# Introduction to MATLAB

## – exercises and solution notes

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### Computer Science Tripos – Part II

Exercise 1: Find a short MATLAB expression to build the matrix

$$B = \left(\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 7 & 5 & 3 & 1 & -1 & -3 \\ 4 & 8 & 16 & 32 & 64 & 128 & 256 \end{array}\right)$$

Answer:  $b = [1:7; 9:-2:-3; 2.^(2:8)]$ 

Exercise 2: Give a MATLAB expression that uses only a single matrix multiplication with B to obtain

(a) the sum of columns 5 and 7 of B

Answer: b \* [0 0 0 0 1 0 1]'

(b) the last row of B

*Answer:* [0 0 1] \* b

(c) a version of B with rows 2 and 3 swapped

Answer: [1 0 0; 0 0 1; 0 1 0] \* b

Exercise 3: Give a MATLAB expression that multiplies two vectors to obtain

(a) the matrix  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ 

*Answer:* [1 1 1]' \* (1:5)

(b) the matrix  $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{pmatrix}$ 

*Answer:* (0:4)' \* [1 1 1]

#### Exercise 4: Modify slide 30 to produce tones of falling frequency instead.

```
Answer: Replace
f = fmin * (fmax/fmin) .^ 1;
with
f = fmax * (fmin/fmax) .^ 1;
```

#### Exercise 5:

(a) Write down the function g(t) that has the shape of a sine wave that increases linearly in frequency from 0 Hz at t = 0 s to 5 Hz at t = 10 s.

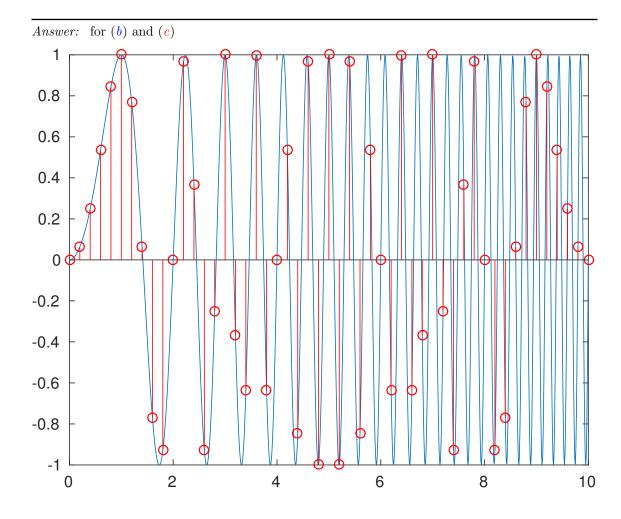
Answer: The instantaneous frequency of function g(t) at time t is

$$f(t) = t \cdot \frac{5 \text{ Hz}}{10 \text{ s}} = \frac{t}{2 \text{ s}^2}$$

and since the phase of a sine wave is  $2\pi$  times the integrated frequency so far, we get

$$g(t) = \sin\left(2\pi \int_0^t f(t') dt'\right) = \sin\left(2\pi \frac{t^2}{4 s^2}\right) = \sin\left(\frac{\pi t^2}{2 s^2}\right)$$

- (b) Plot the graph of this function using MATLAB's plot command.
- (c) Add to the same figure (this can be achieved using the hold command) in a different colour a graph of the same function sampled at 5 Hz, using the stem command.

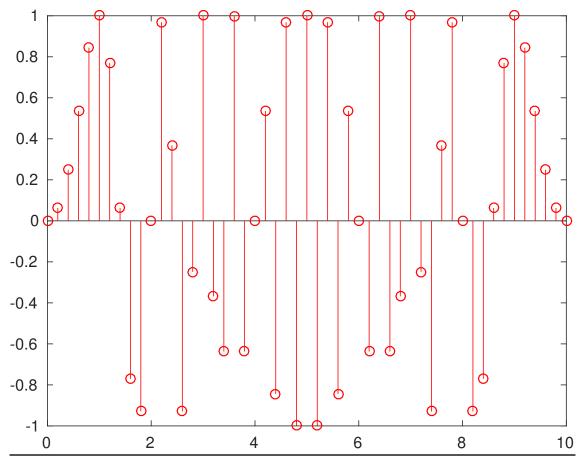


```
t = 0:0.01:10;
f = sin(pi*t.^2/2);
plot(t,f);
hold;
t2 = 0:1/5:10;
stem(t2, sin(pi*t2.^2/2), 'r');
```

(d) [Extra credit] Plot the graph from (c) separately. Can you explain its symmetry? [Hints: sampling theorem, aliasing].

Answer: A sine wave with a frequency f larger than half the sampling frequency  $f_{\rm s}$  cannot be distinguished based on the sample values from a sine wave of frequency  $f_{\rm s}-f$ . In other words, the sample values would have looked the same had we replaced the instantaneous frequency f(t) with  $f_{\rm s}/2-|f_{\rm s}/2-f(t)|$ , and the latter is symmetric around  $f_{\rm s}/2$ , which is in this graph 2.5 Hz and occurs at t=5 s.

[The above is of course just a hand-waving argument, but shall be sufficient for this exercise. There are actually a few more conditions fulfilled here that lead to the exact symmetry of the plot. Firstly, since we started sampling at t=0 s with  $f_{\rm s}=5$  Hz, the positions of the sample values end up being symmetric around t=5 s. Secondly, at the symmetry point t=5 s, the sine wave was at a symmetric peak from where increasing or decreasing the phase has the same result.]



Exercise 6: Use MATLAB to write an audio waveform (8 kHz sampling frequency) that contains a sequence of nine tones with frequencies 659, 622, 659, 622, 659, 494, 587, 523, and 440 Hz. Then add to this waveform a copy of itself in which every other sample has been multiplied by -1. Play the waveform, write it to a WAV file, and use the specgram command to plot its spectrogram with correctly labelled time and frequency axis.

Answer:

```
f = [659 622 659 622 659 494 587 523 440];
fs = 8000;  % sampling frequency
d = 0.5;  % duration per tone
t = 0:1/fs:d-1/fs;
w = sin(2 * pi * f' * t)/2;
w = w'; w = w(:)';
w = [w, w .* (mod((1:length(w)), 2) * 2 - 1)];
audiowrite('matlab_answer-2.wav', w, fs);
specgram(w, [], fs);
```

