

Topics in Concurrency

Lecture 3

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18 February 2015

Recap: Syntax of CCS

- Expressions: Arithmetic a and Boolean b
- Processes:

p	$::=$	nil	nil process
		$(\tau \rightarrow p)$	silent/internal action
		$(\alpha!a \rightarrow p)$	output
		$(\alpha?x \rightarrow p)$	input
		$(b \rightarrow p)$	Boolean guard
		$p_0 + p_1$	non-deterministic choice
		$p_0 \parallel p_1$	parallel composition
		$p \setminus L$	restriction (L a set of channel identifiers)
		$p[f]$	relabelling (f a function on channel identifiers)
		$P(a_1, \dots, a_k)$	process identifier

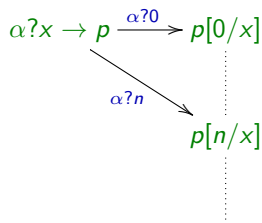
- Process definitions:

$$P(x_1, \dots, x_k) \stackrel{\text{def}}{=} p$$

(free variables of $p \subseteq \{x_1, \dots, x_n\}$)

Towards a more basic language

- Aim: removal of variables to reveal symmetry of input and output
- Transitions for value-passing carry labels τ , $\alpha?n$, $\alpha!n$



- This suggests introducing **prefix** $\alpha?n.p$ (as well as $\alpha!n.p$) and view $\alpha?x \rightarrow p$ as a sum $\sum_n \alpha?n.p[n/x]$ **infinite sum**
- View $\alpha?n$ and $\alpha!n$ as **complementary** actions
- Synchronization can only occur on complementary actions

Pure CCS

- Actions: $a, b, c, \dots$
- Complementary actions: $\bar{a}, \bar{b}, \bar{c}, \dots$
- Internal action: τ
- Notational convention: $\bar{\bar{a}} = a$
- Processes:

$p ::=$	$\lambda.p$	prefix	λ ranges over $\tau, a, \bar{a}$ for any action a
	$\sum_{i \in I} p_i$	sum	I is an indexing set
	$p_0 \parallel p_1$	parallel	
	$p \setminus L$	restriction	L a set of actions
	$p[f]$	relabelling	f a function on actions
	P	process identifier	

- Process definitions:

$$P \stackrel{\text{def}}{=} p$$

Transition rules for pure CCS

- **Nil process** no rules
- **Guarded processes**

$$\lambda.p \xrightarrow{\lambda} p$$

- **Sum**

$$\frac{p_j \xrightarrow{\lambda} p' \quad j \in I}{\sum_{i \in I} p_i \xrightarrow{\lambda} p'_0}$$

- **Parallel composition**

$$\frac{p_0 \xrightarrow{\lambda} p'_0}{p_0 \parallel p_1 \xrightarrow{\lambda} p'_0 \parallel p_1} \qquad \frac{p_1 \xrightarrow{\lambda} p'_1}{p_0 \parallel p_1 \xrightarrow{\lambda} p_0 \parallel p'_1}$$
$$\frac{p_0 \xrightarrow{a} p'_0 \quad p_1 \xrightarrow{\bar{a}} p'_1}{p_0 \parallel p_1 \xrightarrow{\tau} p'_0 \parallel p'_1}$$

- **Restriction**

$$\frac{p \xrightarrow{\lambda} p'}{p \setminus L \xrightarrow{\lambda} p' \setminus L} \quad \lambda \notin L \cup \bar{L} \quad \text{where } \bar{L} = \{\bar{a} \mid a \in L\}$$

- **Relabelling**

$$\frac{p \xrightarrow{\lambda} p'}{p[f] \xrightarrow{f(\lambda)} p'[f]}$$

where f is a function such that $f(\tau) = \tau$ and $f(\bar{a}) = \overline{f(a)}$

- **Identifiers**

$$\frac{p \xrightarrow{\lambda} p' \quad P \stackrel{\text{def}}{=} p}{P \xrightarrow{\lambda} p'}$$

