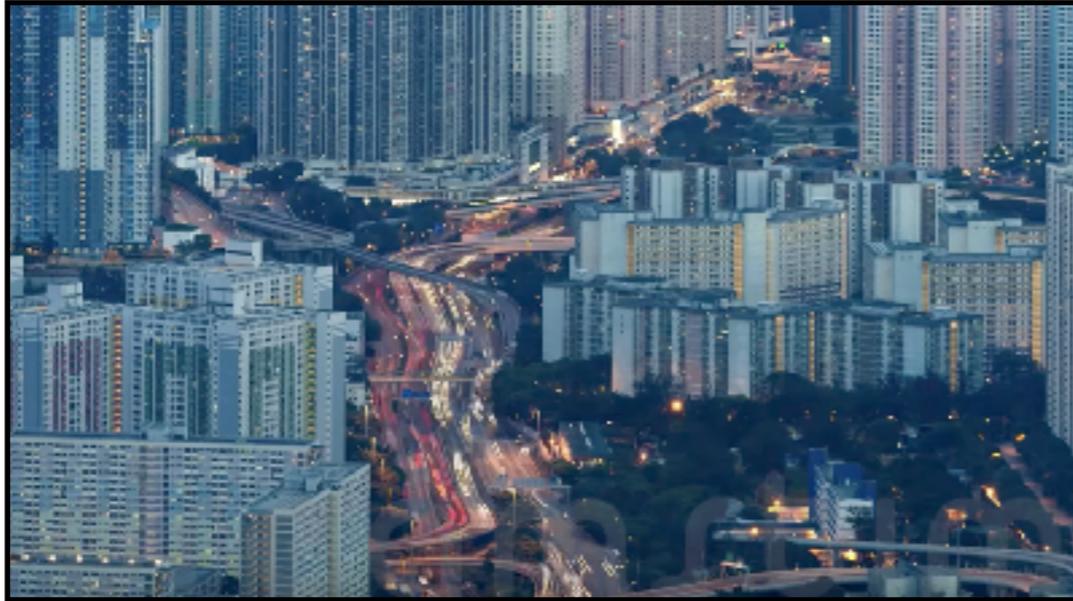


# Mobile and Sensor Systems

Lecture 10: Mobile Robots, Control, and  
Coordination in Robot Teams

Dr. Amanda Prorok

# Robots and Mobile Systems



smart infrastructure / mobility-on-demand



connected vehicles / automated highways



drone swarms / surveillance



truck platoons / long-haul transport

# In this Lecture

- Overview of mobile robot control
  - ▶ Basic principles of kinematics
  - ▶ Overview of classical control architectures
- Coordination in systems with multiple robots
  - ▶ Taxonomy
  - ▶ Distributed estimation
  - ▶ Distributed control

# Autonomous Robots

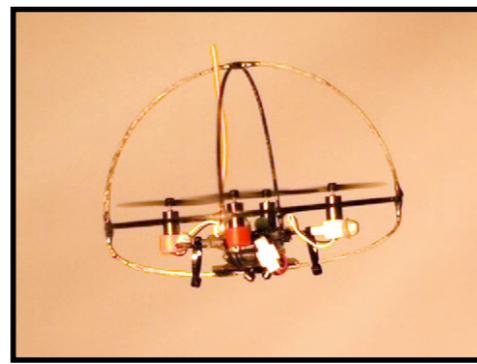
- What is a robot?



microrobots  
[Wood, Harvard]



self-foldable / self-actuated  
[Sung and Rus; MIT]



lightweight aerial robots  
[Kumar et al.; UPenn]



consumer-grade drones



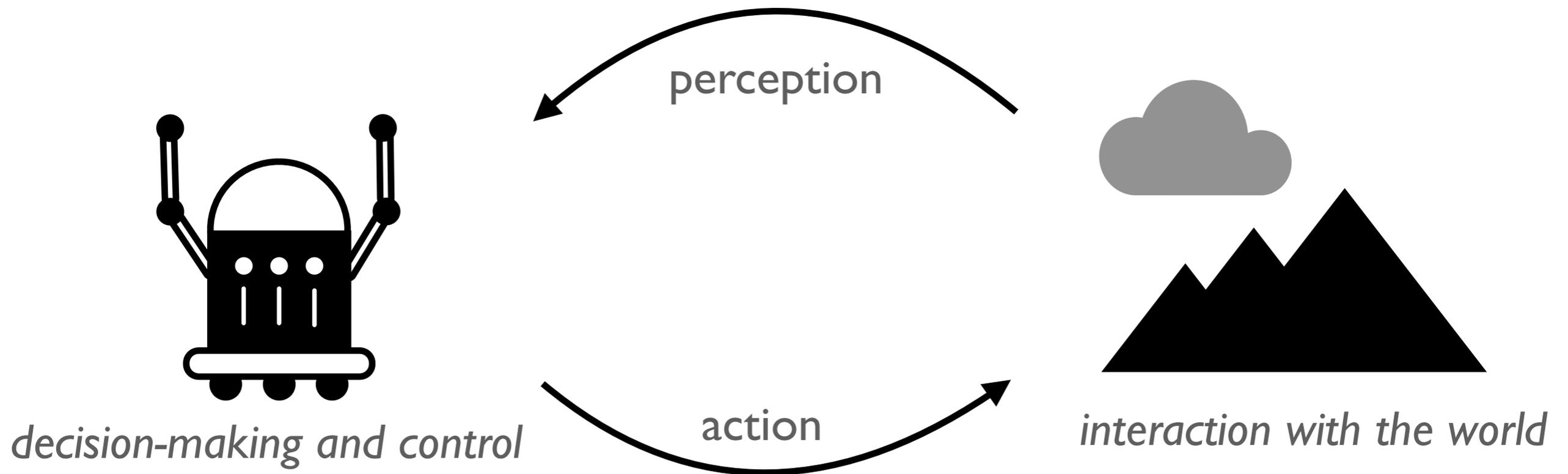
autonomous vehicles  
[Google]

- Challenges:

- ▶ How to model and perceive the world?
- ▶ How to process information and exert control?
- ▶ How to reason and plan in the face of uncertainty?

# Perception-Action Loop

- Basic building block of autonomy!



Three main variants:

1. Reactive (e.g., nonlinear transform of sensor readings)
2. Reactive + memory (eg., filter, state variables)
3. Deliberative (e.g., planning)

# Sensors for Robots

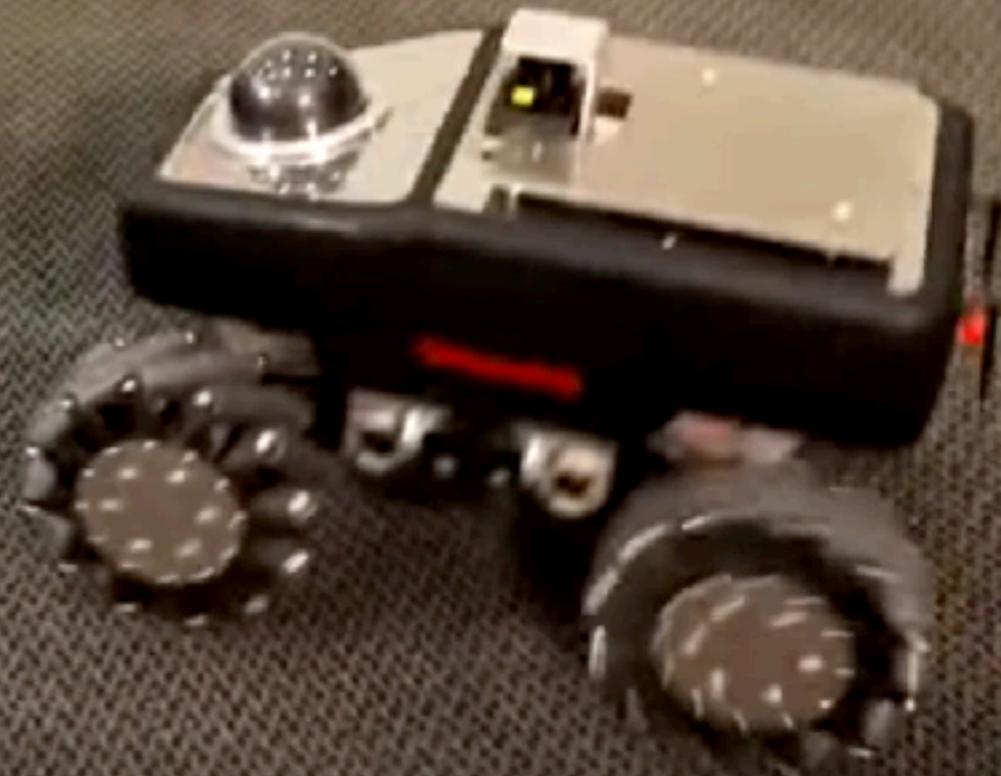
- Proprioceptive vs. exteroceptive
  - ▶ **Proprioceptive:** “*body*” sensors, e.g., motor speed, battery voltage, joint angle
  - ▶ **Exteroceptive:** “*environment*” sensors, e.g., distance measurement, light intensity
- Passive vs. active
  - ▶ **Passive:** “*measure ambient energy*”, e.g., temperature probes, cameras, microphones
  - ▶ **Active:** “*emit energy, and measure the environmental reaction*”, e.g., infrared proximity sensors, ultrasound sensors

# Sensor and Actuators

- Actuators
  - ▶ For different purposes: e.g., locomotion, control of a body part, heating, sound emission.
  - ▶ Examples of electrical-to-mechanical actuators: DC motors, stepper motors, servos, loudspeakers.
- Uncertainty and disturbances
  - ▶ Causes for actuation noise: e.g., wheel slip, slack in mechanism
  - ▶ Causes for sensor noise: e.g., environmental factors, cheap circuitry

