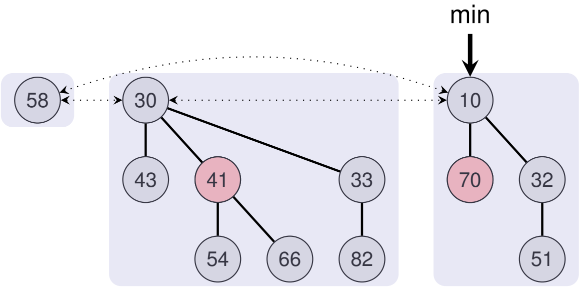


5.2 Fibonacci Heaps

Frank Stajano

Thomas Sauerwald

Lent 2015

UNIVERSITY OF CAMBRIDGE

Priority Queues Overview

Operation	Linked list	Binary heap	Binomial heap
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MINIMUM	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$
EXTRACT-MIN	$\mathcal{O}(n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(\log n)$
DECREASE-KEY	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
DELETE	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Priority Queues Overview

Operation	Linked list	Binary heap	Binomial heap	Fibon. heap
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap	Fibonacci heap
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap **actual cost**	Fibonacci heap **amortized cost**
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap **actual cost**	Fibonacci heap **amortized cost**
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

All these cost bounds hold
if n is the size of the heap.

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap **actual cost**	Fibonacci heap **amortized cost**
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap: $k/2$ DECREASE-KEY
+ $k/2$ INSERT

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap	Fibonacci heap
	actual cost	**amortized cost**
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap: $k/2$ DECREASE-KEY
+ $k/2$ INSERT

- $c_1 = c_2 = \cdots = c_k = \mathcal{O}(\log n)$

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap **actual cost**	Fibonacci heap **amortized cost**
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap: $k/2$ DECREASE-KEY
+ $k/2$ INSERT

- $c_1 = c_2 = \cdots = c_k = \mathcal{O}(\log n)$

$\Rightarrow \sum_{i=1}^{k} c_i = \mathcal{O}(k \log n)$

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap	Fibonacci heap
	actual cost	**amortized cost**
Make-Heap	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Insert	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
Minimum	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
Extract-Min	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
Merge	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
Decrease-Key	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
Delete	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap: $k/2$ Decrease-Key + $k/2$ Insert

- $c_1 = c_2 = \cdots = c_k = \mathcal{O}(\log n)$

$\Rightarrow \sum_{i=1}^{k} c_i = \mathcal{O}(k \log n)$

Fibonacci Heap: $k/2$ Decrease-Key + $k/2$ Insert

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap **actual cost**	Fibonacci heap **amortized cost**
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap: $k/2$ DECREASE-KEY + $k/2$ INSERT

- $c_1 = c_2 = \cdots = c_k = \mathcal{O}(\log n)$

$\Rightarrow \sum_{i=1}^{k} c_i = \mathcal{O}(k \log n)$

Fibonacci Heap: $k/2$ DECREASE-KEY + $k/2$ INSERT

- $\widetilde{c}_1 = \widetilde{c}_2 = \cdots = \widetilde{c}_k = \mathcal{O}(1)$

Binomial Heap vs. Fibonacci Heap: Costs

Operation	Binomial heap **actual cost**	Fibonacci heap **amortized cost**
MAKE-HEAP	$\mathcal{O}(1)$	$\mathcal{O}(1)$
INSERT	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
MINIMUM	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
EXTRACT-MIN	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
MERGE	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DECREASE-KEY	$\mathcal{O}(\log n)$	$\mathcal{O}(1)$
DELETE	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Binomial Heap: $k/2$ DECREASE-KEY + $k/2$ INSERT

- $c_1 = c_2 = \cdots = c_k = \mathcal{O}(\log n)$

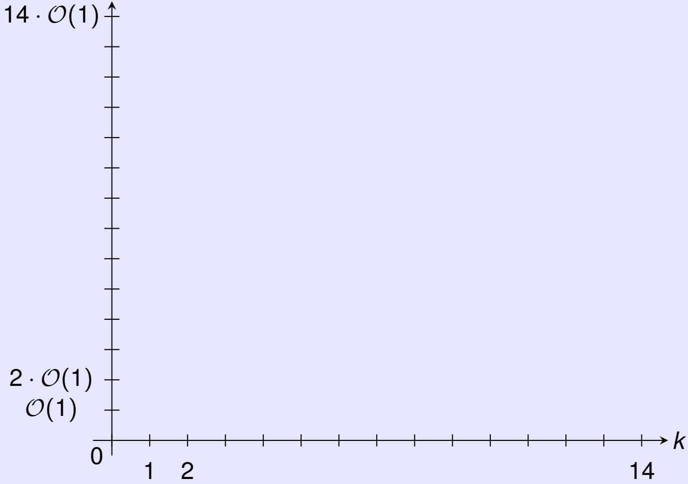
$\Rightarrow \sum_{i=1}^{k} c_i = \mathcal{O}(k \log n)$

