Colour perception and colour spaces

Advanced Graphics and Image Processing

Rafał Mantiuk

Computer Laboratory, University of Cambridge
Colour and colour spaces
Electromagnetic spectrum

- **Visible light**
  - Electromagnetic waves of wavelength in the range 380nm to 730nm
  - Earth’s atmosphere lets through a lot of light in this wavelength band
  - Higher in energy than thermal infrared, so heat does not interfere with vision
Colour

- There is no physical definition of colour – colour is the result of our perception

- For emissive displays / objects

  \[
  \text{colour} = \text{perception}( \text{spectral_emission} )
  \]

- For reflective displays / objects

  \[
  \text{colour} = \text{perception}( \text{illumination} \ast \text{reflectance} )
  \]
Black body radiation

- Electromagnetic radiation emitted by a perfect absorber at a given temperature
- Graphite is a good approximation of a black body
Correlated colour temperature

- The temperature of a black body radiator that produces light most closely matching the particular source

- Examples:
  - Typical north-sky light: 7500 K
  - Typical average daylight: 6500 K
  - Domestic tungsten lamp (100 to 200 W): 2800 K
  - Domestic tungsten lamp (40 to 60 W): 2700 K
  - Sunlight at sunset: 2000 K

- Useful to describe colour of the **illumination** (source of light)
Standard illuminant D65

- Mid-day sun in Western Europe / Northern Europe
- Colour temperature approx. 6500 K
Reflectance

- Most of the light we see is reflected from objects.
- These objects absorb a certain part of the light spectrum.

Spectral reflectance of ceramic tiles

Why not red?
Reflected light

\[ L(\lambda) = I(\lambda)R(\lambda) \]

- Reflected light = illumination * reflectance

The same object may appear to have different color under different illumination.
Fluorescence

A

Normalized Data

1.0

Absorption

Fl 420nm Ex

Fl 450nm Ex

Fl 470nm Ex

Wavelength (nm)

0.0

0.2

0.4

0.6

0.8

1.0

400

500

600

700

B

Normalized Data

1.0

Absorption

Fl 420nm Ex

Fl 450nm Ex

Fl 470nm Ex

Wavelength (nm)

0.0

0.2

0.4

0.6

0.8

1.0

400

500

600

700

From: http://en.wikipedia.org/wiki/Fluorescence
Colour perception

- **Di-chromaticity (dogs, cats)**
  - Yellow & blue-violet
  - Green, orange, red indistinguishable

- **Tri-chromaticity (humans, monkeys)**
  - Red-ish, green-ish, blue-ish
  - Colour-deficiency
    - Most often men, green-red colour-deficiency

www.lam.mus.ca.us/cats/color/  
www.colorcube.com/illusions/clrblnd.html
Colour vision

- Cones are the photoreceptors responsible for color vision
  - Only daylight, we see no colors when there is not enough light
- Three types of cones
  - S – sensitive to short wavelengths
  - M – sensitive to medium wavelengths
  - L – sensitive to long wavelengths

Sensitivity curves – probability that a photon of that wavelengths will be absorbed by a photoreceptor. S, M and L curves are normalized in this plot.
Perceived light

- cone response = \(\text{sum}(\text{sensitivity} \times \text{reflected light})\)

Although there is an infinite number of wavelengths, we have only three photoreceptor types to sense differences between light spectra.

Formally

\[
R_S = \int_{380}^{730} S_S(\lambda) \cdot L(\lambda) d\lambda
\]

Index S for S-cones.
Metamers

- Even if two light spectra are different, they may appear to have the same colour.
- The light spectra that appear to have the same colour are called **metamers**.

**Example:**

\[
\begin{align*}
\text{Example:} & \\
L_1 & = [L_1, M_1, S_1] \\
\parallel & \\
L_2 & = [L_2, M_2, S_2]
\end{align*}
\]
Practical application of metamerism

- Displays do not emit the same light spectra as real-world objects.
- Yet, the colours on a display look almost identical.

