L11: Algebraic Path Problems with applications to Internet Routing Lecture 12

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Properties needed for (S, \oplus, F) to obtain (left) local optima?

Dijkstra's Algorithm

 $\mathbb{INF} \quad \forall a \in S, \ f \in F : \ a \leqslant f(a)$

Proofs from the lecture notes can be extended easily from \otimes to *F*.

Distributed Bellman-Ford

S.INF $\forall a \in S, F \in F : a \neq \overline{0} \implies a < f(a)$

In addition, paths with loops must be eliminated. (Proof sketch in last lecture?)

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Sobrinho's encoding of the Gao/Rexford rules for BGP

Additive component uses min with

- 0 is the type of a <u>downstream</u> route,
- 1 is the type of a peer route, and
- 2 is the type of an <u>upstream</u> route.
- ∞ is the type of no route.

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Sobrinho's encoding ...

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Iultiplicative compone	nt					
	\otimes	0	1	2	∞	
	0	0	∞	∞	∞	
		1				
	2	2	2	2	∞	
	∞	∞	∞	∞	∞	

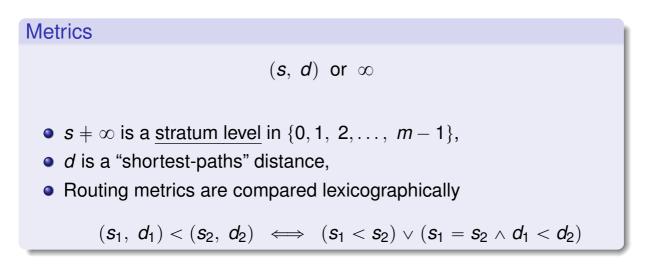
• This is INF, but not associative:

а	b	С	$\pmb{a} \otimes (\pmb{b} \otimes \pmb{c})$	$(\boldsymbol{a} \otimes \boldsymbol{b}) \otimes \boldsymbol{c}$
2	0	1	∞	2
2	0	2	∞	2
2	1	1	∞	2
2	1	2	∞	2

- Models just the "local preference" component of BGP.
- Can we improve on this with structures of the form (S, \oplus, F) ?

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Stratified Shortest-Paths Metrics



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Stratified Shortest-Paths Policies

Labels have form
$$(f, d)$$

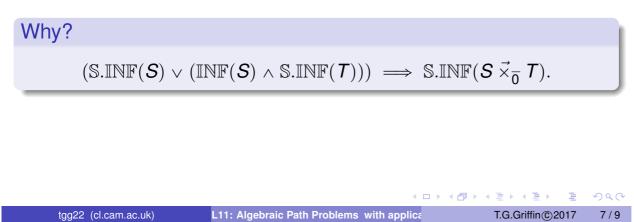
 $(f, d) \triangleright (s, d') \equiv \langle f(s), d + d' \rangle$
 $(f, d) \triangleright (\infty) \equiv \infty$
where
 $\langle s, t \rangle = \begin{cases} \infty & (\text{if } s = \infty) \\ (s, t) & (\text{otherwise}) \end{cases}$
Yes, a reduction!

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Constraint on Policies

(*f*, *d*)

- Either *f* is inflationary and 0 < d,
- or *f* is strictly inflationary and $0 \le d$.



Some properties for algebraic structures of the form (S, \oplus, F)

property	definition
\mathbb{D}	$\forall a, b \in S, f \in F : f(a \oplus b) = f(a) \oplus f(b)$
\mathbb{C}	$\forall a, b \in S, f \in F : f(a) = f(b) \implies a = b$
$\mathbb{C}_{\overline{0}}$	$\forall a, b \in S, \ f \in F \ : \ f(a) = f(b) \implies (a = b \lor f(a) = \overline{0})$
\mathbb{K}	$\forall a, b \in S, f \in F : f(a) = f(b)$
$\mathbb{K}_{\overline{0}}$	$\forall a, b \in S, \ f \in F : \ f(a) \neq f(b) \implies (f(a) = \overline{0} \lor f(b) = \overline{0})$

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All Inflationary Policy Functions for Three Strata

	0	1	2	D	\mathbb{C}^∞	\mathbb{K}^{∞}		0	1	2	D	\mathbb{C}^∞	\mathbb{K}^{∞}	
а	0	1	2	*	*		m	2	1	2				
b	0	1	∞	*	*		n	2	1	∞		*		
С	0	2	2	*			ο	2	2	2	*		*	
d	0	2	∞	*	*		р	2	2	∞	*		*	
е	0	∞	2		*		q	2	∞	2			*	
f	0	∞	∞	*	*	*	r	2	∞	∞	*	*	*	
g	1	1	2	*			S	∞	1	2		*		
h	1	1	∞	*		*	t	∞	1	∞		*	*	
i	1	2	2	*			u	∞	2	2			*	
j	1	2	∞	*	*		v	∞	2	∞		*	*	
k	1	∞	2		*		w	∞	∞	2		*	*	
Ι	1	∞	∞	*	*	*	x	∞	∞	∞	*	*	*	
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Both $\mathbb D$ and $\mathbb C_{\overline 0}$

This makes combined algeb	ra d	istri	ibutive!
	0	1	2
а	0	1	2
b	0	1	∞
d	0	2	∞
f	0	∞	∞
j	1	2	∞
I	1	∞	∞
r	2	∞	∞
х	∞	∞	∞

Why?

$(\mathbb{D}(S) \land \mathbb{D}(T) \land \mathbb{C}_{\overline{0}}(S)) \implies \mathbb{D}(S \times_{\overline{0}} T)$

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