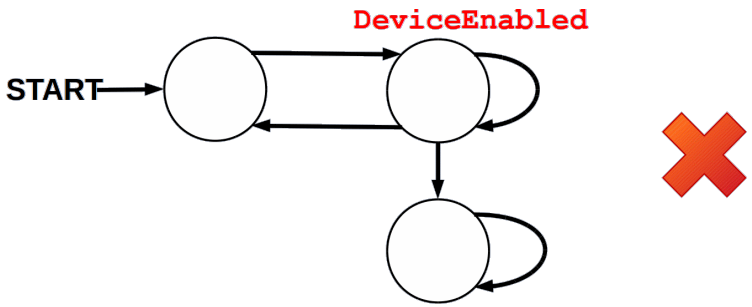
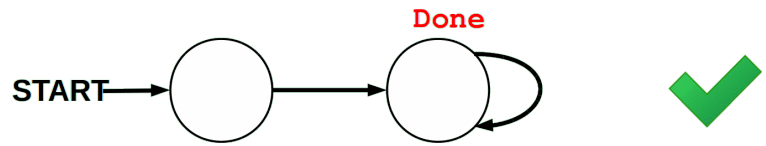


LTL formulas and models (2)

- "Eventually the state becomes permantently `Done`"
- **F** (**G** `Done`)



LTL formulas and models (2)

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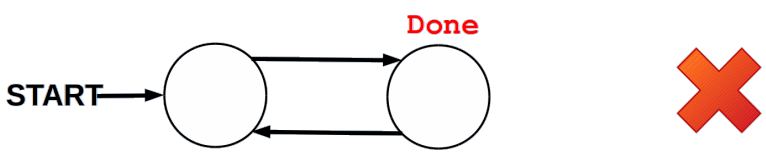

LTL formulas and models (2)

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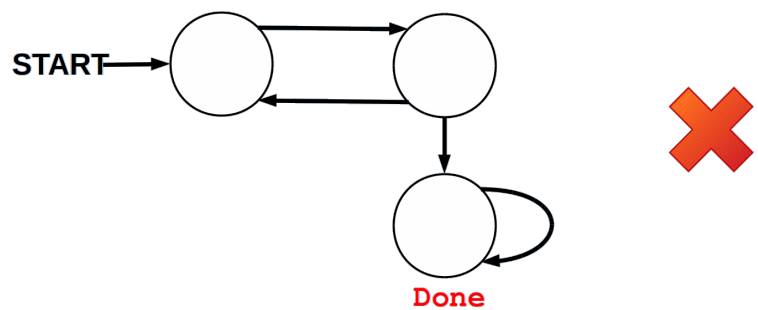
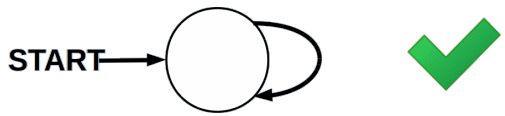


LTL formulas and models (3)

- "Every **Req** is followed by an **Ack**"
- **G** (**Req** $\Rightarrow$ (**F** **Ack**))



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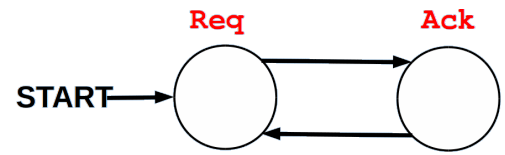

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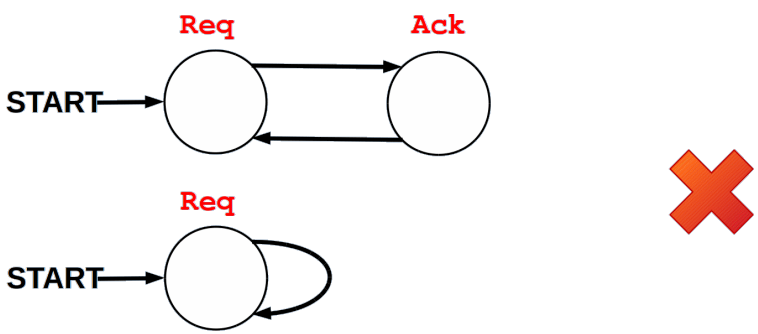
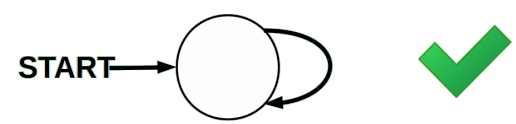


LTL formulas and models (4)

- "If **Enabled** infinitely often then **Running** infinitely often."
- **G** (**F** **Enabled**) $\Rightarrow$ **G** (**F** **Running**)