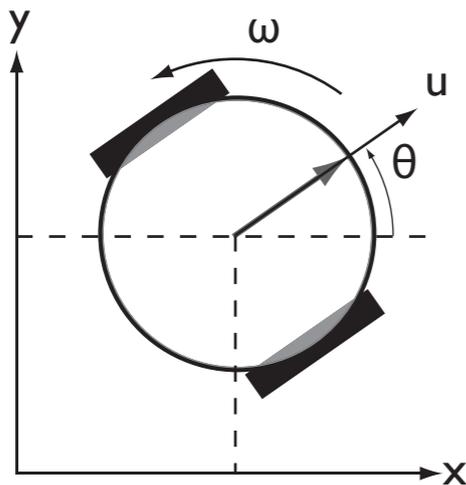
# Degrees of Freedom

- Most actuators control a single degree of freedom (DOF)
  - ▶ a motor shaft controls one rotational DOF
  - ▶ a sliding part on a plotter controls one translational DOF
- Every robot has a specific number of DOF
- If there is an actuator for every DOF, then all of the DOF are controllable
- Usually not all DOF are controllable
  - ▶ **Holonomic robot:** When the number of controllable DOF is equal to robot's total DOF
  - ▶ **Non-holonomic robot:** When the number of controllable DOF is less than robot's total DOF
  - ▶ When it is larger, the robot is 'redundant'



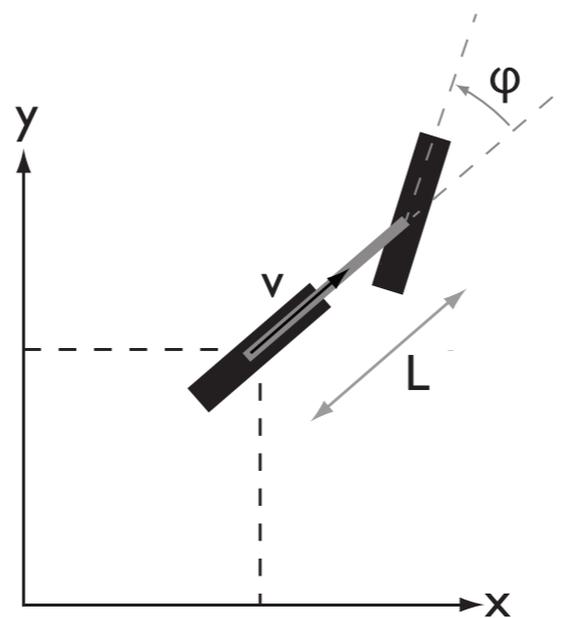
# Forward Kinematics

- Differential equations describe robot motion
- How does robot state change over time as a function of control inputs?



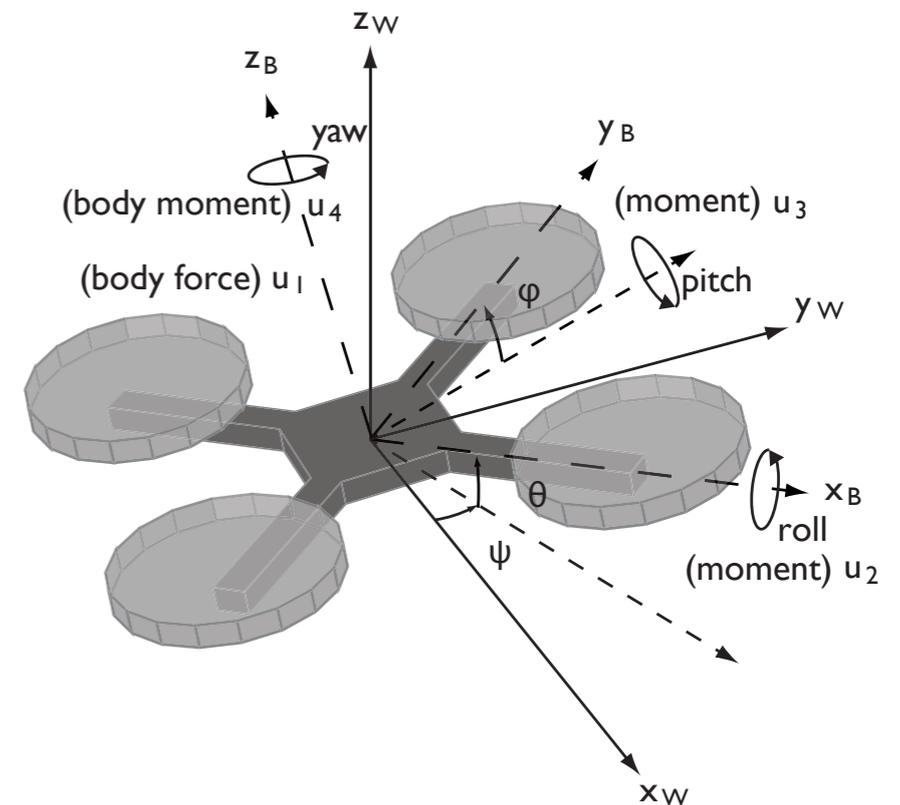
$$\begin{cases} \dot{x} = u \cdot \cos \theta \\ \dot{y} = u \cdot \sin \theta \\ \dot{\theta} = \omega \end{cases}$$

differential-drive  
3 DOF (2 controllable)



$$\begin{cases} \dot{x} = v \cdot \cos \theta \\ \dot{y} = v \cdot \sin \theta \\ \dot{\theta} = v \cdot \frac{\tan \phi}{L} \end{cases}$$

Bicycle  
3 DOF (2 controllable)



$$\begin{cases} \ddot{\mathbf{r}} = -g\mathbf{z}_W + \frac{u_1}{m}\mathbf{z}_B \\ \dot{\boldsymbol{\omega}} = I^{-1} \left( -\boldsymbol{\omega} \times I\boldsymbol{\omega} + \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} \right) \end{cases}$$

inertia matrix

Quadrotor  
6 DOF (4 controllable)

# Forward Kinematics (body frame)

Actuators of differential-drive:

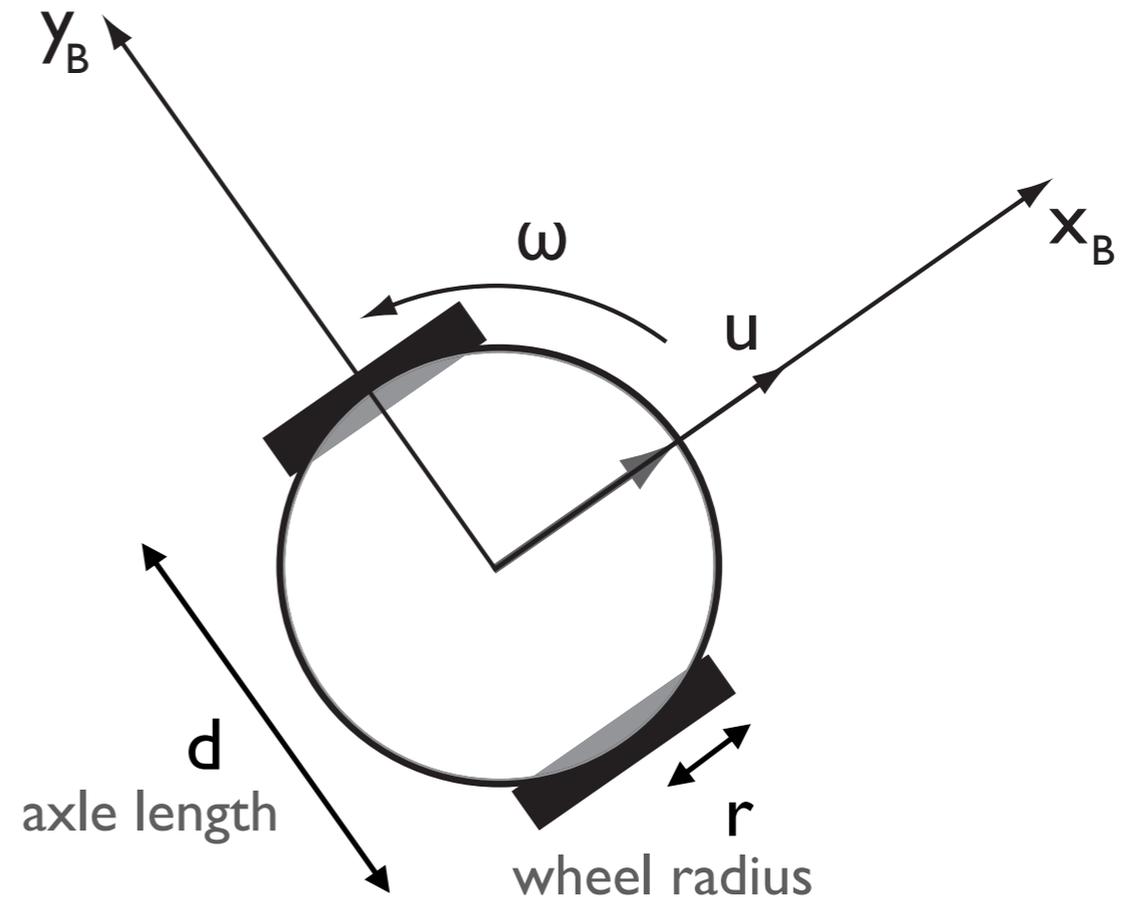
- Left wheel speed  $\dot{\phi}_l$
- Right wheel speed  $\dot{\phi}_r$

Forward velocity:

$$u = \frac{r\dot{\phi}_r}{2} + \frac{r\dot{\phi}_l}{2}$$

Rotational velocity:

$$\omega = \frac{r\dot{\phi}_r}{d} - \frac{r\dot{\phi}_l}{d}$$



Motion:

