Last time: generic programming

```
val show : 'a data → 'a → string
```

This time: staging

$.\!<$ e $>\!.$

Review: abstraction

Lambda abstraction

$\lambda x : A.M$
$\Lambda A :: K.M$
$\lambda A :: K.B$

First-class ∀ and ∃

```
type t = { f: 'a. ... }
type t = E : 'a s → t
```

Modular abstraction

```
module F(X : T) = ...
```

Abstraction of type equalities

```
a ≡ b
```

Interfaces to computation

```
m >>= k    f ⊗ p    f >>> g
```

Abstraction over data shape

```
val show :
  'a data → 'a → string
```

The cost of ignorance

Fewer opportunities for optimization

```
let both_eq1 : int * int → int * int → bool =
  fun (x1, y1) (x2, y2) →
   x1 = x2 && y1 = y2


let both_eq2 : (int → int → bool) →
  int * int → int * int → bool =
  fun eq (x1, y1) (x2, y2) →
    eq x1 x2 && eq y1 y2

both_eq2 (fun x y → x = y)


type eq = { eq: 'a. 'a → 'a → bool }
let both_eq {eq} (x1, y1) (x2, y2) =
  eq x1 x2 && eq y1 y2
```

The cost of ignorance

Interpretative overhead

```
let print_int_pair (x,y) =
  print_char '(';
  print_int x;
  print_char ',';
  print_int y;
  print_char ')'


let print_int_pair2 (x,y) =
  Printf.sprintf "(%d,%d)" x y


let print_int_pair3 (x,y) =
  print_string (gshow (pair int int) (x, y))
```

Abstraction wants to be free

```
let pow2 x = x * x                 (* x^2 *)
let pow3 x = x * x * x             (* x^3 *)
let pow5 x = x * x * x * x * x     (* x^5 *)
```

```
let rec pow x n =                  (* x^n *)
  if n = 0 then 1
  else x * pow x (n - 1)
```

```
val pow : int → int → int
```

Power, staged

```
let rec pow x n =
  if n = 0 then .< 1 >.
  else .<.~x * .~(pow x (n - 1)) >.
```

Power, staged

```
let rec pow x n =
  if n = 0 then .< 1 >.
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val pow : int code → int → int code
```

Power, staged

```
let rec pow x n =
  if n = 0 then .< 1 >.
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val pow : int code → int → int code

let pow_code n = .< fun x → .~(pow .<x>. n) >.
```

Power, staged

```
let rec pow x n =
  if n = 0 then .< 1 >.
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val pow : int code → int → int code

let pow_code n = .< fun x → .~(pow .<x>. n) >.

val pow_code : int → (int → int) code
```

Power, staged

```
let rec pow x n =
  if n = 0 then .< 1 >.
  else .<.~x * .~(pow x (n - 1)) >.

val pow : int code → int → int code

let pow_code n = .< fun x → .~(pow .<x>. n) >.

val pow_code : int → (int → int) code

# pow_code 3;;
.<fun x → x * x * x * 1>.
```

Power, staged

```
let rec pow x n =
  if n = 0 then .< 1 >.
  else .<.~x * .~(pow x (n - 1)) >.

val pow : int code → int → int code

let pow_code n = .< fun x → .~(pow .<x>. n) >.

val pow_code : int → (int → int) code

# pow_code 3;;
.<fun x → x * x * x * 1>.

# let pow3' = !. (pow_code 3);;
val pow3' : int → int = <fun>
```

Power, staged

```
let rec pow x n =
  if n = 0 then .< 1 >.
  else .< .~x * .~(pow x (n - 1)) >.

val pow : int code → int → int code

let pow_code n = .< fun x → .~(pow .<x>. n) >.

val pow_code : int → (int → int) code

# pow_code 3;;
.<fun x → x * x * x * 1>.

# let pow3' = !. (pow_code 3);;
val pow3' : int → int = <fun>

# pow3' 4;;
- : int = 64
```

MetaOCaml basics

Quoting

```
let x = "w" in
let y = "x" in
  print_string (x ^ y)
```

```
let x = "w" in
let y = x in
  print_string ("x ^ y")
```

```
let x = "w" in
let y = x in
  print_string (x ^ y)
```

```
let x = "w" in
let y = x in
  print_string ("x" ^ y)
```

Quoting **prevents evaluation**.

Quoting code

MetaOCaml: multi-stage programming with code quoting.

Stages: current (available now) and delayed (available later).
(Also double-delayed, triple-delayed, etc.)

