

Sorting Algorithms

Bubble, Insertion, ...
Merge, Quick, Heap, ...

comparisons
 $O(n^2)$
 $O(n \cdot \log n)$

Sorting Networks

"Basic" Constructions (Batcher)

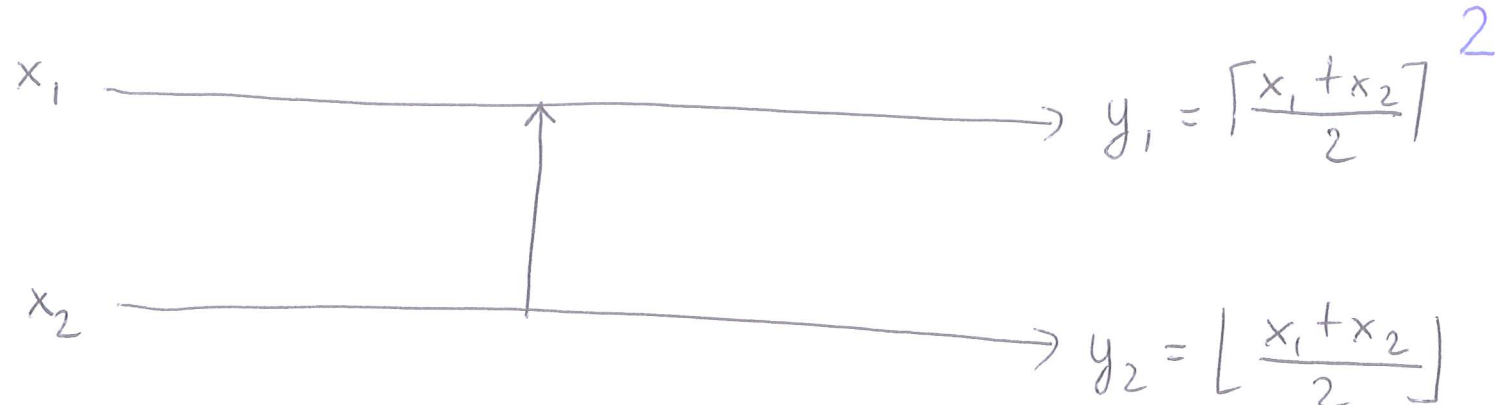
comparators
 $O(n \cdot \log^2 n)$

depth
 $O(\log^2 n)$

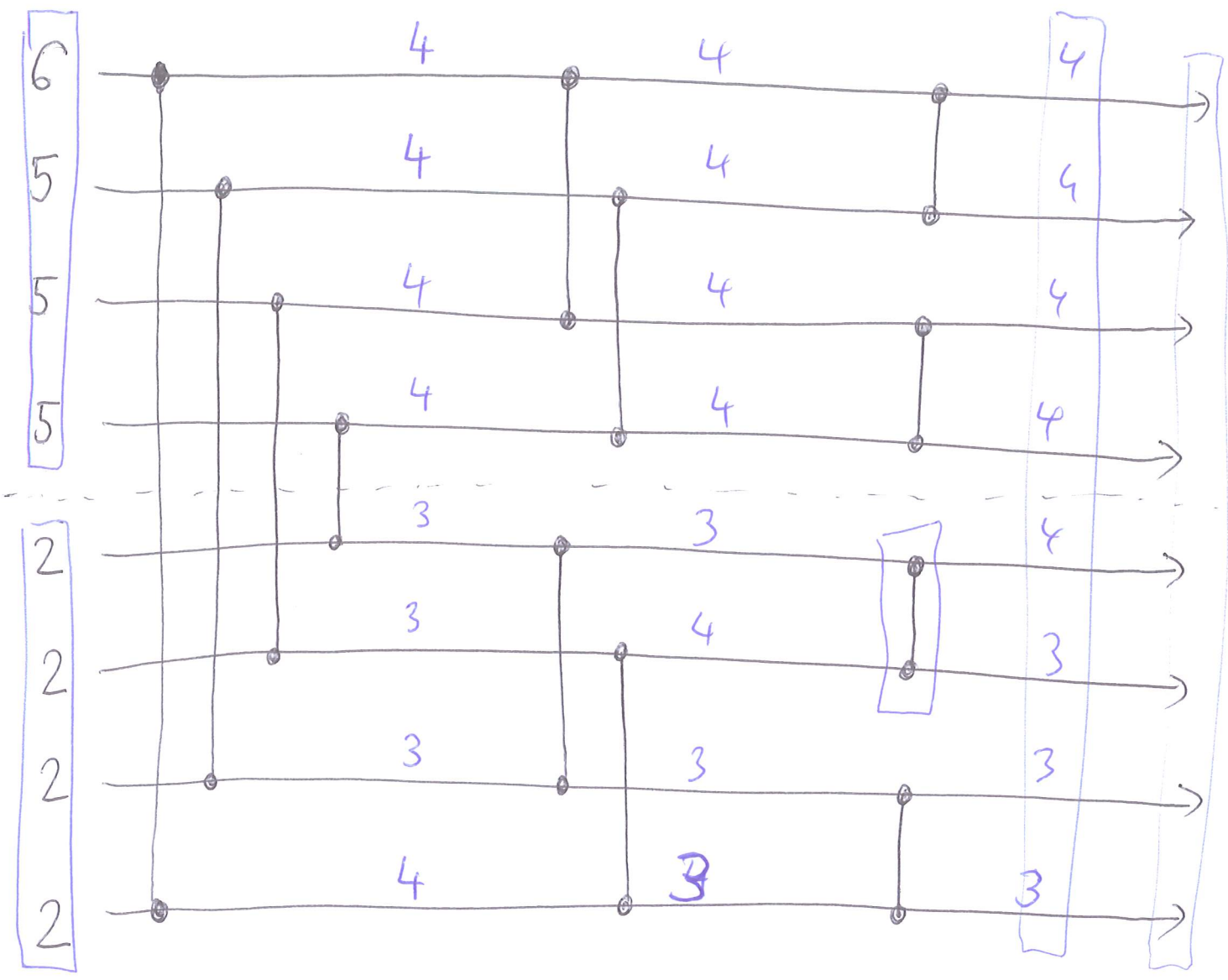
Lower Bound?