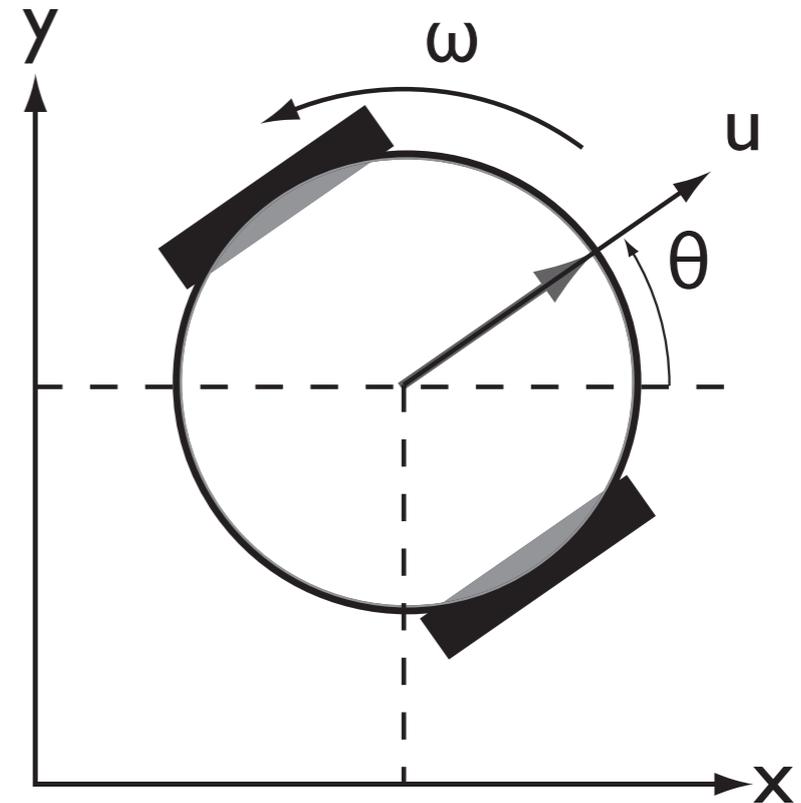
$$\dot{x}_B = u$$

$$\dot{y}_B = 0$$

$$\dot{\theta}_B = \omega$$

# Forward Kinematics (world frame)

- Rotation of coordinates
  - From body to world frames, the axes rotate by  $\theta$



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T(\theta)} \begin{bmatrix} \dot{x}_B \\ \dot{y}_B \\ \dot{\theta}_B \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 0 \\ \omega \end{bmatrix} = \begin{bmatrix} u \cos \theta \\ u \sin \theta \\ \omega \end{bmatrix}$$

# Inverse Kinematics I

- We would like to control the robot velocities:  $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$
- We inverse the previous equations:

$$\begin{bmatrix} u \\ 0 \\ \omega \end{bmatrix} = T^{-1}(\theta) \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

- yielding 
$$\begin{aligned} u &= \dot{x} \cos \theta + \dot{y} \sin \theta \\ \omega &= \dot{\theta} \end{aligned}$$
- under the constraint (remember than our robot is non-holonomic):

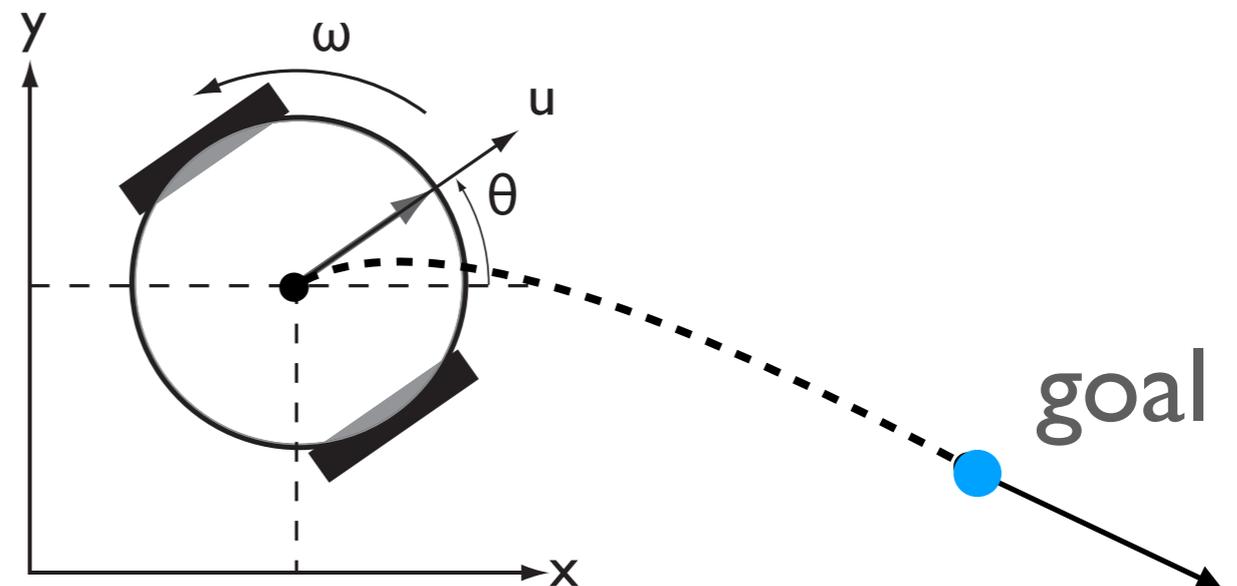
$$\dot{x} \sin \theta = \dot{y} \cos \theta$$

- and finally 
$$\begin{aligned} \dot{\phi}_l &= u - \frac{\omega d}{2r} & \implies & \dot{\phi}_l = \dot{x} \cos \theta + \dot{y} \sin \theta - \frac{\dot{\theta} d}{2r} \\ \dot{\phi}_r &= u + \frac{\omega d}{2r} & & \dot{\phi}_r = \dot{x} \cos \theta + \dot{y} \sin \theta + \frac{\dot{\theta} d}{2r} \end{aligned}$$

# Inverse Kinematics II

- We would like to control the robot to reach a goal pose:  $\begin{bmatrix} x_G \\ y_G \\ \theta_G \end{bmatrix}$
- Ideally (if the robot would be holonomic), we would set

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = K \begin{bmatrix} x_G - x \\ y_G - y \\ \theta_G - \theta \end{bmatrix}$$



- To satisfy our constraint, we need to be creative. Cubic Bézier curves, for example, would satisfy our constraint if we set

$$\mathbf{p}_1 = \begin{bmatrix} x \\ y \end{bmatrix} \quad \mathbf{p}_2 = \begin{bmatrix} x + K_1 \cos \theta \\ y + K_1 \sin \theta \end{bmatrix} \quad \mathbf{p}_3 = \begin{bmatrix} x_G + K_2 \cos \theta_G \\ y_G + K_2 \sin \theta_G \end{bmatrix} \quad \mathbf{p}_4 = \begin{bmatrix} x_G \\ y_G \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{B}(t | \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) \quad \text{with curvature: } \dot{\theta} = \frac{\dot{x}\ddot{y} - \ddot{x}y}{\dot{x}^2 + \dot{y}^2}$$

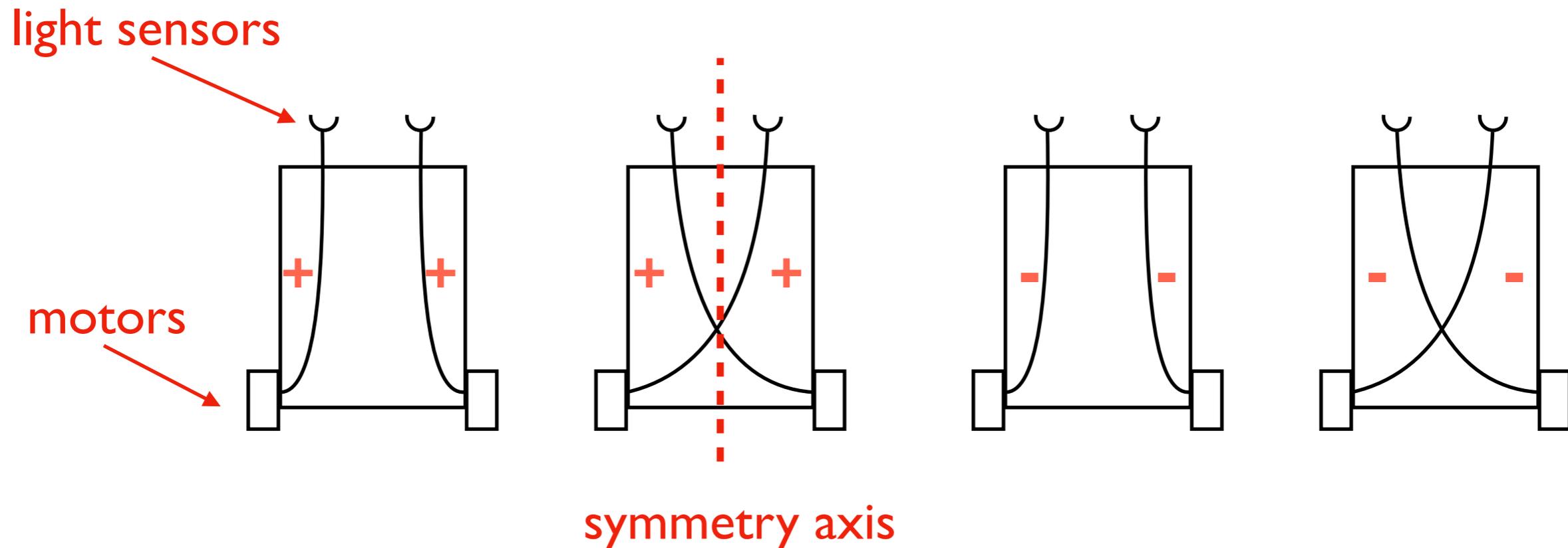
# Closed-Loop vs Open-Loop

- Once we have a path that enables the robot to reach its goal, we need to follow that path:
  - ▶ **Open-loop:** Robot follows path blindly by applying the pre-computed control inputs
  - ▶ **Closed-loop:** Robot can follow path for a small duration, then observe if anything changed in the world, recompute a new adapted path (repeatedly)
- Closed-loop is much more robust to external perturbations:
  - ▶ Noisy sensors: wrong estimate of the goal position, wrong estimate of the robot position.
  - ▶ Unforeseen events, dynamic obstacles, e.g., someone walks in front of the robot.