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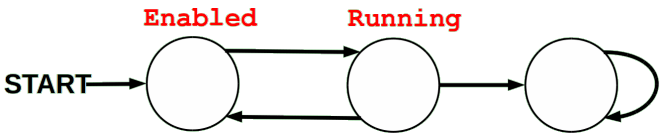

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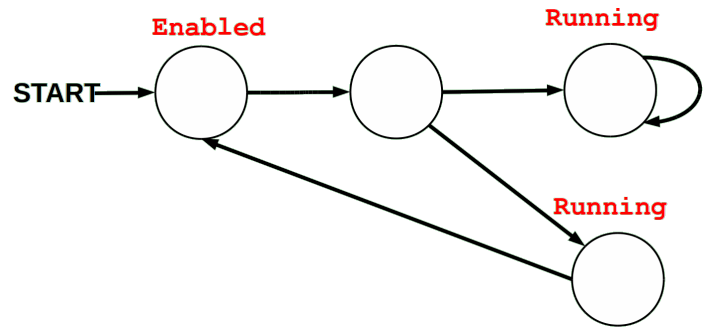

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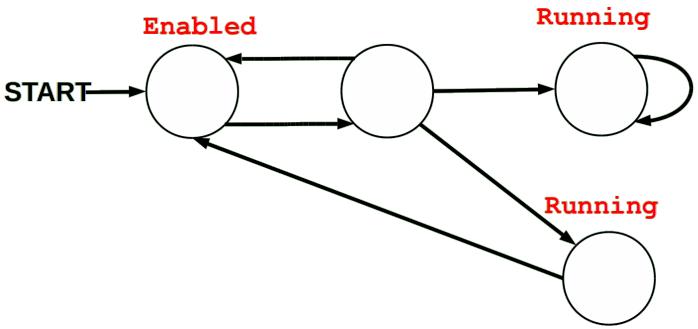

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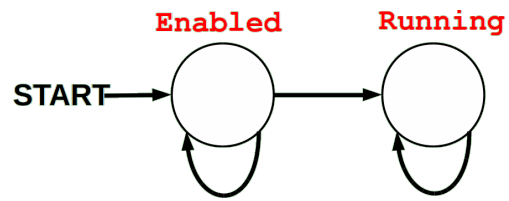
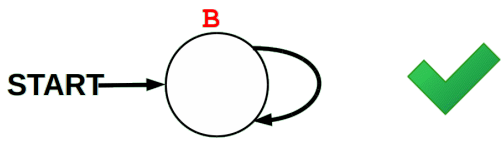


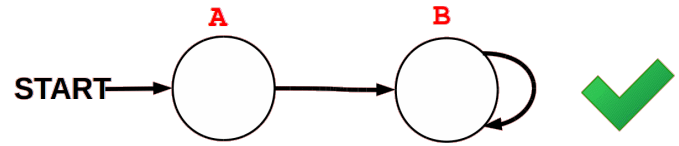
LTL formulas and models (5)

- The lift example is a little unwieldy, so here are some examples of **U** in isolation instead.
- **A U B**



LTL formulas and models (5)

- **A U B**



LTL formulas and models (5)

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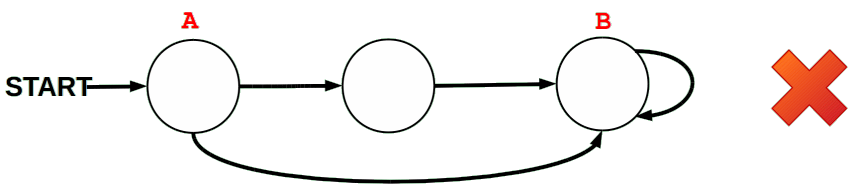

LTL formulas and models (5)

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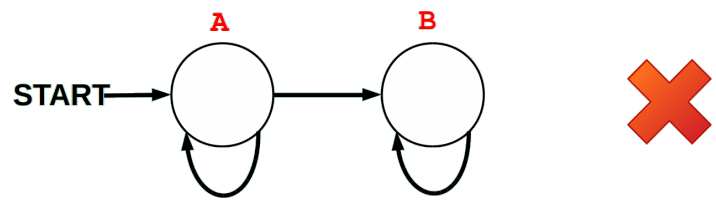

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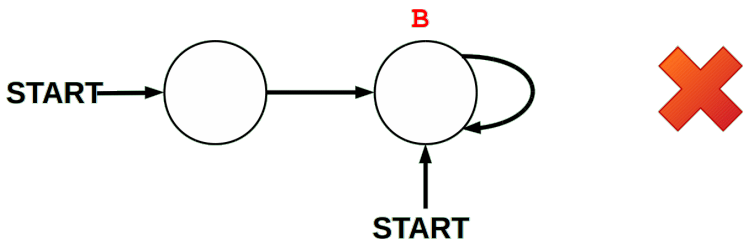

LTL proofs

- Example proofs of LTL implications and equivalences.
- Based on board-work during lecture 3.