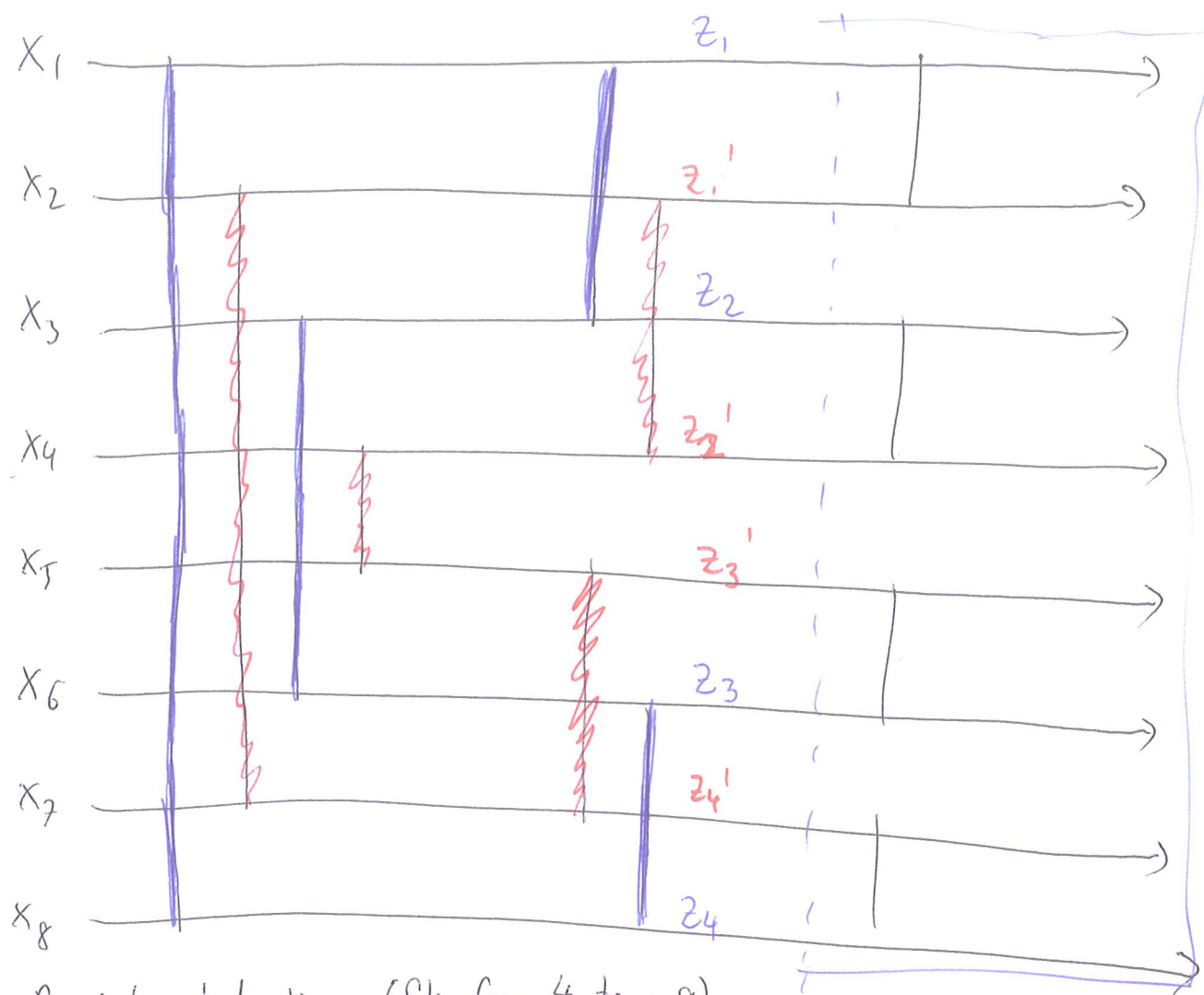
$$\# \text{ comparisons / comparators} \geq \log_2 (n!) = \underline{\Omega(n \cdot \log n)}$$

$$\text{depth} \geq \frac{\# \text{ comparators}}{n/2} = \underline{\Omega(\log n)}$$



Merging two step-property sequences:





Proof by Induction: (Step from 4 to $n=8$)

x_1, x_2, x_3, x_4 step property & x_5, x_6, x_7, x_8 step property

$\Rightarrow x_1, x_3$ has step property & x_6, x_8 has step property

$\Rightarrow$ Induction Hypothesis $\Rightarrow \left. \begin{array}{l} z_1, z_2, z_3, z_4 \text{ has step property} \\ z_1', z_2', z_3', z_4' \text{ has step property} \end{array} \right\} \textcircled{1}$

$$Z = z_1 + z_2 + z_3 + z_4$$

$$= \underline{x_1 + x_3} + \underline{x_6 + x_8}$$

$$\stackrel{\text{Fact 1}}{=} \left\lceil \frac{1}{2}(x_1 + x_2 + x_3 + x_4) \right\rceil + \left\lfloor \frac{1}{2}(x_5 + x_6 + x_7 + x_8) \right\rfloor$$

$$Z' = \left\lceil \quad \quad \right\rceil + \left\lfloor \quad \quad \right\rfloor$$

$$\Rightarrow |Z - Z'| \leq 1 \} \textcircled{2}$$

Case 1: $z = z'$

Fact 2

$\Rightarrow z_1, z_1', z_2, z_2', \dots$ satisfies step property

Case 2: $|z - z'| = 1$

Fact 3

$\Rightarrow$ (see also Example before)