# Control Architectures

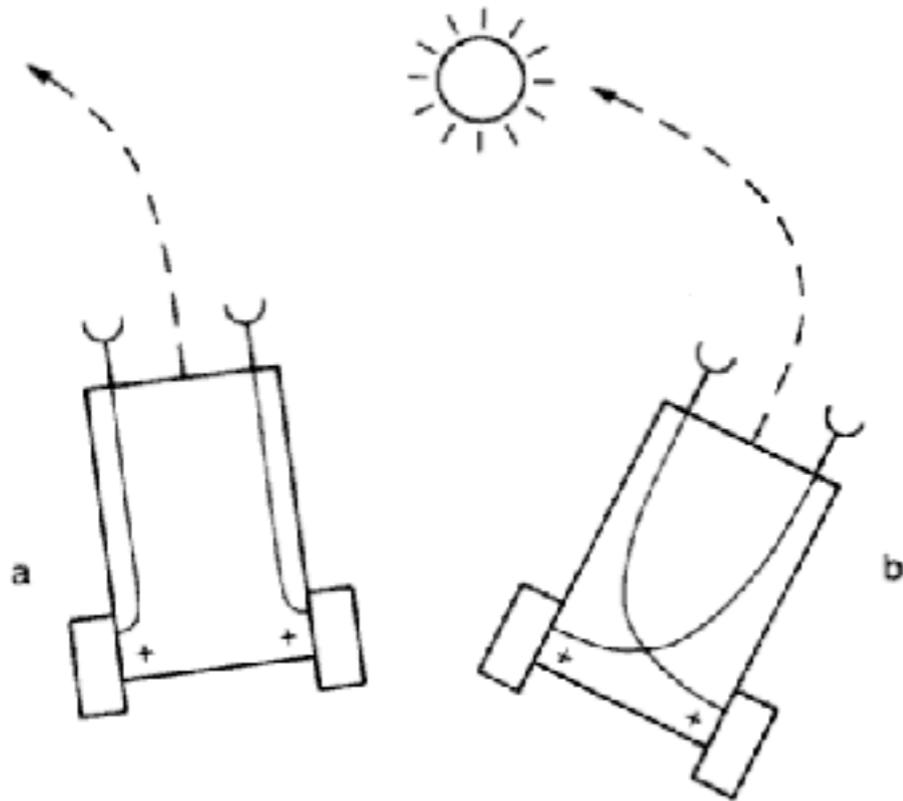
- Sensing: proximal vs. distal
  - ▶ **Proximal** architectures are close to sensor input (e.g., Braitenberg; ANN), whereas **distal** are composed of behavioral blocks (e.g., rule-based, motor-schema).
- Planning: reactive vs. deliberative
  - ▶ **Reactive:** control uses current estimate of world, time-invariant rules produce action; simple and fast to compute
  - ▶ **Deliberative:** predictions of future states are made; sequences of actions are planned that minimize some metric (e.g., collisions, energy consumption); computationally involved

# Braitenberg Vehicle I

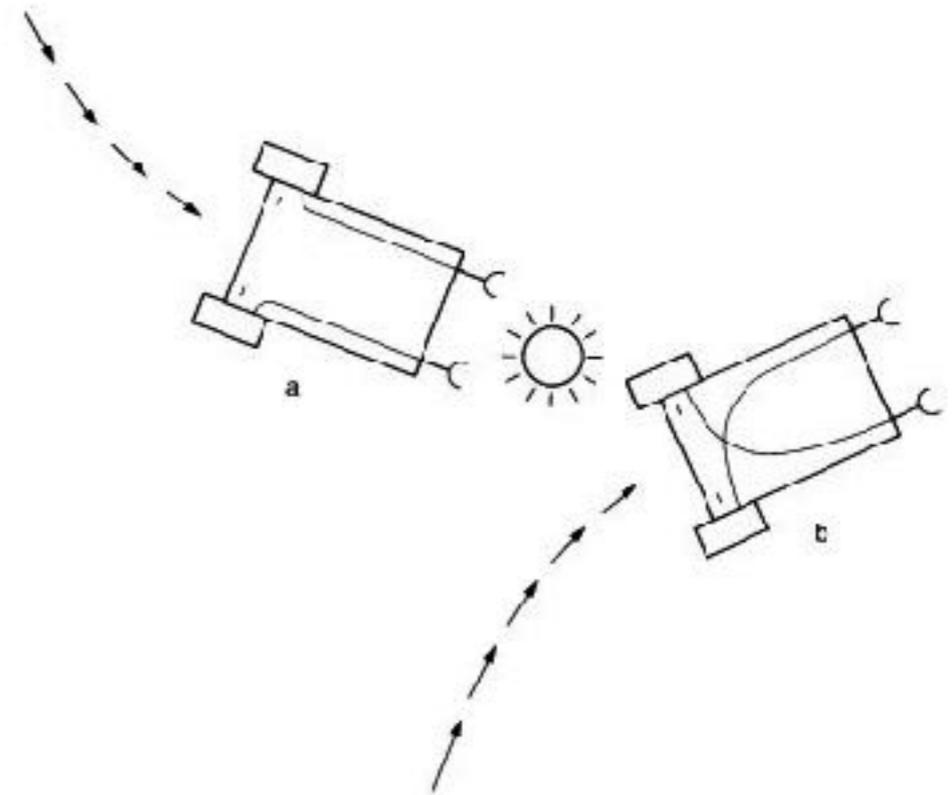


- Difference (gradient) between sensors (across symmetry axis)
- Sensors can (+) excite or (-) inhibit motors
- Original idea worked with light sensors