On the display:

In real world:

\[
\begin{align*}
[L_1, M_1, S_1] & \quad || \\
[L_2, M_2, S_2] &
\end{align*}
\]
Observation

- Any colour can be matched using three linear independent reference colours
- May require “negative” contribution to test colour
- Matching curves describe the value for matching monochromatic spectral colours of equal intensity
  - With respect to a certain set of primary colours
Standard Colour Space CIE-XYZ

- **CIE Experiments [Guild and Wright, 1931]**
  - Colour matching experiments
  - Group ~12 people with „normal“ colour vision
  - 2 degree visual field (fovea only)

- **CIE 2006 XYZ**
  - Derived from LMS color matching functions by Stockman & Sharpe
  - S-cone response differs the most from CIE 1931

- **CIE-XYZ Colour Space**
  - **Goals**
    - Abstract from concrete primaries used in experiment
    - All matching functions are positive
    - Primary „Y” is roughly proportionally to light intensity (luminance)
Standard Colour Space CIE-XYZ

- Standardized imaginary primaries CIE XYZ (1931)
  - Could match all physically realizable colour stimuli
  - Y is roughly equivalent to luminance
    - Shape similar to luminous efficiency curve
  - Monochromatic spectral colours form a curve in 3D XYZ-space

Cone sensitivity curves can be obtained by a linear transformation of CIE XYZ
CIE chromaticity diagram

- **chromaticity** values are defined in terms of $x$, $y$, $z$

\[
x = \frac{X}{X + Y + Z}, \quad y = \frac{Y}{X + Y + Z}, \quad z = \frac{Z}{X + Y + Z}
\]

- ignores luminance
- can be plotted as a 2D function
- pure colours (single wavelength) lie along the outer curve
- all other colours are a mix of pure colours and hence lie inside the curve
- points outside the curve do not exist as colours
Achromatic/chromatic vision mechanisms

Light spectra

S M L
Achromatic/chromatic vision mechanisms

Light spectra

S M L

Luminance does NOT explain the brightness of light! [Koenderink et al. Vision Research 2016]

Sensitivity of the achromatic mechanism
Achromatic/chromatic vision mechanisms

Light spectra

S M L
Green-red chromatic Luminance achromatic
Achromatic/chromatic vision mechanisms

Light spectra

S M L

Blue-yellow chromatic Green-red chromatic Luminance achromatic
Achromatic/chromatic vision mechanisms

Luminance

- Luminance – measure of light weighted by the response of the achromatic mechanism. Units: cd/m²

\[ L_V = \int_{350}^{700} kL(\lambda)V(\lambda)d\lambda \]

\[ k = \frac{1}{683.002} \]
Visible vs. displayable colours

- All physically possible and visible colours form a solid in XYZ space
- Each display device can reproduce a subspace of that space
- A chromacity diagram is a slice taken from a 3D solid in XYZ space
- Colour Gamut – the solid in a colour space
  - Usually defined in XYZ to be device-independent
Standard vs. High Dynamic Range

- **HDR** cameras/formats/displays attempt capture/represent/reproduce (almost) all visible colours
  - They represent scene colours and therefore we often call this representation *scene-referred*

- **SDR** cameras/formats/devices attempt to capture/represent/reproduce only colours of a standard sRGB colour gamut, mimicking the capabilities of CRTs monitors
  - They represent display colours and therefore we often call this representation *display-referred*
From rendering to display

1. HDR / physical rendering
2. Tone mapping
3. Display encoding (EOTF / inverse display model)
4. Scene-referred colours
5. Display-referred colours
6. Digital signal
7. Emitted light
From rendering to display

HDR / physical Rendering

Tone mapping

Scene-referred colours

Display-referred colours

Display encoding

EOTF / Inverse display model

8-12 bit integers encoded for efficiency

Digital signal

Gamma-corrected colour space

Linear colour space

floating point values relative to physical values

Ejected light
From rendering to display

- HDR / physical Rendering
- Tone mapping
  - Scene-referred colours
  - Display-referred colours
    - floating point values relative to physical values
- Display encoding
  - EOTF / Inverse display model
    - Linear colour space
    - Gamma-corrected colour space
    - 8-12 bit integers encoded for efficiency
- Display model
- Inverse display model
- Digital signal
- Emitted light
Display encoding for SDR: gamma correction

- Gamma correction is often used to encode luminance or tri-stimulus color values (RGB) in imaging systems (displays, printers, cameras, etc.)

\[ V_{out} = a \cdot V_{in}^{\gamma} \]

Gain (usually = 2.2)

Inverse:
\[ V_{in} = \left( \frac{1}{a \cdot V_{out}} \right)^{\frac{1}{\gamma}} \]

(colour) Luminance
Physical signal

Luma
Digital signal (0-1)

Colour: the same equation applied to red, green and blue colour channels.
Why is gamma needed?

