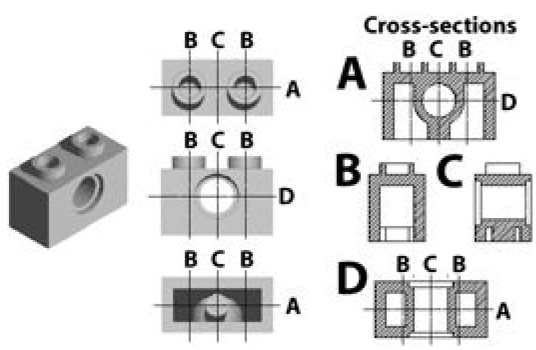
Exercises for Advanced Graphics (Lectures 5-8)

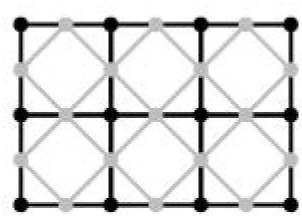
All work to be submitted by email in a single PDF, no less than 48 hours before supervision.

These exercises are partly drawn from past exam questions.

1. **Constructive Solid Geometry**
    a. List three ways of combining objects using Constructive Solid Geometry (CSG), and describe how an object built using CSG can be represented using a binary tree.
    b. Given the intersection points of a ray with each primitive in the tree, explain how these points are passed up the tree by each type of combination node to produce a list of intersection points for the whole CSG object.
    c. Show how the Lego™ brick below can be constructed using Constructive Solid Geometry (CSG). You may assume the following primitives: sphere, cylinder, cone, torus, box.



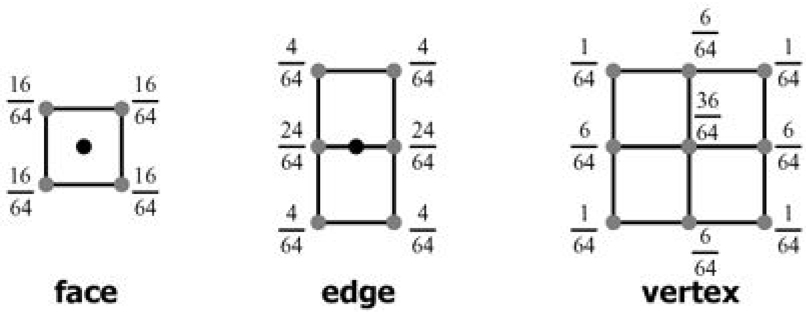
2. **Doo-Sabin and Reif-Peters Subdivision**
The Reif-Peters subdivision scheme is illustrated at right. Reif-Peters uses a different approach to Doo-Sabin: in it the new points are generated halfway between existing points and connected up into a mesh as illustrated in the diagram on the right (black: original mesh; grey: new mesh). Note that (i) the existing points do not form part of the new mesh and (ii) each new point's position is simply the average of the positions of the two existing points at either end of the corresponding line segment.



    a. Doing the Reif-Peters subdivision twice produces a mesh which looks similar to that produced by doing the Doo-Sabin subdivision once. You can treat two steps of Reif-Peters as if it were a single step of a Doo-Sabin-like subdivision scheme. Calculate the weights on the original points for each new point after two steps of Reif-Peters.

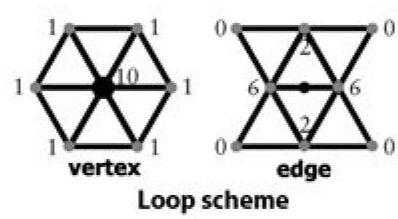
b. For the Reif-Peters scheme, explain what happens around extraordinary vertices and what happens around extraordinary polygons, giving examples.

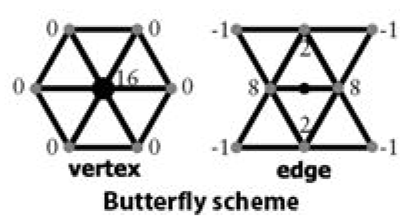
3. **Catmull-Clark Subdivision**
The Catmull-Clark bivariate subdivision scheme is a bivariate generalisation of the univariate 1/8 [1, 4, 6, 4, 1] subdivision scheme. It creates new vertices as blends of old vertices in the following ways:



a. Provide similar diagrams for the bivariate generalisation of the univariate four-point interpolating subdivision scheme 1/16 [-1, 0, 9, 16, 9, 0,-1].
b. Explain what problems arise around extraordinary vertices (vertices of valency other than four) for this bivariate interpolating scheme and suggest a possible way of handling the creation of new edge vertices when the old vertex at one end of the edge has a valency other than four.

4. **Triangular Subdivision**
The Loop and Butterfly subdivision schemes can both operate on triangular meshes, in which all of the polygons have three sides. Both schemes subdivide the mesh by introducing new vertices at the midpoints of edges, splitting every original triangle into four smaller triangles, as shown at right. Each scheme has rules for calculating the locations of the new "edge" and "vertex" vertices based on the locations of the old vertices. These rules are shown below. All weights should be multiplied by 1/16.





a. Which of the two schemes produces a limit surface which interpolates the original data points?

b. Which of the four rules must be modified when there is an extraordinary vertex? For each of the four rules either explain why it must be modified or explain why it does not need to be modified.
c. Suggest appropriate modifications where necessary to accommodate extraordinary vertices.

5. **Terms and Concepts**

Voronoi Diagrams

a. What is *equiangularity*?
b. What is the *empty circle property*?
c. Describe how to use hardware acceleration to swiftly compute Voronoi diagrams. What are the limitations of this approach?

Topology

d. Define the *Euler characteristic*
e. Define the term *angle deficit*
f. State the *Poincaré Theorem*, which links the geometry of a surface to its topology
g. State Descartes' *Theorem of Total Angle Deficit*, which links angle deficit across a surface to its Euler charateristic

Curvature

h. The *one-ring* of a vertex is the (usually ordered) set of vertices which lie exactly one edge away from a given vertex on a polyhedral surface. Given a vertex V with one-ring $\{v_0,\ldots,v_{n-1}\}$, give a formula for the discrete curvature of the surface at V.

Monte Carlo

Define what is meant by *Monte Carlo integration* and explain its use:

i. In Ambient Occlusion
j. In Screen Space Ambient Occlusion
k. In Photon Mapping (twice!)