# Braitenberg Vehicle II

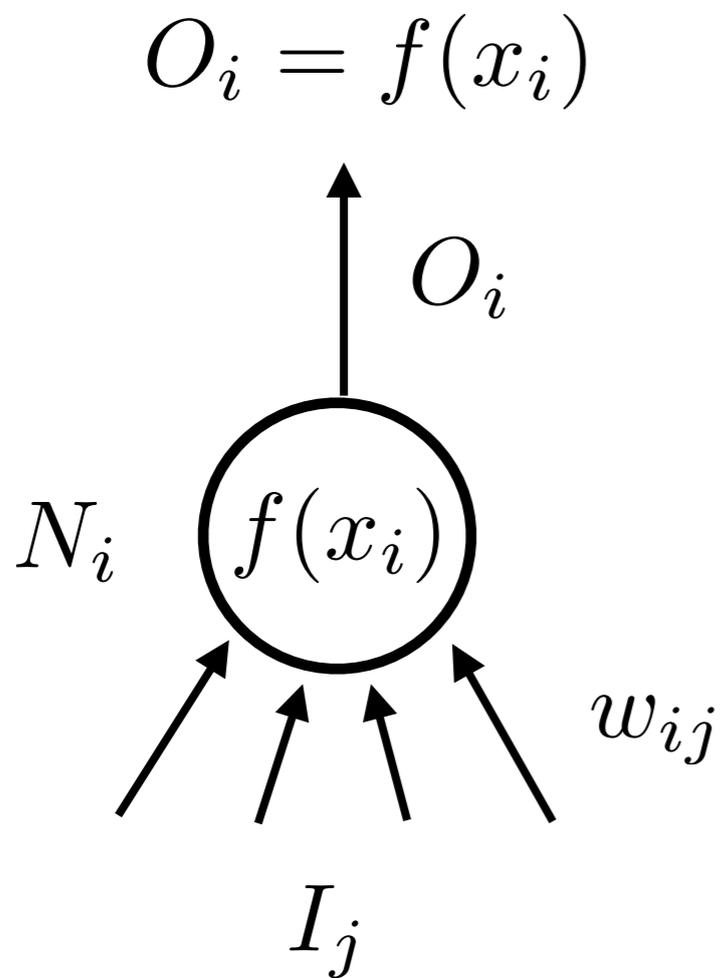


Excitatory connections



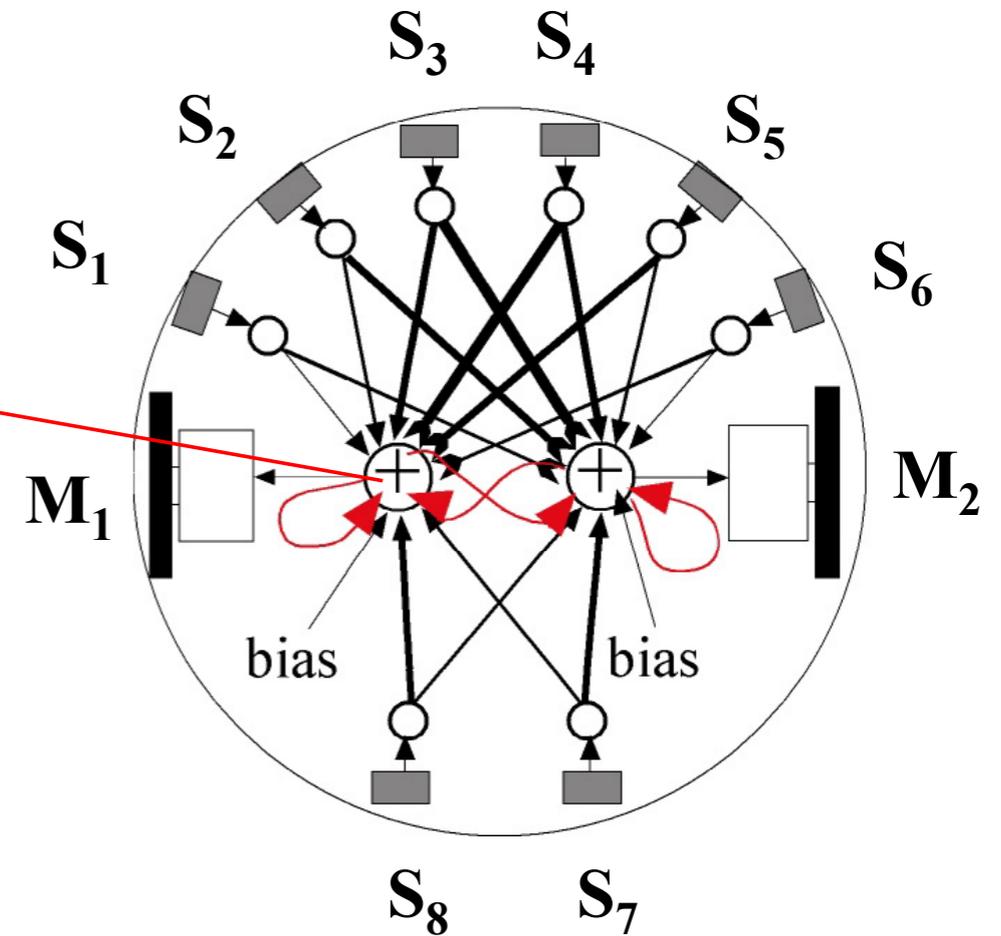
Inhibitory connections

# Neural Network



$$f(x) = \tanh(x)$$

neuron  $N_i$  with transfer function  $f$



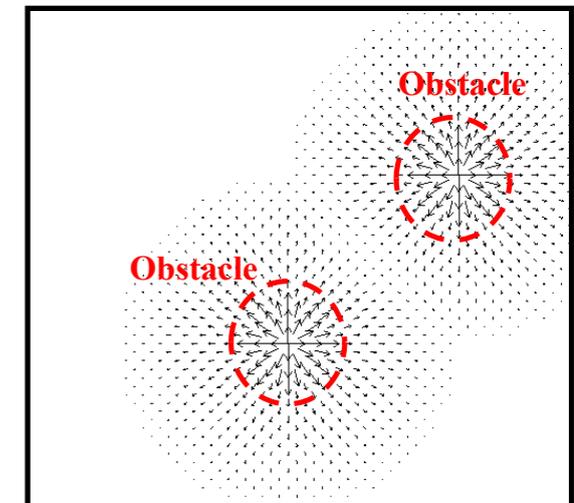
$$x_i = \sum_{j=1}^m w_{ij} I_j + I_0$$

# Rule-Based

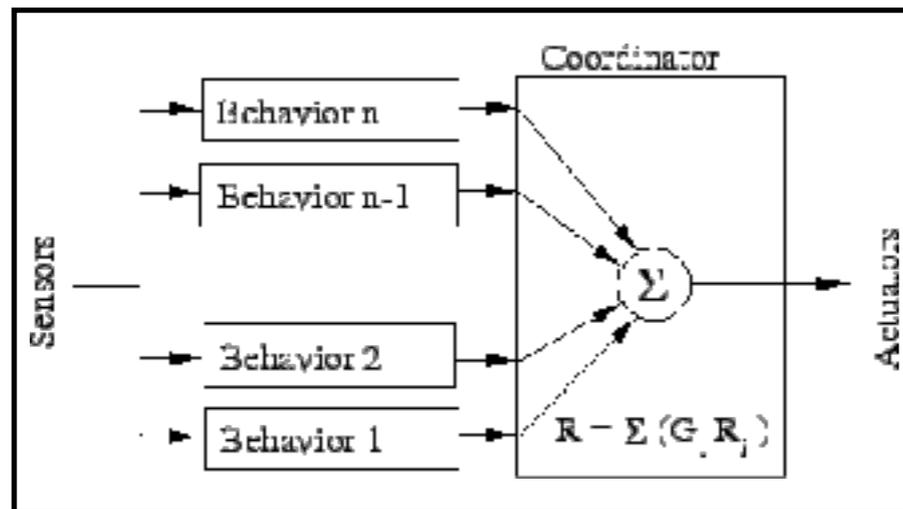
```
forever do:  
  rule 1:  
    if (proximity sensors on left active) then:  
      turn right  
  rule 2:  
    if (proximity sensors on right active) then:  
      turn left  
  rule 3:  
    if (no proximity sensors active) then:  
      move forwards
```

# Other Classical Paradigms

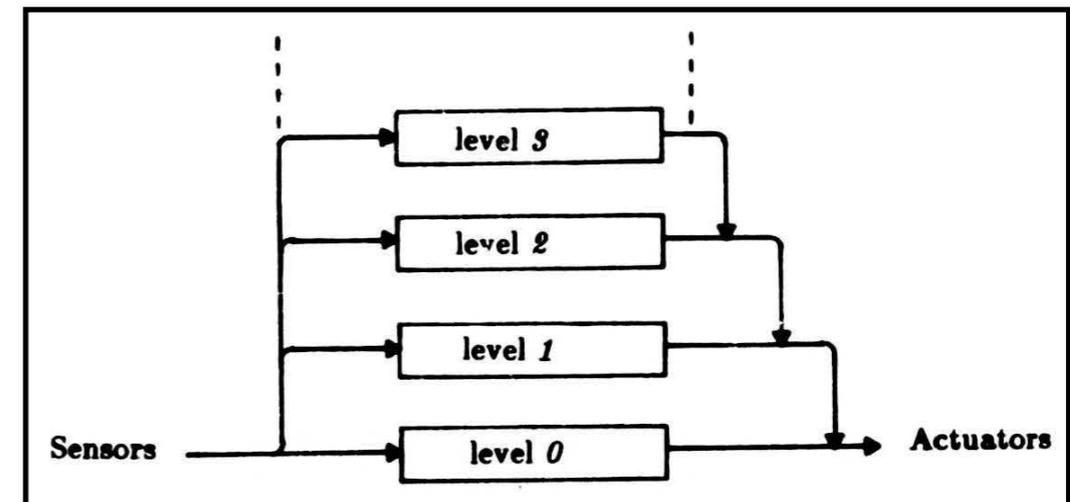
- Potential Field (Khatib, 1986)
- Motor Schema (Arkin, 1989)
- Subsumption Architecture (Brooks, 1986)



potential field



motor schema



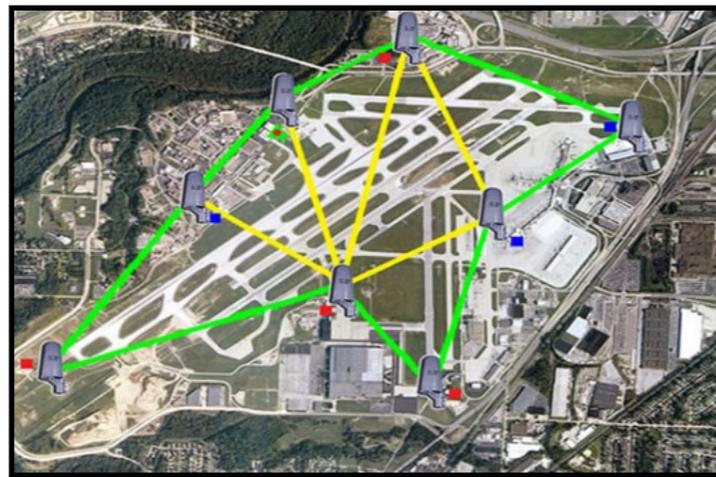
subsumption architecture

# Multi-Robot Systems

- Terms used: robot swarms / robot teams / robot networks
- Why?
  - Distributed nature of many problems
  - Overall performance greater than sum of individual efforts
  - Redundancy
- Numerous commercial, civil, military applications



search & rescue



surveillance / monitoring

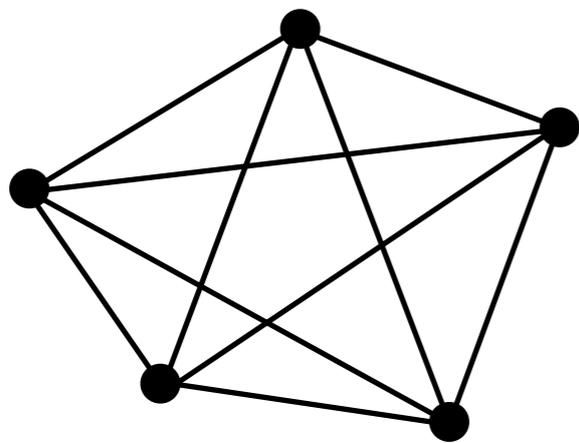


product pickup / delivery

# Taxonomy

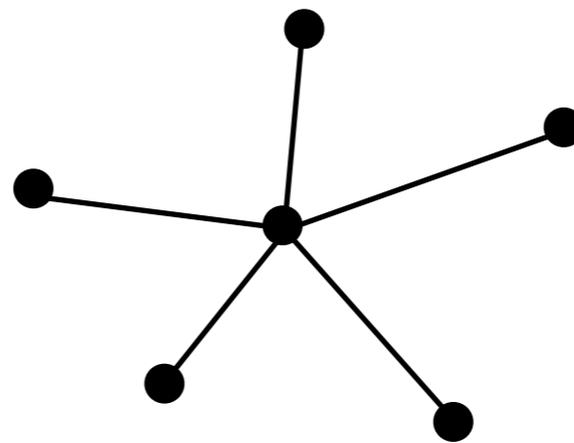
- Architecture: centralized vs. decentralized
  - ▶ **Centralized:** one control/estimation unit communicates with all robots to issue commands; requires synchronized, reliable communication channels; single-point failures
  - ▶ **Decentralized:** scalable, robust to failure; often asynchronous; sub-optimal performance (w.r.t centralized)
- Communication: explicit vs. implicit
  - ▶ **Implicit:** observable states; information exchanged through observation
  - ▶ **Explicit:** unobservable states; need to be communicated explicitly
- Heterogeneity: homogenous vs. heterogeneous
  - ▶ Robot teams can leverage inter-robot complementarities

