High dynamic range and tone mapping
Advanced Graphics and Image Processing
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Cornell Box: need for tone-mapping in graphics
Rendering Photograph

Real-world scenes are more challenging
- The match could not be achieved if the light source in the top of the box was visible
- The display could not reproduce the right level of brightness

Dynamic range

Dynamic range (contrast)
- As ratio:
  \[ C = \frac{L_{\text{max}}}{L_{\text{min}}} \]
  Usually written as C:1, for example 1000:1.
- As “orders of magnitude” or \( \log_{10} \) units:
  \[ C_{10} = \log_{10} \frac{L_{\text{max}}}{L_{\text{min}}} \]
- As stops:
  \[ C_2 = \log_2 \frac{L_{\text{max}}}{L_{\text{min}}} \]
  One stop is doubling of halving the amount of light

High dynamic range (HDR)
Why do we need tone mapping?

- To reduce dynamic range
- To customize the look
  - colour grading
- To simulate human vision
  - for example night vision
- To adapt displayed images to a display and viewing conditions
- To make rendered images look more realistic
- To map from scene- to display-referred colours

Different tone mapping operators achieve different goals.

From scene- to display-referred colours

The primary purpose of tone mapping is to transform an image from scene-referred to display-referred colours.

Basic tone-mapping and display coding

The simplest form of tone-mapping is the exposure/brightness adjustment:

\[ R_d = \frac{R_s \cdot L_{\text{white}}}{L_{\text{white}}} \]

- \( R_d \) for red, the same for green and blue
- No contrast compression, only for a moderate dynamic range

The simplest form of display coding is the “gamma”

\[ R' = (R_d)^{\gamma} \]

- \( \gamma \) typically 2.2
- For SDR displays only

Techniques

- Arithmetic of HDR images
- Display model
- Tone-curve
- Color transfer
- Base-detail separation
- Glare
Arithmetic of HDR images

- How does the basic arithmetic operations
  - Addition
  - Multiplication
  - Power function
  affect the appearance of an HDR image?
- We work in the luminance space (NOT luma)
- The same operations can be applied to linear RGB
  - Or to luminance-only and the colour can be transferred

Multiplication – brightness change

$$ T(L_p) = B \cdot L_p $$

- Multiplication makes the image brighter or darker
- It does not change the dynamic range!

Power function – contrast change

$$ T(L_p) = \left( \frac{L_p}{L_{\text{white}}} \right)^{\text{gamma}} $$

- Power function stretches or shrinks image dynamic range
- It is usually performed relative to a reference white colour/luminance
- Apparent brightness changes is the side effect of pushing tones towards or away from the white point

Addition – black level

$$ T(L_p) = L_p + F $$

- Addition elevates black level, adds „fog“ to an image
- It affects mostly darker tones
- It reduces image dynamic range

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Display-adaptive tone mapping

- Tone-mapping can account for the physical model of a display
  - How a display transforms pixel values into emitted light
  - Useful for ambient light compensation
  - Has a similar role as display encoding, but can account for viewing conditions
(Forward) Display model

- **GOG**: Gain-Gamma-Offset
- **Display black level**

\[
L = (L_{\text{peak}} - L_{\text{black}}) V^\gamma + L_{\text{black}} + L_{\text{refl}}
\]

Inverse display model

Symbols are the same as for the forward display model

\[
V = \left( \frac{L - L_{\text{black}} - L_{\text{refl}}}{L_{\text{peak}} - L_{\text{black}}} \right)^{\frac{1}{\gamma}}
\]

Note: This display model does not address any colour issues. The same equation is applied to red, green and blue color channels. The assumption is that the display primaries are the same as for the sRGB color space.

Example: Ambient light compensation

- We are looking at the screen in bright light
- We assume that the dynamic of the input is 2.6 (≈400:1)

\[
L_{\text{peak}} = 100 \text{ [cd m}^{-2}\text{]} \quad \lambda = 0.005 \quad \text{Modern screens have reflectivity of around 0.5%}
\]

\[
L_{\text{refl}} = 0.1 \text{ [cd m}^{-2}\text{]} \quad E_{\text{amb}} = 2000 \text{ [lux]} \quad L_{\text{refl}} = 0.005 \quad \pi L_{\text{refl}} = 2000 \times 3.1416 \text{ [cd m}^{-2}\text{]}
\]

We assume that the dynamic of the input is 2.6 (≈400:1)

\[
r_{\text{in}} = 2.6 \quad r_{\text{out}} = \log_{10} \left( \frac{L_{\text{peak}}}{L_{\text{black}} + L_{\text{refl}}} \right) = 1.77
\]

First, we need to compress contrast to fit the available dynamic range, then compensate for ambient light

\[
L_{\text{out}} = \left( \frac{L_{\text{in}}}{r_{\text{in}}} \right) \quad \text{The resulting value is in luminance, must be mapped to display luma / gamma corrected values}
\]

The simplest, but not the best tone mapping
Best tone-mapping is the one which does not do anything, i.e. slope of the tone-mapping curves is equal to 1.

But in practice contrast (slope) must be limited due to display limitations.

Global tone-mapping is a compromise between clipping and contrast compression.