- **Gamma-corrected** pixel values give a scale of brightness levels that is more perceptually uniform
- At least 12 bits (instead of 8) would be needed to encode each color channel without gamma correction
- And accidentally it was also the response of the CRT gun

![Linear encoding](image)

<table>
<thead>
<tr>
<th>Linear encoding $V_S$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear intensity $I$</td>
<td>0.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<- Pixel value (luma)
<- Luminance
Luma – gray-scale pixel value

- **Luma** - pixel brightness in *gamma corrected* units
  \[ L' = 0.2126R' + 0.7152G' + 0.0722B' \]
  - \( R', G', \text{ and } B' \) are *gamma-corrected* colour values
  - Prime symbol denotes *gamma corrected*
  - Used in image/video coding

- **Note that relative luminance if often approximated with**
  \[ L = 0.2126R + 0.7152G + 0.0722B \]
  \[ = 0.2126(R')^\gamma + 0.7152(G')^\gamma + 0.0722(B')^\gamma \]
  - \( R, G, \text{ and } B \) are *linear* colour values
  - Luma and luminance are different quantities despite similar formulas
Standards for display encoding

<table>
<thead>
<tr>
<th>Display type</th>
<th>Colour space</th>
<th>EOTF</th>
<th>Bit depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Dynamic Range</td>
<td>ITU-R 709</td>
<td>2.2 gamma / sRGB</td>
<td>8 to 10</td>
</tr>
<tr>
<td>High Dynamic Range</td>
<td>ITU-R 2020</td>
<td>ITU-R 2100 (PQ/HLG)</td>
<td>10 to 12</td>
</tr>
</tbody>
</table>

**Colour space**
What is the XYZ of “pure” red, green and blue

**Electro-Optical Transfer Function**
How to efficiently encode each primary colour

![Graph showing colour space and EOTF](image-url)
How to transform between linear RGB colour spaces?

From ITU-R 709 RGB to XYZ:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
0.4124 & 0.3576 & 0.1805 \\
0.2126 & 0.7152 & 0.0722 \\
0.0193 & 0.1192 & 0.9505
\end{bmatrix}
\cdot
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{R709toXYZ}
\]
How to transform between RGB colour spaces?

- From ITU-R 709 RGB to ITU-R 2020 RGB:
  \[
  \begin{bmatrix}
  R \\
  G \\
  B 
  \end{bmatrix}_{\text{R2020}} = M_{\text{XYZtoR2020}} \cdot M_{\text{R709toXYZ}} \cdot \begin{bmatrix}
  R \\
  G \\
  B 
  \end{bmatrix}_{\text{R709}}
  \]

- From ITU-R 2020 RGB to ITU-R 709 RGB:
  \[
  \begin{bmatrix}
  R \\
  G \\
  B 
  \end{bmatrix}_{\text{R709}} = M_{\text{XYZtoR709}} \cdot M_{\text{R2020toXYZ}} \cdot \begin{bmatrix}
  R \\
  G \\
  B 
  \end{bmatrix}_{\text{R2020}}
  \]

Where:

\[
M_{\text{R709toXYZ}} = \begin{bmatrix}
0.4124 & 0.3576 & 0.1805 \\
0.2126 & 0.7152 & 0.0722 \\
0.0193 & 0.1192 & 0.9505
\end{bmatrix}
\]

and

\[
M_{\text{XYZtoR709}} = M_{\text{R709toXYZ}}^{-1}
\]

\[
M_{\text{R2020toXYZ}} = \begin{bmatrix}
0.6370 & 0.1446 & 0.1689 \\
0.2627 & 0.6780 & 0.0593 \\
0.0000 & 0.0281 & 1.0610
\end{bmatrix}
\]

and

\[
M_{\text{XYZtoR2020}} = M_{\text{R2020toXYZ}}^{-1}
\]
Representing colour

- We need a way to represent colour in the computer by some set of numbers
  - A) preferably a small set of numbers which can be quantised to a fairly small number of bits each
    - Gamma corrected RGB, sRGB and CMYK for printers
  - B) a set of numbers that are easy to interpret
    - Munsell’s artists’ scheme
    - HSV, HLS
  - C) a set of numbers in a 3D space so that the (Euclidean) distance in that space corresponds to approximately perceptually uniform colour differences
    - CIE Lab, CIE Luv
RGB space

- Most display devices that output light mix red, green and blue lights to make colour
  - televisions, CRT monitors, LCD screens
- Nominally, RGB space is a cube
- The device puts physical limitations on:
  - the range of colours which can be displayed
  - the brightest colour which can be displayed
  - the darkest colour which can be displayed
**RGB in XYZ space**

- CRTs and LCDs mix red, green, and blue to make all other colours.
- The red, green, and blue **primaries** each map to a point in CIE xy space.
- Any colour within the resulting triangle can be displayed.
  - Any colour outside the triangle cannot be displayed.
  - For example: CRTs cannot display very saturated purple, turquoise, or yellow.
CMY space