Fibonacci Heap: $k/2$ DECREASE-KEY + $k/2$ INSERT

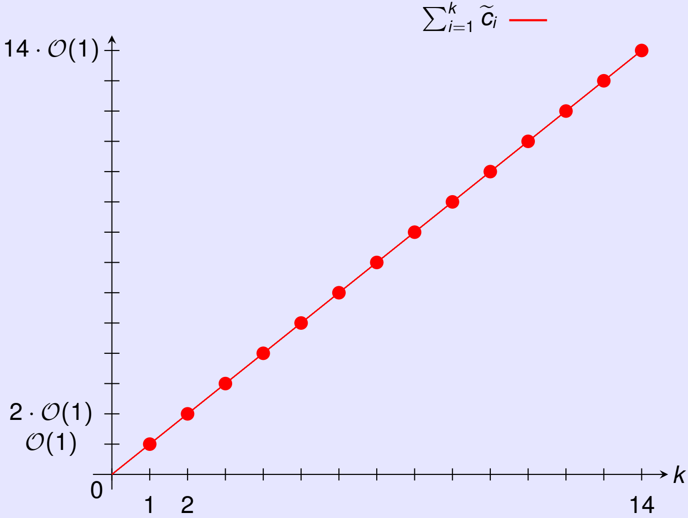
- $\widetilde{c}_1 = \widetilde{c}_2 = \cdots = \widetilde{c}_k = \mathcal{O}(1)$

$\Rightarrow \sum_{i=1}^{k} c_i \leq \sum_{i=1}^{k} \widetilde{c}_i = \mathcal{O}(k)$