Brackets

```
.<e>.
```

Escaping (within brackets)

```
.~e
```

Running code

```
!. e
```

Cross-stage persistence

```
.<x>.
```

Quoting and escaping: some examples

```
.<3 >.

.<1 + 2 >.

.<[1; 2; 3] >.

.<x + y >.

.<fun x → x >.

.<(.~f) 3 >.

.<.~(f 3)>.

.<fun x → .~(f .< x >.) >.
```

Quoting: typing

$$\Gamma \vdash^{n} \texttt{e} : \tau$$

$$\frac{\Gamma \vdash^{n+} \texttt{e} : \tau}{\Gamma \vdash^{n} \texttt{.<e>.} : \tau\ \text{code}}\ \text{T-bracket}$$

$$\frac{\Gamma^{+} \vdash^{n} \texttt{e} : \tau\ \text{code}}{\Gamma \vdash^{n} \texttt{!. e} : \tau}\ \text{T-run}$$

$$\frac{\Gamma \vdash^{n} \texttt{e} : \tau\ \text{code}}{\Gamma \vdash^{n+} \texttt{.\~{}e} : \tau}\ \text{T-escape}$$

$$\frac{\Gamma(x) = \tau^{(n-m)}}{\Gamma \vdash^{n} : \tau}\ \text{T-var}$$

Quoting: open code

Open code

```
let pow_code n = .< fun x → .~(pow .<x>. n) >.
```

Cross-stage persistence

```
let print_int_pair (x,y) =
  Printf.printf "(%d,%d)" x y

let pairs = .< [(3, 4); (5, 6)] >.

.< List.iter print_int_pair .~pairs >.
```

Quoting: scoping

Scoping is **lexical**, just as in OCaml.

```
.< fun x → .~( let x = 3 in .<x>. ) >.

let x = 3 in .< fun x → .~( .<x>. ) >.
```

MetaOCaml renames variables to avoid clashes:

```
.< let x = 3 in
   .~(let y = .<x>. in
      .< fun x → .~y + x >.) >.
```

Quoting: scoping

Scoping is **lexical**, just as in OCaml.

```
.< fun x → .~( let x = 3 in .<x>. ) >.

let x = 3 in .< fun x → .~( .<x>. ) >.
```

MetaOCaml renames variables to avoid clashes:

```
# .< let x = 3 in
    .~(let y = .<x>. in
       .< fun x → .~y + x >.) >.;;
- : (int → int) code =
.<let x_1 = 3 in fun x_2  → x_1 + x_2>.
```

Learning from mistakes

Error: quoting nonsense

```
.< 1 + "two" >.
```

Error: quoting nonsense

```
# .< 1 + "two" >.;;
Characters 7-12:
  .< 1 + "two" >.;;
         ^^^^^
Error: This expression has type string but an
    expression was expected of type int
```

Error: looking into the future

```
.< fun x → .~( x ) >.
```

Error: looking into the future

```
# .< fun x -> .~( x ) >.;;
Characters 14-19:
  .< fun x -> .~( x ) >.;;
                ^^^^^
Wrong level: variable bound at level 1 and
    used at level 0
```

Error: escape from nowhere

```
let x = .< 3 >. in .~x
```

Error: escape from nowhere

```
# let x = .< 3 >. in .~x;;
Characters 22-23:
  let x = .< 3 >. in .~x;;
                      ^
Wrong level: escape at level 0
```

Error: running open code

```
.< fun x → .~(!. .<x>. ) >.
```

Error: running open code

```
# .< fun x → .~(!. .<x>. ) >.;;
Exception:
Failure
 "The code built at Characters 7-8:\n
  .< fun x → .~(!. .< x >. ) >.;;\n
         ^\n
 is not closed: identifier x_2 bound at
 Characters 7-8:\n
  .< fun x → .~(!. .< x >. ) >.;;\n
         ^\n
 is free".
```

Learning by doing

Power again

Reducing the number of multiplications:

$$
\begin{aligned}
x^0 &= 1 \\
x^{2n+2} &= (x^{n+1})^2 \\
x^{2n+1} &= x(x^{2n})
\end{aligned}
$$

```
let even x = x mod 2 = 0
let sqr x = x * x

let rec pow x n =
  if n = 0 then 1
  else if even n then sqr (pow x (n / 2))
  else x * pow (n - 1) x
```