Very common in digital cameras
- Mimic the response of analog film
- Analog film has been engineered over many years to produce good tone-reproduction
- Fast to compute

Simple formula for a sigmoidal tone-curve:

\[ R'(x,y) = \frac{R(x,y)^a}{\left(\frac{L_{rm}}{2}\right)^a + R(x,y)^a} \]

where \( L_{rm} \) is the geometric mean (or mean of logarithms):

\[ L_{rm} = \exp\left(\frac{1}{N} \sum_{j=1}^{N} \ln L(x_j, y_j)\right) \]

and \( L(x, y) \) is the luminance of the pixel \((x, y)\).
Histogram equalization

1. Compute normalized cumulative image histogram
   \[ c(I) = \frac{1}{N} \sum_{i=1}^{N} h(i) = c(I-1) + \frac{1}{N} h(I) \]
   - For HDR, operate in the log domain

2. Use the cumulative histogram as a tone-mapping function
   \[ Y_{\text{out}} = c(Y_{\text{in}}) \]
   - For HDR, map the log-10 values to the \([-dr_{\text{out}}, 0]\) range
   - where \(dr_{\text{out}}\) is the target dynamic range (of a display)

Histogram equalization distributes contrast distortions relative to the “importance” of a brightness level.

Histogram adjustment with a linear ceiling


- Truncate the bins that exceed the ceiling;
- Distribute the removed counts to all bins;
- Repeat until converges

Ceiling, based on the maximum permissible contrast.
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Colour transfer in tone-mapping

- Many tone-mapping operators work on luminance
  - For speed
  - To avoid colour artefacts
- Colours must be transferred later form the original image
- Colour transfer in the linear RGB colour space:

\[
R_{out} = \left( \frac{R_{in}}{L_{in}} \right) \cdot L_{out} \quad \text{Saturation parameter}
\]

\[
\text{Resulting luminance}
\]

- The same formula applies to green (G) and blue (B) linear colour values

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Illumination & reflectance separation

\[
Y = I \times R
\]

Input

Image

Illumination

Reflectance
Illumination and reflectance

- Reflectance:
  - White ≈ 90%
  - Black ≈ 3%
  - Dynamic range < 100:1
  - Reflectance critical for object & shape detection

- Illumination:
  - Sun ≈ 10^9 cd/m²
  - Lowest perceivable luminance ≈ 10^-6 cd/m²
  - Dynamic range 10,000:1 or more
  - Visual system partially discounts illumination

Reflectance & Illumination TMO

- Hypothesis: Distortions in reflectance are more apparent than the distortions in illumination
- Tone mapping could preserve reflectance but compress illumination

\[ L_d = R \cdot T(I) \]

- for example:

\[ L_d = R \cdot \left( \frac{I}{L_{white}} \right)^c \cdot L_{white} \]

How to separate the two?

- (Incoming) illumination – slowly changing
  - except very abrupt transitions on shadow boundaries
- Reflectance – low contrast and high frequency variations

Gaussian filter

- First order approximation

\[ f(x) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \]

- Blurs sharp boundaries
- Causes halos

Bilateral filter

- Better preserves sharp edges

\[ I_p = \sum_{k \in B} f(p-t) g(L_p - L_k) L_p \]

- Still some blurring on the edges
- Reflectance is not perfectly separated from illumination near edges

Weighted-least-squares (WLS) filter

- Stronger smoothing and still distinct edges

- Can produce stronger effects with fewer artifacts
- See „Advanced image processing” lecture

[Durand & Dorsey, SIGGRAPH 2002]

[Farbman et al., SIGGRAPH 2008]
Retinex

- Retinex algorithm was initially intended to separate reflectance from illumination [Land 1964]
- There are many variations of Retinex, but the general principle is to eliminate from an image small gradients, which are attributed to the illumination

1st step: compute gradients in log domain
2nd step: set to 0 gradients less than the threshold
3rd step: reconstruct an image from the vector field

\[ \nabla^2 I = \text{div } G \]

For example by solving the Poisson equation

Gradient domain HDR compression

- Similarly to Retinex, it operates on log-gradients
- But the function amplifies small contrast instead of removing it

<table>
<thead>
<tr>
<th>Retinex</th>
<th>Gradient domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \nabla G_{in} ]</td>
<td>[ \nabla G_{out} ]</td>
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</table>

- Contrast compression achieved by global contrast reduction
- Enhance reflectance, then compress everything

Glare

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Techniques

Glare Illusion

- Photography
- Painting

Computer Graphics:
- HDR rendering in games
Scattering of the light in the eye


Examples of simulated glare

[From Ritschel et al, Eurographics 2009]

Point Spread Function of the eye

- What portion of the light is scattered towards a certain visual angle
- To simulate:
  - construct a digital filter
  - convolve the image with that filter


Ciliary corona and lenticular halo


Temporal glare

Dynamic glare

[From Ritschel et al. Eurographics 2009]

Selective application of glare

A) Glare applied to the entire image

\[ l_g = I \ast G \]

Glare kernel (PSF)

B) Glare applied only to the clipped pixels

\[ l_g = I + I_{\text{clipped}} \ast G - I_{\text{clipped}} \]

where \( I_{\text{clipped}} = \begin{cases} 1 & \text{for } I > 1 \\ 0 & \text{otherwise} \end{cases} \)

Better image quality

Selective application of glare

A) Glare applied to the entire image
B) Glare applied to clipped pixels only

Glare (or bloom) in games

- Convolution with large, non-separable filters is too slow
- The effect is approximated by a combination of Gaussian filters
  - Each filter with different “sigma”
  - The effect is meant to look good, not be accurate model of light scattering
- Some games simulate camera rather than the eye

Does the exact shape of the PSF matter?
- The illusion of increased brightness works even if the PSF is very different from the PSF of the eye

References
- Comprehensive book on HDR Imaging
- Overview of HDR imaging & tone-mapping
- Review of recent video tone-mapping
  - A comparative review of tone-mapping algorithms for high dynamic range video
- Selected papers on tone-mapping