LTL proofs (1)

- Prove $(M \models \mathbf{G}\ \phi) \Rightarrow (M \models \mathbf{G}\ (\mathbf{F}\ \phi))$
- Reminder:
  $$M \models \phi \ \Leftrightarrow\ \forall \pi\ s.\ s \in S_0 \wedge \mathrm{Path}\ R\ s\ \pi \Rightarrow [\![\phi]\!]_M(\pi)$$
- So sufficient to prove $[\![\mathbf{G}\ \phi]\!]_M(\pi) \Rightarrow [\![\mathbf{G}\ (\mathbf{F}\ \phi)]\!]_M(\pi)$ (for arbitrary π)

$[\![\mathbf{G}\ \phi]\!]_M(\pi)$
$\equiv \forall i.\ [\![\phi]\!]_M(\pi\downarrow i)$
$\equiv \forall i.\ [\![\phi]\!]_M(\pi\downarrow(i+0))$
$\Rightarrow \forall i.\ \exists j.\ [\![\phi]\!]_M(\pi\downarrow(i+j))$
$\equiv \forall i.\ \exists j.\ [\![\phi]\!]_M((\pi\downarrow i)\downarrow j)$
$\equiv \forall i.\ [\![\mathbf{F}\ \phi]\!]_M(\pi\downarrow i)$
$\equiv [\![\mathbf{G}\ (\mathbf{F}\ \phi)]\!]_M(\pi)$ □

LTL proofs (2)

- Prove $(M \models \mathbf{G}\ \phi) \equiv (M \models \mathbf{G}\ (\mathbf{G}\ \phi))$
- Reminder:
  $$M \models \phi \ \Leftrightarrow\ \forall \pi\ s.\ s \in S_0 \wedge \mathrm{Path}\ R\ s\ \pi \Rightarrow [\![\phi]\!]_M(\pi)$$
- So sufficient to prove $[\![\mathbf{G}\ \phi]\!]_M(\pi) \equiv [\![\mathbf{G}\ (\mathbf{G}\ \phi)]\!]_M(\pi)$

$[\![\mathbf{G}\ \phi]\!]_M(\pi)$
$\equiv \forall i.\ [\![\phi]\!]_M(\pi\downarrow i)$
$\equiv \forall i.\ \forall j.\ [\![\phi]\!]_M(\pi\downarrow(i+j))$
$\equiv \forall i.\ \forall j.\ [\![\phi]\!]_M((\pi\downarrow i)\downarrow j)$
$\equiv \forall i.\ [\![\mathbf{G}\ \phi]\!]_M(\pi\downarrow i)$
$\equiv [\![\mathbf{G}\ (\mathbf{G}\ \phi)]\!]_M(\pi)$ □

LTL proofs (3)

- Prove $(M \models \mathbf{F}\ (\mathbf{G}\ \phi)) \Rightarrow (M \models \mathbf{G}\ (\mathbf{F}\ \phi))$
- Reminder:
  $$\boxed{M \models \phi \ \Leftrightarrow\ \forall \pi\ s.\ s \in S_0 \wedge \mathsf{Path}\ R\ s\ \pi \Rightarrow [\![\phi]\!]_M(\pi)}$$
- So sufficient to prove $[\![\mathbf{F}\ (\mathbf{G}\ \phi)]\!]_M(\pi) \Rightarrow [\![\mathbf{G}\ (\mathbf{F}\ \phi)]\!]_M(\pi)$

$[\![\mathbf{F}\ (\mathbf{G}\ \phi)]\!]_M(\pi)$
$\equiv \exists j.\ [\![\mathbf{G}\ \phi]\!]_M(\pi\downarrow j)$
$\equiv \exists j.\ \forall i.\ [\![\phi]\!]_M((\pi\downarrow j)\downarrow i)$
$\equiv \exists j.\ \forall i.\ [\![\phi]\!]_M((\pi\downarrow i)\downarrow j)$
$\Rightarrow \forall i.\ \exists j.\ [\![\phi]\!]_M((\pi\downarrow i)\downarrow j)$
$\equiv \forall i.\ [\![\mathbf{F}\ \phi]\!]_M(\pi\downarrow i)$
$\equiv [\![\mathbf{G}\ (\mathbf{F}\ \phi)]\!]_M(\pi)$ $\square$