Power again, staged

Reducing the number of multiplications:

$$x^0 = 1$$
$$x^{2n+2} = (x^{n+1})^2$$
$$x^{2n+1} = x(x^{2n})$$

```
let even x = x mod 2 = 0
let sqr x = .< let y = .~x in y * y >.

let rec pow' x n =
  if n = 0 then .<1>.
  else if even n then sqr (pow' x (n / 2))
  else .<.~x * .~(pow' x (n - 1)) >.
```

Power again, staged

```
let rec pow' x n =
  if n = 0 then .<1>.
  else if even n then sqr (pow' x (n / 2))
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Power again, staged

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let rec pow' x n =
  if n = 0 then .<1>.
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val pow' : int code → int → int code
```

Power again, staged

```
 let rec pow' x n =
   if n = 0 then .<1>.
   else if even n then sqr (pow' x (n / 2))
   else .<.~x * .~(pow' x (n - 1)) >.

val pow' : int code → int → int code

let pow_code' n = .< fun x → .~(pow' .<x>. n) >.
```

Power again, staged

```
 let rec pow' x n =
   if n = 0 then .<1>.
   else if even n then sqr (pow' x (n / 2))
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val pow' : int code → int → int code

let pow_code' n = .< fun x → .~(pow' .<x>. n) >.

val pow_code' : int → (int → int) code
```

Power again, staged

```
let rec pow' x n =
  if n = 0 then .<1>.
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  else .<.~x * .~(pow' x (n - 1)) >.

val pow' : int code → int → int code

let pow_code' n = .< fun x → .~(pow' .<x>. n) >.

val pow_code' : int → (int → int) code

# pow_code' 5;;
- : (int → int) code =
.< fun x →
    x * (let y = let y' = x * 1
                 in y' * y'
         in y * y)>.
```

The staging process, idealized

1. Write the program as usual:

```
val program : t_sta → t_dyn → t
```

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val back: ('a code → 'b code) → ('a → 'b) code
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```
val s : t_sta
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5. Apply code generator to static inputs:

```
val specialized_code : (t_dyn → t) code
```

The staging process, idealized

1. Write the program as usual:

```
val program : t_sta → t_dyn → t
```

2. Add staging annotations:

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```

3. Compile using `back`:

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val back: ('a code → 'b code) → ('a → 'b) code
val code_generator : t_sta → (t_dyn → t)
```

4. Construct static inputs:

```
val s : t_sta
```

5. Apply code generator to static inputs:

```
val specialized_code : (t_dyn → t) code
```

6. Run specialized code to build a specialized function:

```
val specialized_function : t_dyn → t
```

Inner product

```
let dot
  : int → float array → float array → float
  = fun n l r →
    let rec loop i =
      if i = n then 0.
      else l.(i) *. r.(i)
        +. loop (i + 1)
    in loop 0
```

Inner product, loop unrolling

```
let dot'
 : int → float array code → float array code → float code
 = fun n l r →
    let rec loop i =
      if i = n then .< 0. >.
      else .< ((.~l).(i) *. (.~r).(i))
                +. .~(loop (i + 1)) >.
    in loop 0
```

Inner product, loop unrolling

```
# .< fun l r → .~(dot' 3 .<l>. .<r>.) >.;;
- : (float array → float array → float) code =
.< fun l r →
     (l.(0) *. r.(0)) +.
       ((l.(1) *. r.(1)) +. ((l.(2) *. r.(2)) +. 0.))>.
```

Inner product, eliding no-ops

```
let dot''
 : float array → float array code → float code =
 fun l r →
   let n = Array.length l in
   let rec loop i =
     if i = n then .< 0. >.
     else match l.(i) with
       0.0 → loop (i + 1)
     | 1.0 → .<(.~r).(i) +. .~(loop (i + 1)) >.
     | x → .<(x *. (.~r).(i)) +. .~(loop (i + 1)) >.
   in loop 0
```

Inner product, eliding no-ops

```
# .< fun r → .~(dot'' [| 1.0; 0.0; 3.5 |] .<r>.) >.;;
- : (float array → float) code =
.< fun r → r.(0) +. ((3.5 *. r.(2)) +. 0.)>.
```

Binding-time analysis

Classify **variables** into **dynamic** (`'a code`) / **static** (`'a`)

```
let dot'
  : int → float array code → float array code → float code
    =
  fun n l r →
```

dynamic: `l`, `r`

static: `n`

Classify **expressions** into static (no dynamic variables) / dynamic

```
if i = n then 0
else l.(i) *. r.(i)
```

dynamic: `l.(i) *. r.(i)`

static: `i = n`

Goal: reduce static expressions during code generation.