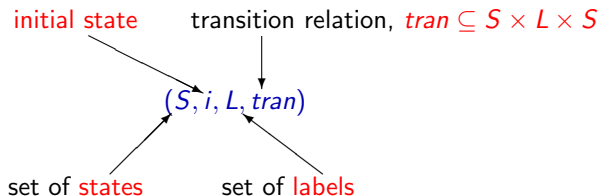
Transition systems

- Given a CCS process p , can construct its *transition system*
- A *transition system* is:

$$(S, i, L, \text{tran})$$

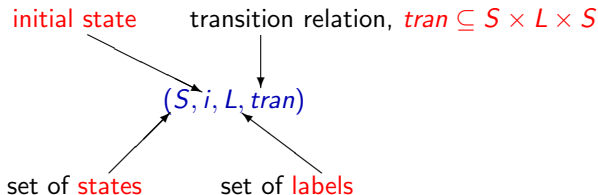
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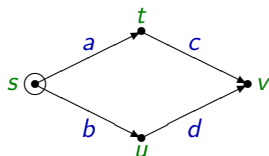


Transition systems

- Given a CCS process p , can construct its *transition system*
- A *transition system* is:



- Graphically:



$$\begin{aligned} S &= \{s, t, u, v\} \\ i &= s \\ L &= \{a, b, c, d\} \\ tran &= \{ (s, a, t), \\ &\quad (s, b, u), \\ &\quad (t, c, v), \\ &\quad (u, d, v) \} \end{aligned}$$

Transition systems from CCS

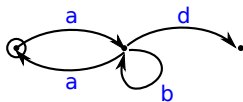
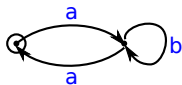
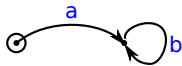
- The transition system for a CCS process p has reachable terms as states and transitions as generated by the operational semantics
- Example: $a.\bar{b}.\text{nil} + \bar{b}.a.\text{nil}$
- Example: $a \parallel b$ (note: dropped .nil)
- Example: $a \parallel \bar{a}$
- Example: $(a \parallel \bar{a}) \setminus a$
- Example: $(a \parallel \bar{b})[f]$ where $f(a) = w$ and $f(b) = w$
- Example: $a[f] \parallel \bar{b}[f]$ where $f(a) = w$ and $f(b) = w$
- Example: P where

$$P \stackrel{\text{def}}{=} a.Q$$

$$Q \stackrel{\text{def}}{=} b.Q + a.P$$

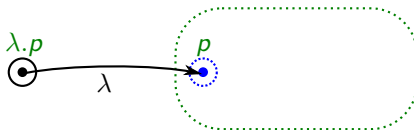
Realising transition systems

Give pure CCS terms for:



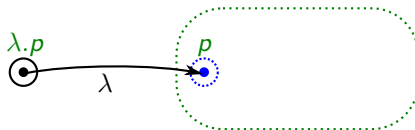
CCS operations on transition systems

- $\lambda.p$:

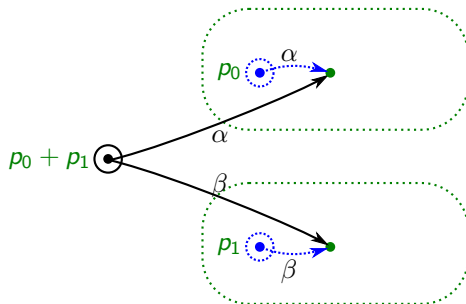


CCS operations on transition systems

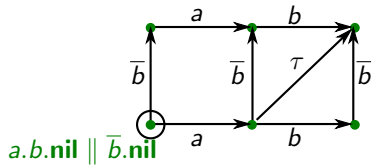
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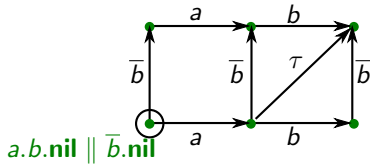
- $p_0 + p_1$:



