L11: Algebraic Path Problems with applications to Internet Routing Lecture 12

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Properties needed for $(S, \oplus, F)$ to obtain (left) local optima?

Dijkstra's Algorithm

$$\mathbb{INF} \quad \forall a \in S,\ f \in F \ :\ a \leqslant f(a)$$

Proofs from the lecture notes can be extended easily from $\otimes$ to F.

Distributed Bellman-Ford

$$\mathbb{S}.\mathbb{INF} \quad \forall a \in S,\ F \in F \ :\ a \neq \overline{0} \implies a < f(a)$$

In addition, paths with loops must be eliminated. (Proof sketch in last lecture?)

Sobrinho's encoding of the Gao/Rexford rules for BGP

Additive component uses min with

- 0 is the type of a <u>downstream</u> route,
- 1 is the type of a <u>peer</u> route, and
- 2 is the type of an <u>upstream</u> route.
- ∞ is the type of no route.

Sobrinho's encoding ...

Multiplicative component

$\otimes$	0	1	2	∞
0	0	∞	∞	∞
1	1	∞	∞	∞
2	2	2	2	∞
∞	∞	∞	∞	∞

- This is $\mathbb{INF}$, but not associative:

a	b	c	$a \otimes (b \otimes c)$	$(a \otimes b) \otimes c$
2	0	1	∞	2
2	0	2	∞	2
2	1	1	∞	2
2	1	2	∞	2

- Models just the "local preference" component of BGP.
- Can we improve on this with structures of the form $(S, \oplus, F)$?

Stratified Shortest-Paths Metrics

Metrics

$$(s,\ d) \ \text{ or } \ \infty$$

- $s \neq \infty$ is a stratum level in $\{0, 1,\ 2, \ldots,\ m-1\}$,
- d is a "shortest-paths" distance,
- Routing metrics are compared lexicographically

$$(s_1,\ d_1) < (s_2,\ d_2) \iff (s_1 < s_2) \vee (s_1 = s_2 \wedge d_1 < d_2)$$

Stratified Shortest-Paths Policies

Labels have form $(f,\ d)$

$$(f,\ d) \rhd (s,\ d') \equiv \langle f(s),\ d + d' \rangle$$

$$(f,\ d) \rhd (\infty) \equiv \infty$$

where

$$\langle s,\ t\rangle = \begin{cases} \infty & (\text{if } s = \infty) \\ (s,\ t) & (\text{otherwise}) \end{cases}$$

Yes, a reduction!

Constraint on Policies

$$(f,\ d)$$

- Either f is inflationary and $0 < d$,
- or f is strictly inflationary and $0 \leqslant d$.

Why?

$$(\mathbb{S}.\mathbb{INF}(S) \vee (\mathbb{INF}(S) \wedge \mathbb{S}.\mathbb{INF}(T))) \implies \mathbb{S}.\mathbb{INF}(S \mathbin{\vec{\times}_{\overline{0}}} T).$$

Some properties for algebraic structures of the form $(S,\ \oplus,\ F)$

property	definition
$\mathbb{D}$	$\forall a, b \in S,\ f \in F\ :\ f(a \oplus b) = f(a) \oplus f(b)$
$\mathbb{C}$	$\forall a, b \in S,\ f \in F\ :\ f(a) = f(b) \implies a = b$
$\mathbb{C}_{\overline{0}}$	$\forall a, b \in S,\ f \in F\ :\ f(a) = f(b) \implies (a = b \vee f(a) = \overline{0})$
$\mathbb{K}$	$\forall a, b \in S,\ f \in F\ :\ f(a) = f(b)$
$\mathbb{K}_{\overline{0}}$	$\forall a, b \in S,\ f \in F\ :\ f(a) \neq f(b) \implies (f(a) = \overline{0} \vee f(b) = \overline{0})$

All Inflationary Policy Functions for Three Strata

	0	1	2	D	$\mathbb{C}_\infty$	$\mathbb{K}_\infty$		0	1	2	D	$\mathbb{C}_\infty$	$\mathbb{K}_\infty$
a	0	1	2	$\star$	$\star$		**m**	2	1	2			
b	0	1	∞	$\star$	$\star$		**n**	2	1	∞		$\star$	
c	0	2	2	$\star$			**o**	2	2	2	$\star$		$\star$
d	0	2	∞	$\star$	$\star$		**p**	2	2	∞	$\star$		$\star$
e	0	∞	2		$\star$		**q**	2	∞	2			$\star$
f	0	∞	∞	$\star$	$\star$	$\star$	**r**	2	∞	∞	$\star$	$\star$	$\star$
g	1	1	2	$\star$			**s**	∞	1	2		$\star$	
h	1	1	∞	$\star$		$\star$	**t**	∞	1	∞		$\star$	$\star$
i	1	2	2	$\star$			**u**	∞	2	2			$\star$
j	1	2	∞	$\star$	$\star$		**v**	∞	2	∞		$\star$	$\star$
k	1	∞	2		$\star$		**w**	∞	∞	2		$\star$	$\star$
l	1	∞	∞	$\star$	$\star$	$\star$	**x**	∞	∞	∞	$\star$	$\star$	$\star$

Both $\mathbb{D}$ and $\mathbb{C}_{\vec{0}}$

This makes combined algebra distributive!

	0	1	2
a	0	1	2
b	0	1	∞
d	0	2	∞
f	0	∞	∞
j	1	2	∞
l	1	∞	∞
r	2	∞	∞
x	∞	∞	∞

Why?

$$(\mathbb{D}(S) \wedge \mathbb{D}(T) \wedge \mathbb{C}_{\vec{0}}(S)) \implies \mathbb{D}(S \vec{\times}_{\vec{0}} T)$$