# Communication Topologies



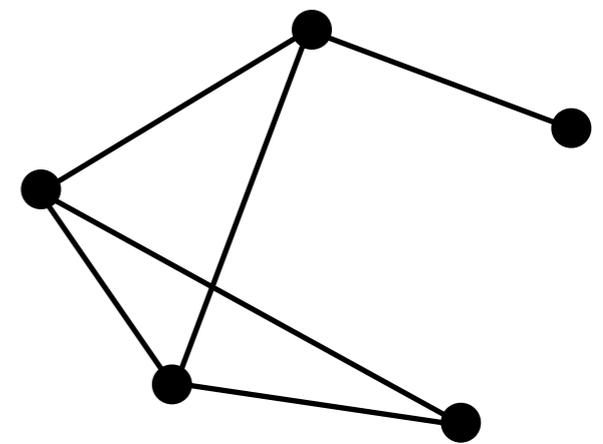
fully connected

centralized / decentralized  
coordination



star topology

centralized / decentralized  
coordination



random mesh

decentralized  
coordination

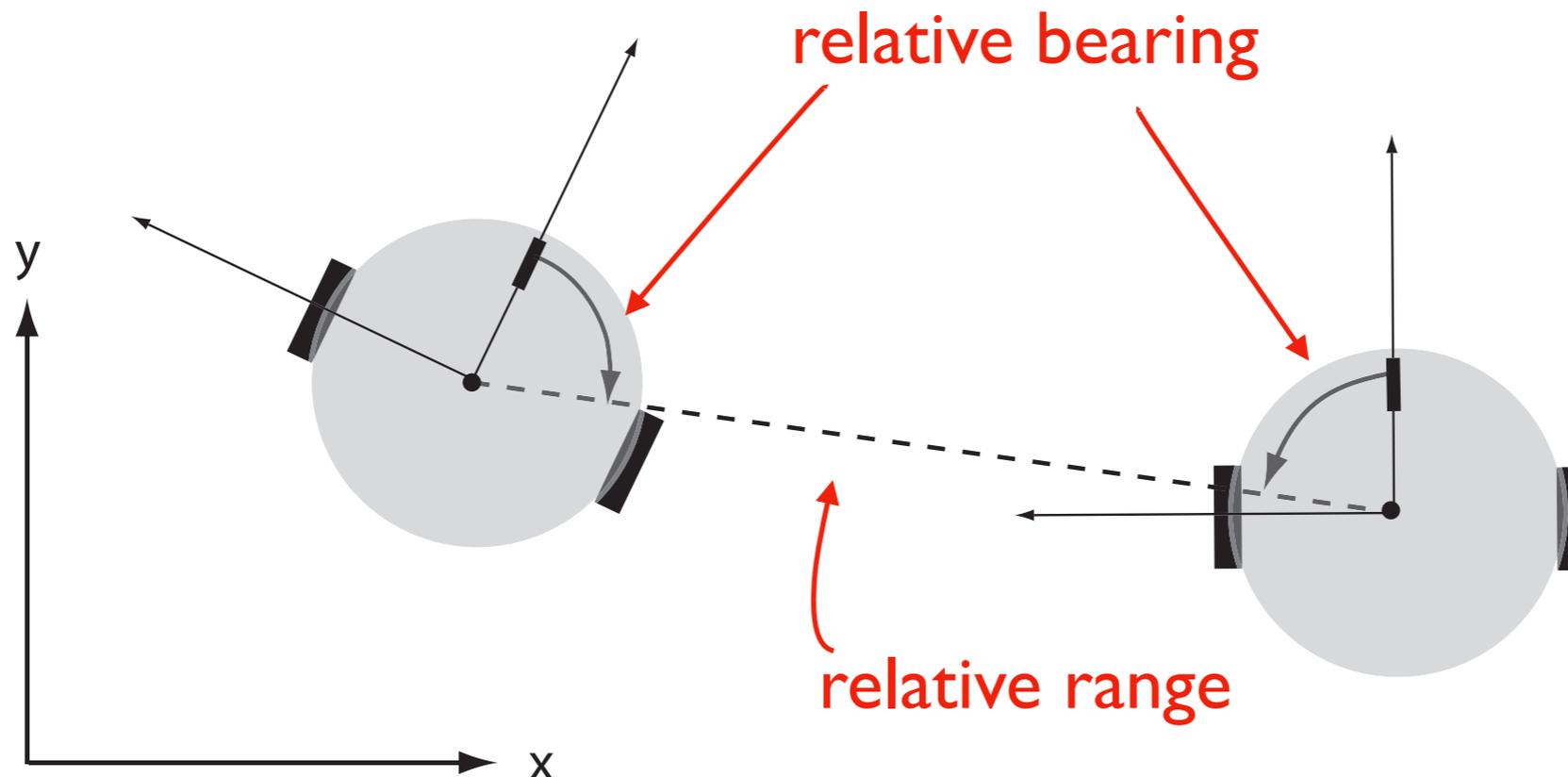
# Decentralization

- Goal: Achieve similar (or same) performance as would be achievable with an ideal, centralized system.
- Challenges:
  - ▶ Communication: delays and overhead
  - ▶ Input: asynchronous; with rumor propagation
  - ▶ Sub-optimality with respect to the centralized solution
- Advantages:
  - ▶ No single-point failure
  - ▶ Can converge to optimum as time progresses
  - ▶ ‘Any-comm’ algorithms exist (with graceful degradation)

# Distributed Estimation

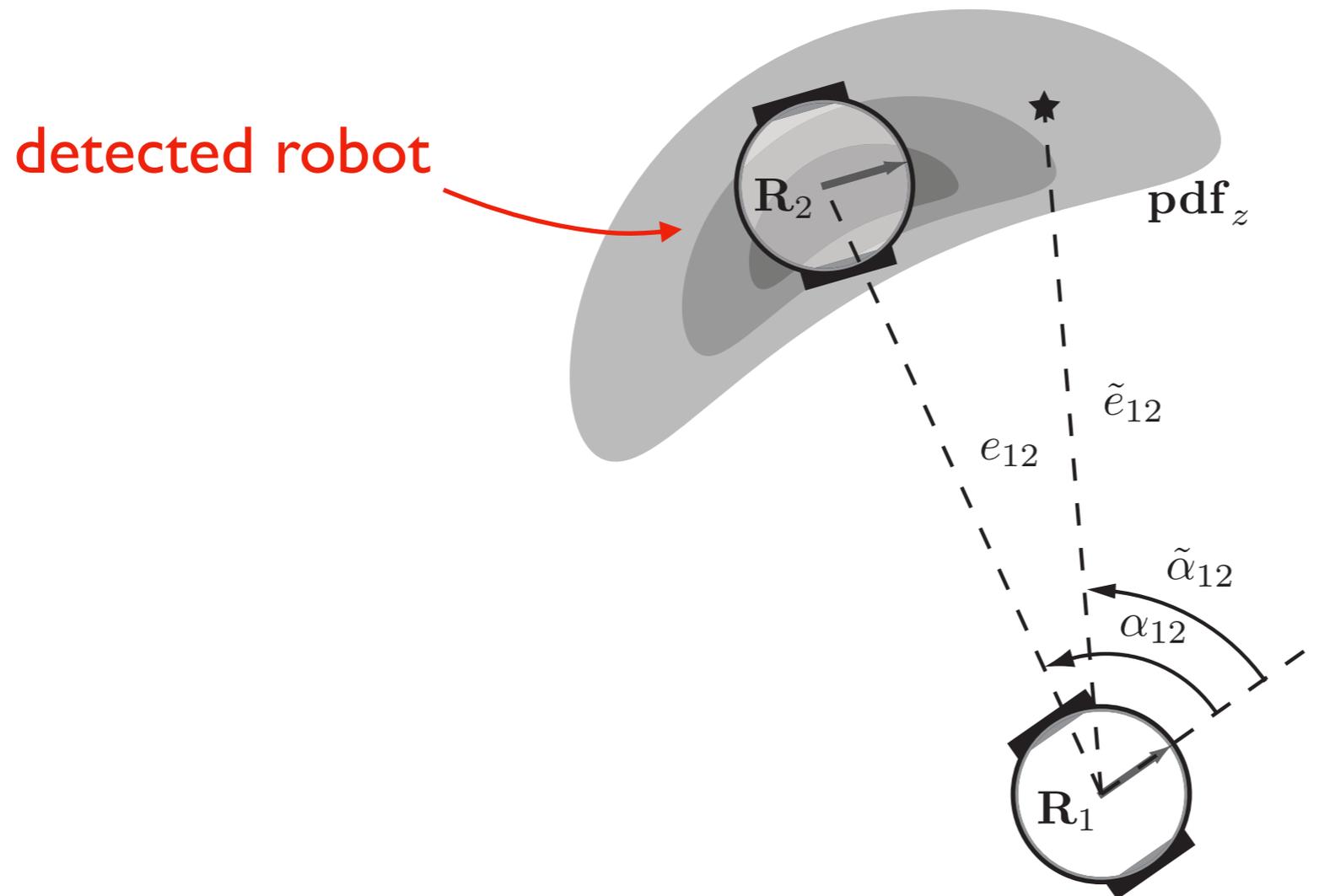
- Goal: Estimate a local or global variable in distributed manner
- Filters can be distributed
  - ▶ Examples: Kalman filter, particle filter
  - ▶ Method: fuse relative observations of other robots
  - ▶ Correct implementation considers relative observations as dependent measurements; the whole history of measurements needs to be tracked (to avoid rumor propagation)!
- Other mechanisms:
  - ▶ Opportunistic mechanisms
  - ▶ Consensus (agreement mechanism)