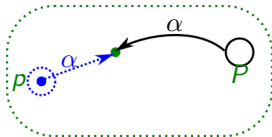
- $a.b \parallel \bar{b}$:



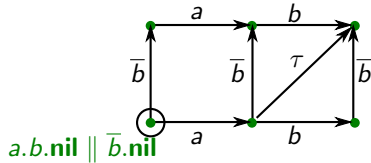
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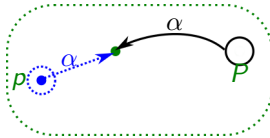
- P where $P \stackrel{\text{def}}{=} p$:



- $a.b \parallel \bar{b}$:



- P where $P \stackrel{\text{def}}{=} p$:



$p \setminus L, p[f]: \dots$

A denotational semantics!

From value-passing to pure

A translation giving a pure CCS process $\hat{p}$ from a value-passing CCS closed term p

p	$\hat{p}$
nil	nil

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$p_0 \parallel p_1$	$\hat{p}_0 \parallel \hat{p}_1$
$p \setminus L$	$\hat{p} \setminus \{\alpha m \mid \alpha \in L \ \& \ m \in \mathbf{Num}\}$

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$p \setminus L$	$\hat{p} \setminus \{\alpha m \mid \alpha \in L \ \& \ m \in \mathbf{Num}\}$
$P(a_1, \dots, a_k)$	$P_{m_1, \dots, m_k}$ where a_i evaluates to m_i

For every definition $P(x_1, \dots, x_k)$, we have a collection of definitions $P_{m_1, \dots, m_k}$ indexed by $m_1, \dots, m_k \in \mathbf{Num}$.

Examples

Let $\text{translate}(p) = \hat{p}$:

$$\text{translate}(\alpha?x \rightarrow (x \leq 5 \rightarrow (\beta!x \rightarrow \mathbf{nil})))$$

$$= \sum_{n \in \mathbf{Num}} \alpha n. \text{translate}(x \leq 5 \rightarrow (\beta!x \rightarrow \mathbf{nil}))$$

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$$\text{translate}(n \leq 5 \rightarrow (\beta!n \rightarrow \mathbf{nil}))$$

$$= \begin{cases} \overline{\beta n. \mathbf{nil}} & \text{if } n \leq 5 \\ \mathbf{nil} & \text{if } n > 5 \end{cases}$$

Theorem

- $p \xrightarrow{\lambda} p'$ implies $\widehat{p} \xrightarrow{\widehat{\lambda}} \widehat{p}'$
- if $\widehat{p} \xrightarrow{\lambda} q$ then there exist p', κ such that $p \xrightarrow{\kappa} p'$ and $\lambda = \widehat{\kappa}$ and $q = \widehat{p}'$

where $\widehat{\alpha?n} = \alpha n$ and $\widehat{\alpha!n} = \overline{\alpha n}$

Recursion: an alternative

- Instead of a process

$$P \text{ where } P \stackrel{\text{def}}{=} p$$

we can use

$$\text{rec}(P = p)$$

with rule

$$\frac{p[\text{rec}(P = p)/P] \xrightarrow{\lambda} p'}{\text{rec}(P = p) \xrightarrow{\lambda} p'}$$

- Example: $\text{rec}(P = a.\mathbf{nil} + b.P)$

Recursion: an alternative

- Instead of a process

$$P \text{ where } P \stackrel{\text{def}}{=} p \text{ and } Q = q$$

we can use the notation

$$\text{rec}_1(P = p, Q = q)$$

and for Q we can use

$$\text{rec}_2(P = p, Q = q)$$

Recursion: an alternative

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- Generally, instead of P_j where $P_i = p_i$ is a collection of definitions indexed by $i \in I$, can use

$$\text{rec}_j(P_i = p_i)_{i \in I}$$

which is also written

$$\text{rec}_j(\vec{P} = \vec{p})$$

Proofs of correctness

- By satisfying formulas in a logic
- By satisfying an equivalence