- printers make colour by mixing coloured inks
- the important difference between inks (CMY) and lights (RGB) is that, while lights emit light, inks absorb light
  - cyan absorbs red, reflects blue and green
  - magenta absorbs green, reflects red and blue
  - yellow absorbs blue, reflects green and red
- CMY is, at its simplest, the inverse of RGB
- CMY space is nominally a cube
**CMYK space**

- In real printing we use black (key) as well as **CMY**

- Why use black?
  - Inks are not perfect absorbers
  - Mixing $C + M + Y$ gives a muddy grey, not black
  - Lots of text is printed in black: trying to align $C, M$ and $Y$ perfectly for black text would be a nightmare
Munsell’s colour classification system

- **three axes**
  - hue ➤ the dominant colour
  - value ➤ bright colours/dark colours
  - chroma ➤ vivid colours/dull colours

- can represent this as a 3D graph
Munsell’s colour classification system

- any two adjacent colours are a standard “perceptual” distance apart
  - worked out by testing it on people
  - a highly irregular space
    - e.g. vivid yellow is much brighter than vivid blue

invented by Albert H. Munsell, an American artist, in 1905 in an attempt to systematically classify colours
Colour spaces for user-interfaces

- **RGB** and **CMY** are based on the physical devices which produce the coloured output.
- **RGB** and **CMY** are difficult for humans to use for selecting colours.
- Munsell’s colour system is much more intuitive:
  - hue — what is the principal colour?
  - value — how light or dark is it?
  - chroma — how vivid or dull is it?
- Computer interface designers have developed basic transformations of **RGB** which resemble Munsell’s human-friendly system.
**HSV: hue saturation value**

- three axes, as with Munsell
  - hue and value have same meaning
  - the term “saturation” replaces the term “chroma”

- designed by Alvy Ray Smith in 1978
- algorithm to convert *HSV* to *RGB* and back can be found in Foley et al., Figs 13.33 and 13.34
**HLS: hue lightness saturation**

† a simple variation of *HSV*
  † hue and saturation have same meaning
  † the term “lightness” replaces the term “value”

† designed to address the complaint that *HSV* has all pure colours having the same lightness/value as white
  † designed by Metrick in 1979
  † algorithm to convert *HLS* to *RGB* and back can be found in Foley et al., Figs 13.36 and 13.37
Perceptual uniformity

- MacAdam ellipses & visually indistinguishable colours

In CIE xy chromatic coordinates

In CIE u’v’ chromatic coordinates
CIE L*u*v* and u’v’

- Approximately perceptually uniform
- u’v’ chromacity
  \[ u' = \frac{4X}{X + 15Y + 3Z} \]
  \[ v' = \frac{9Y}{X + 15Y + 3Z} \]

CIE LUV

- Lightness
  \[ L^* = \begin{cases} 
    \left( \frac{29}{3} \right)^3 \frac{Y}{Y_n}, & Y/Y_n \leq \left( \frac{6}{29} \right)^3 \\
    116(Y/Y_n)^{1/3} - 16, & Y/Y_n > \left( \frac{6}{29} \right)^3 
  \end{cases} \]

- Chromacity coordinates
  \[ u^* = 13L^* \cdot (u' - u'_n) \]
  \[ v^* = 13L^* \cdot (v' - v'_n) \]

- Hue and chroma
  \[ C_{uv} = \sqrt{(u^*)^2 + (v^*)^2} \]
  \[ h_{uv} = \text{atan2}(v^*, u^*) \]

- Colours less distinguishable when dark

sRGB in CIE L*u*v*
CIE L*a*b* colour space

- Another approximately perceptually uniform colour space

\[ L^* = 116f \left( \frac{Y}{Y_n} \right) - 16 \]
\[ a^* = 500 \left( f \left( \frac{X}{X_n} \right) - f \left( \frac{Y}{Y_n} \right) \right) \]
\[ b^* = 200 \left( f \left( \frac{Y}{Y_n} \right) - f \left( \frac{Z}{Z_n} \right) \right) \]

\[ f(t) = \begin{cases} \sqrt{t} & \text{if } t > \delta^3 \\ \frac{t}{3\delta^2} + \frac{4}{29} & \text{otherwise} \end{cases} \]
\[ \delta = \frac{6}{29} \]

- Chroma and hue

\[ C^* = \sqrt{a^{*2} + b^{*2}}, \quad h^\circ = \arctan \left( \frac{b^*}{a^*} \right) \]
**Lab space**

- this visualization shows those colours in *Lab* space which a human can perceive

- again we see that human perception of colour is not uniform
  - perception of colour diminishes at the white and black ends of the $L$ axis
  - the maximum perceivable chroma differs for different hues
Colour - references

- Chapters „Light” and „Colour” in

- Textbook on colour appearance

- Comprehensive review of colour research