# Collaborative Localization I



- Collaborative localization uses relative inter-robot observations
- Robots communicate their position estimate
- Fuse relative observation by transforming position into local frame

# Collaborative Localization II

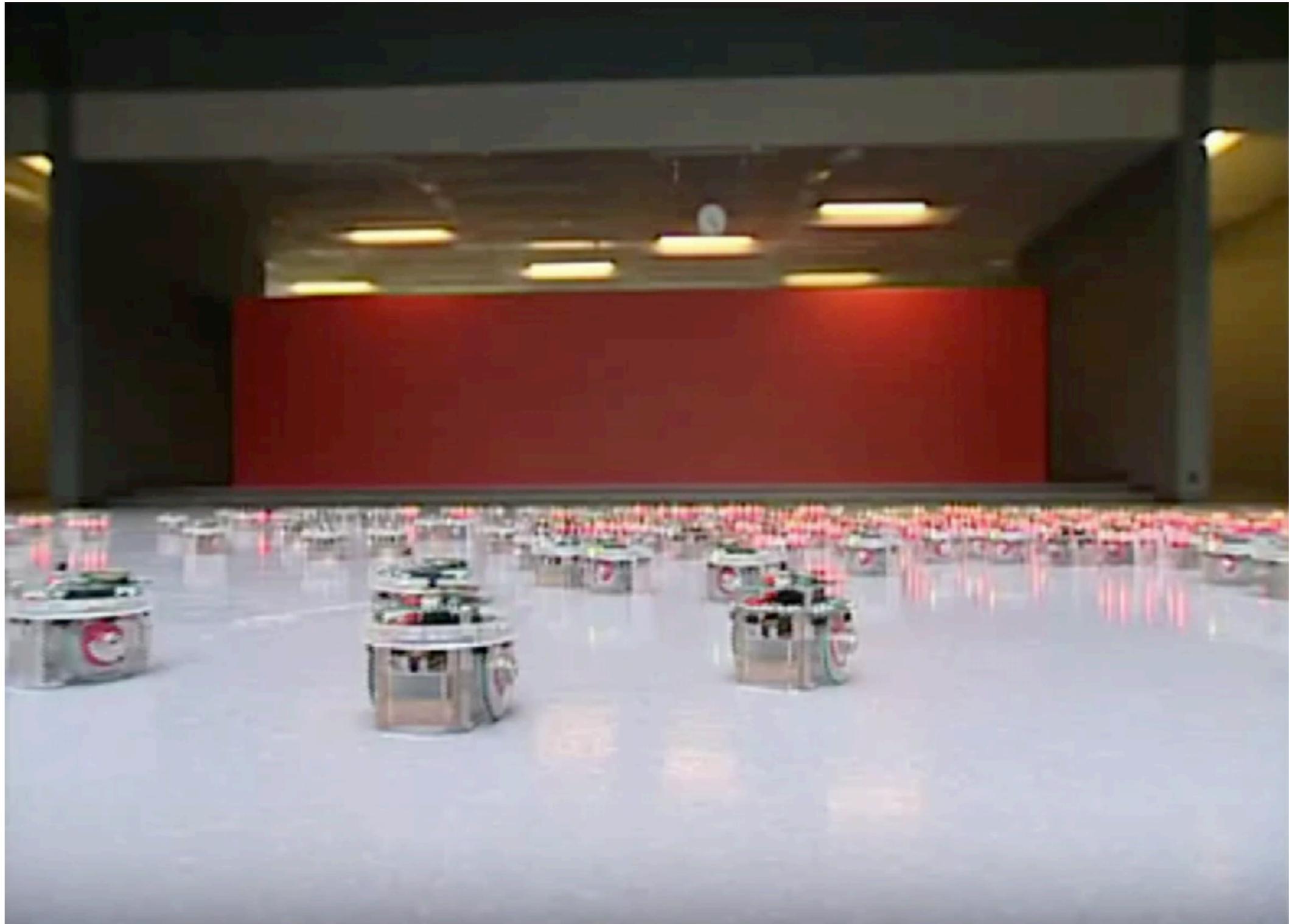


- This example considers a particle filter (Kalman filter also possible)
- Detected robot weights its particles using belief of detecting robot
- Particles re-sampled according to new weights (standard filter)

# Collaborative Localization III

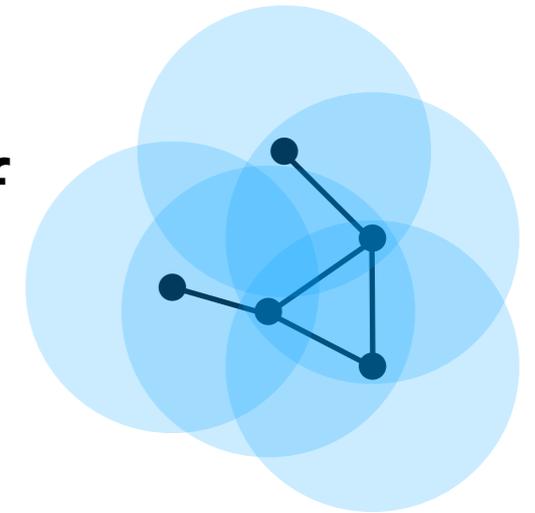


# Coordination



# Distributed Coordination

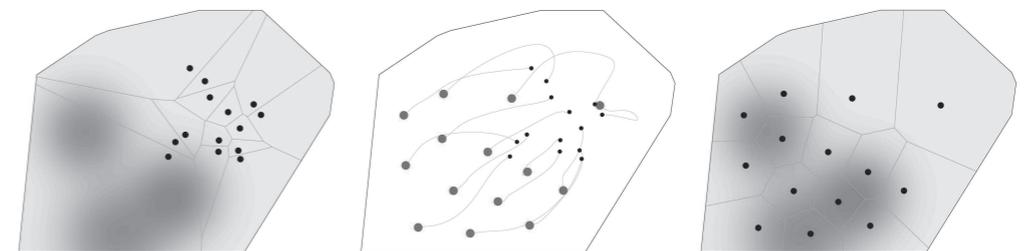
- Coordinated motion: *formations, flocking*
  - ▶ Potential field (sum of local forces)
  - ▶ Network control: Use graph as an abstraction of communication network; use proximity graphs
  - ▶ Leader-follower formations



disc-graph

- Allocation problems: *role / resource distribution*
  - ▶ Market-based algorithms
  - ▶ Threshold-based algorithms

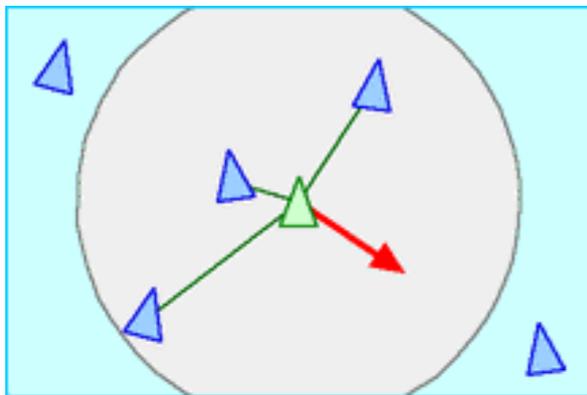
- Coverage: *coverage of spatial areas*
  - ▶ Lloyds algorithm



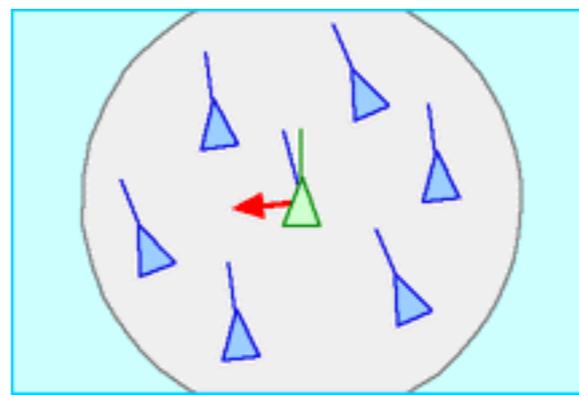
gradient-based coverage control

# Formation Control / Flocking

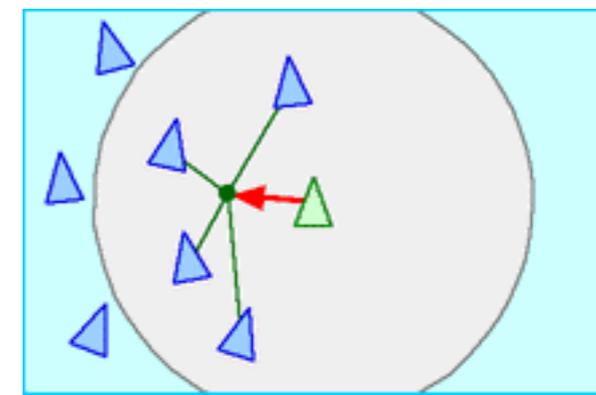
Reynolds' boids (1987)



separation



alignment

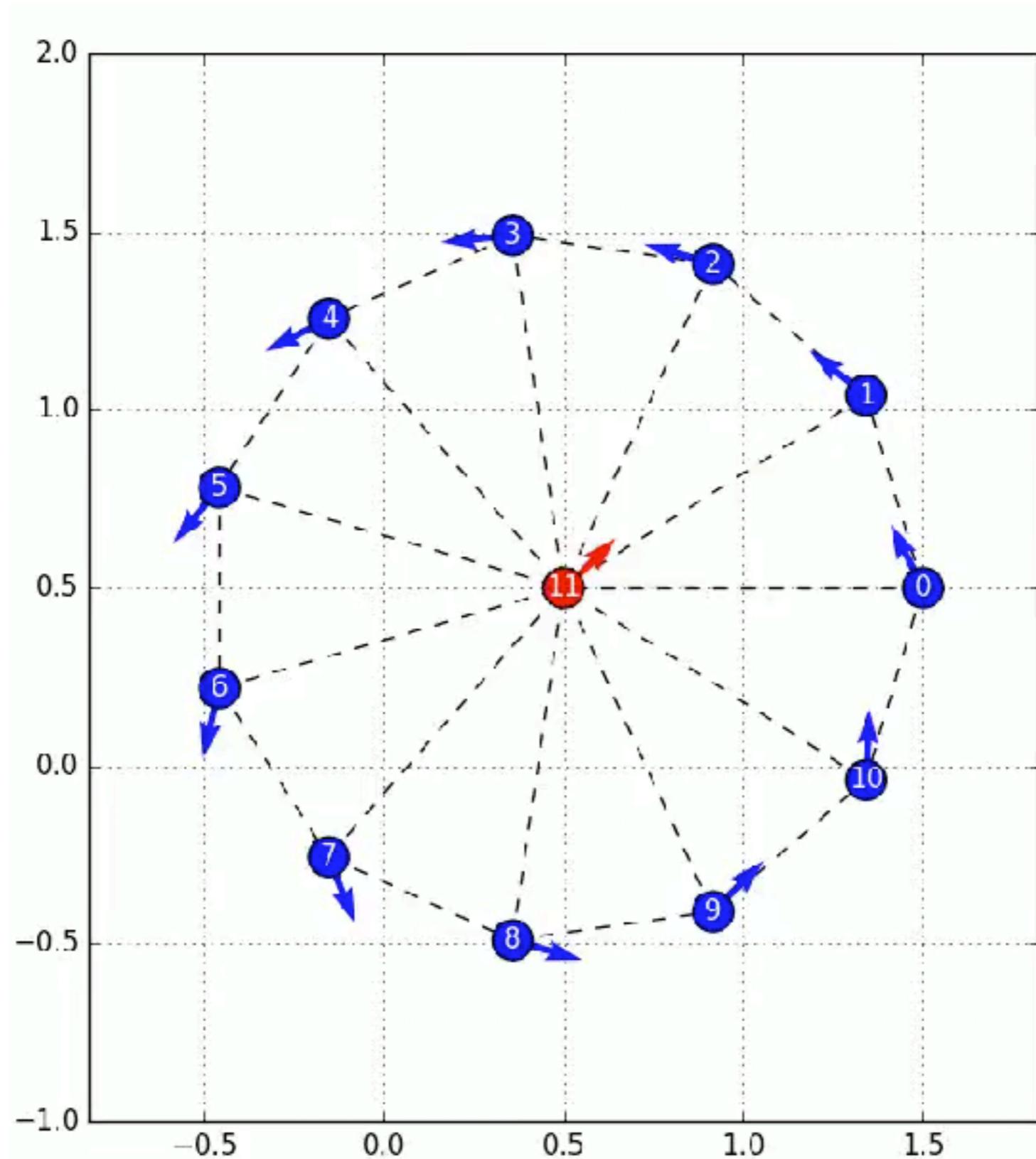


cohesion

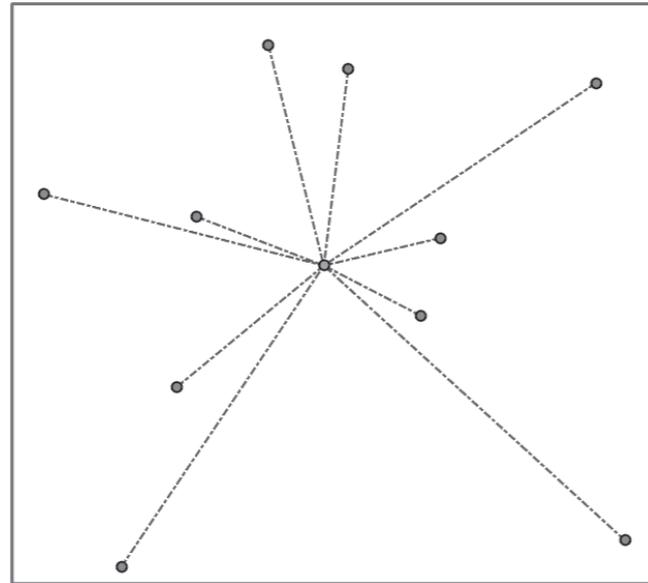
- A boid reacts only to its neighbors
- Neighborhood defined by distance and angle (region of influence)
- Each boid follows 3 steering rules based on positions and velocities of neighbors



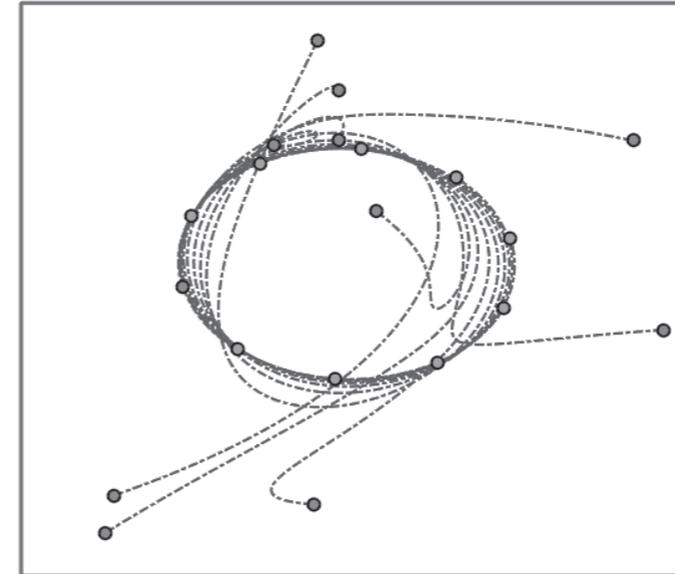
# Consensus for Flocking



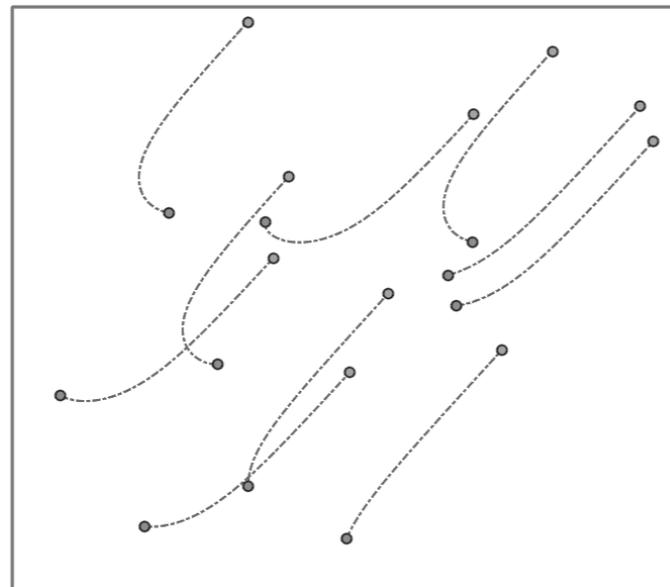
# Applications of Consensus



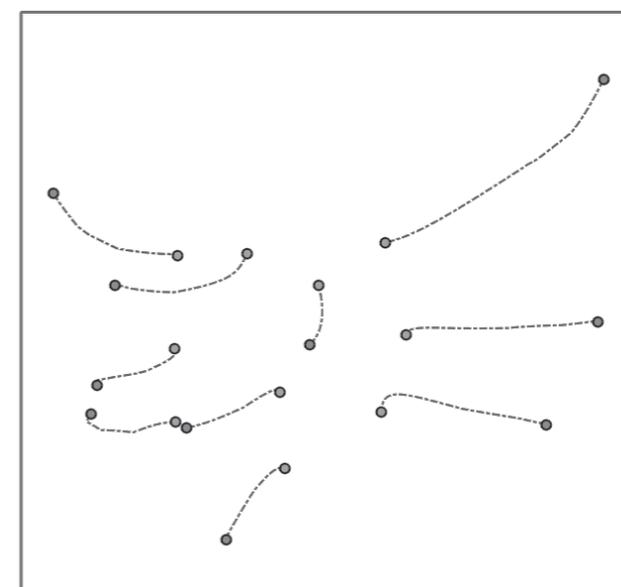
rendezvous



cyclic pursuit



flocking



formation

# Summary

- Mobile robot control
  - ▶ Kinematic principles and control architectures
- Multi-robot systems: estimation and coordination
  - ▶ Collaborative localization as an example
  - ▶ Flocking / formation control as an example
- What we did not talk about (there is much more!):
  - ▶ Noise and uncertainty
  - ▶ Planning algorithms
  - ▶ Learning algorithms (AI)

# References

## Fundamental concepts:

- Elements of Robotics, F Mondada et al., 2018
- Autonomous Mobile Robots, R Siegwart et al., 2004

## State of the art:

- The grand challenges of Science Robotics, *Science*, Yang et al. 2018

## Further reading:

- Probabilistic Robotics, S Thrun et al, 2005
- Springer Handbook of Robotics, B Siciliano et al., 2008
- Graph Theoretic Methods in Multi-agent Networks, Egerstedt et al., 2010

## Seminal papers:

- Motor Schema-Based Mobile Robot Navigation, RC Arkin, 1989
- A Robust Layered Control System for a Mobile Robot, RA Brooks, 1985
- Real-time obstacle avoidance for manipulators and mobile robots, O Khatib, 1986

## **Internships:**

If you are interested - contact me!

asp45@

## **Course:**

Mobile Robot Systems

Lent 2018-19, as part of the new Paper 10

The course will be open to both Part II and Part III students (the max. student number will be capped)