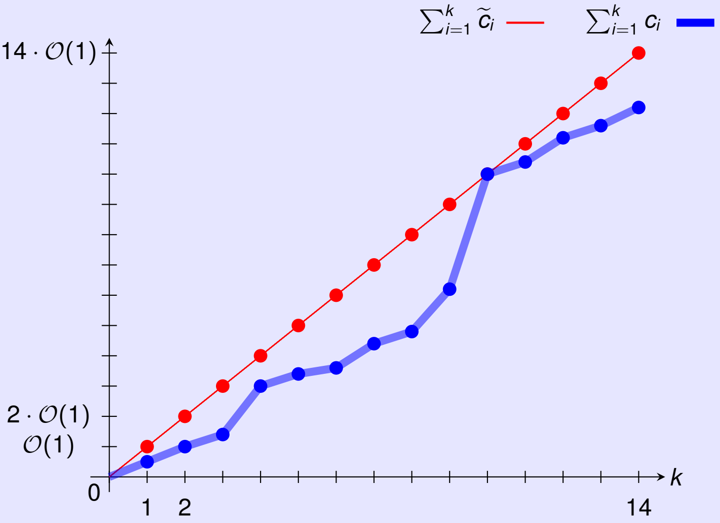
Actual vs. Amortized Cost



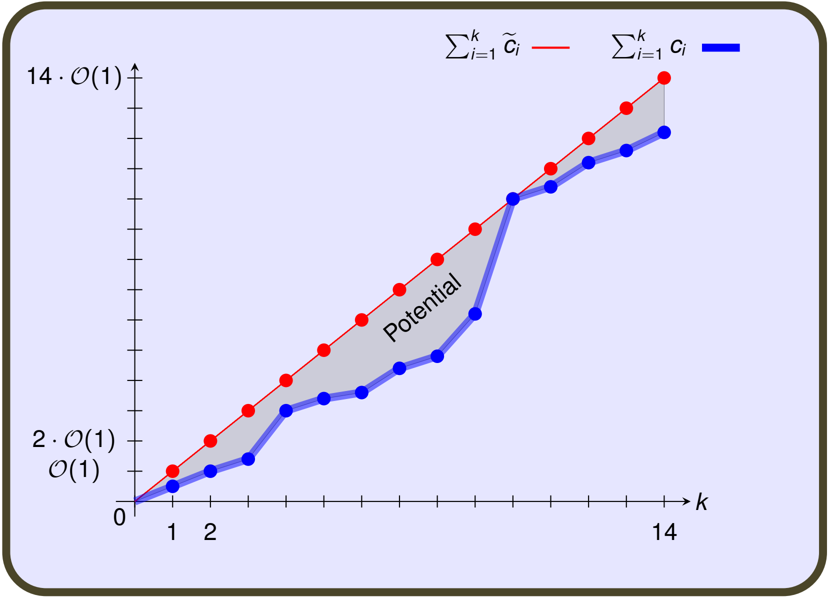
Actual vs. Amortized Cost



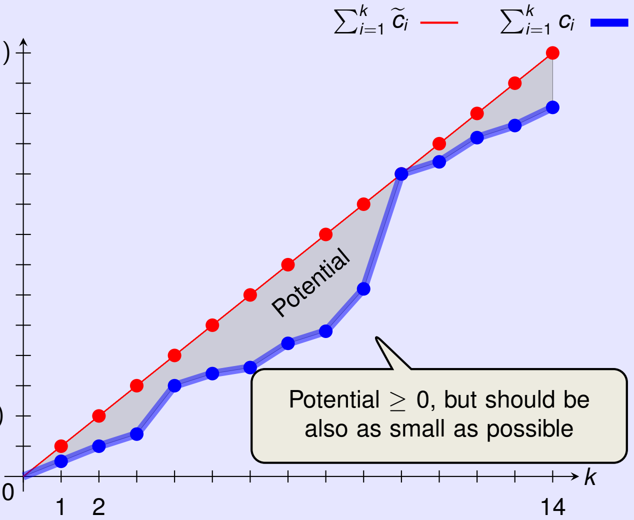
Actual vs. Amortized Cost



Actual vs. Amortized Cost



Actual vs. Amortized Cost



Potential ≥ 0, but should be also as small as possible

Outline

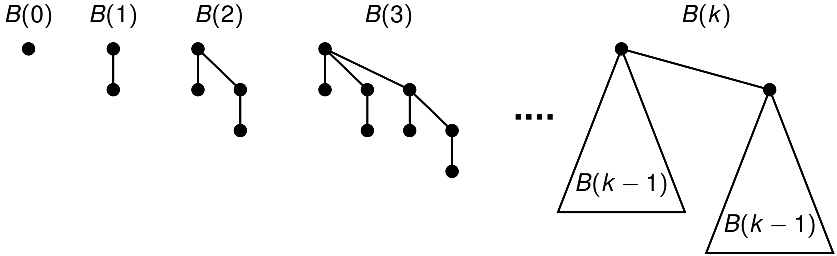
Structure

Operations

Glimpse at the Analysis

Reminder: Binomial Heaps

Binomial Trees

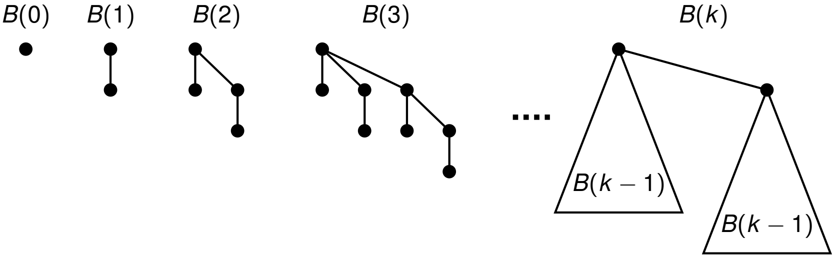


Binomial Heaps

- Binomial Heap is a collection of binomial trees of different orders, each of which obeys the heap property

Reminder: Binomial Heaps

Binomial Trees

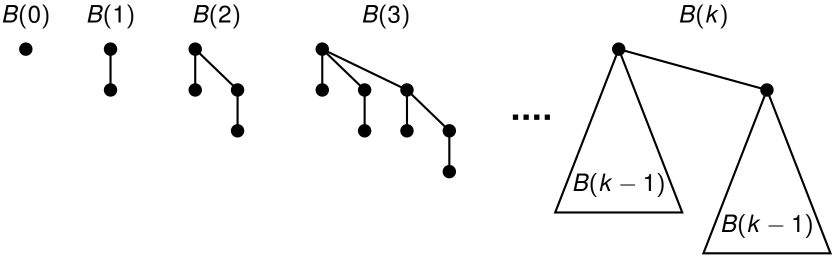


Binomial Heaps

- Binomial Heap is a collection of binomial trees of different orders, each of which obeys the heap property
- Operations:

Reminder: Binomial Heaps

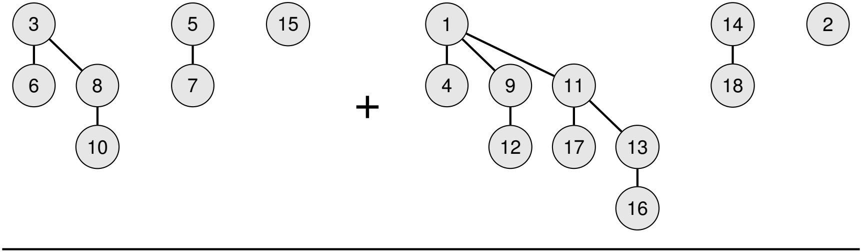
Binomial Trees



Binomial Heaps

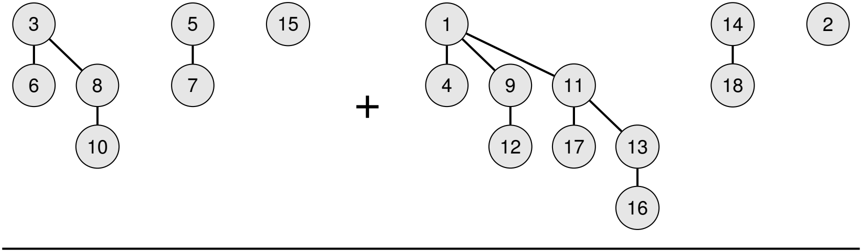
- Binomial Heap is a collection of binomial trees of different orders, each of which obeys the heap property
- Operations:
  - MERGE: Merge two binomial heaps using Binary Addition Procedure
  - INSERT: Add $B(0)$ and perform a MERGE
  - EXTRACT-MIN: Find tree with minimum key, cut it and perform a MERGE
  - DECREASE-KEY: The same as in a binary heap

Merging two Binomial Heaps



$$\begin{array}{cccccl}
0 & 0 & 1 & 1 & 1 & = 7 \\
0 & 1 & 0 & 1 & 1 & = 11 \\
{\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & \\
\hline
1 & 0 & 0 & 1 & 0 & = 18
\end{array}$$

Merging two Binomial Heaps



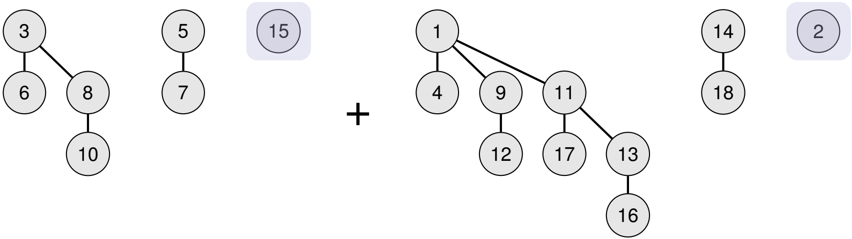
0 0 1 1 1 = 7

0 1 0 1 1 = 11

1 1 1 1

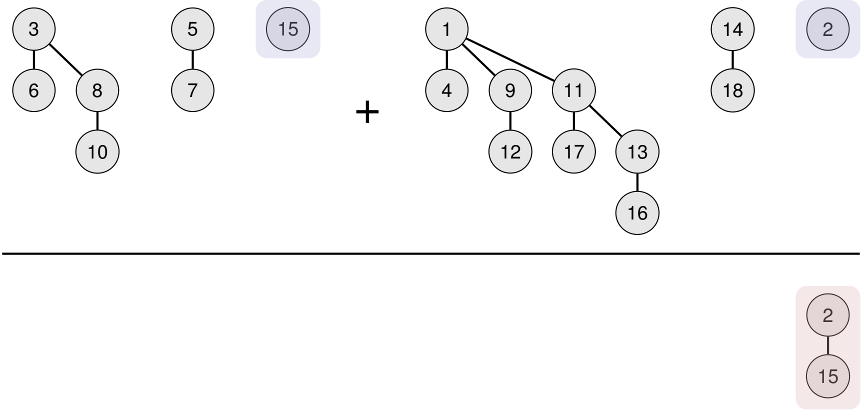
1 0 0 1 0 = 18

Merging two Binomial Heaps



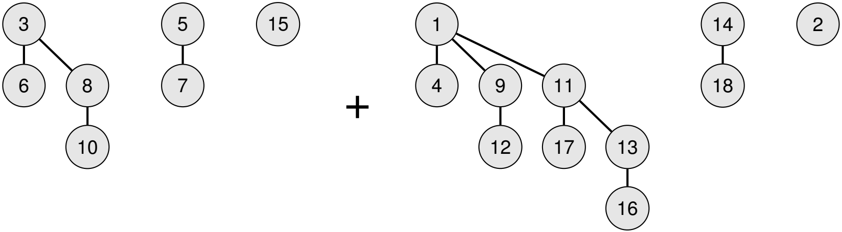
$$
\begin{array}{cccccl}
0 & 0 & 1 & 1 & 1 & = 7 \\
0 & 1 & 0 & 1 & 1 & = 11 \\
{\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & \\
\hline
1 & 0 & 0 & 1 & 0 & = 18
\end{array}
$$

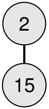
Merging two Binomial Heaps



```
 0  0  1  1  1   = 7
 0  1  0  1  1   = 11
  1  1  1  1
---------------
 1  0  0  1  0   = 18
```

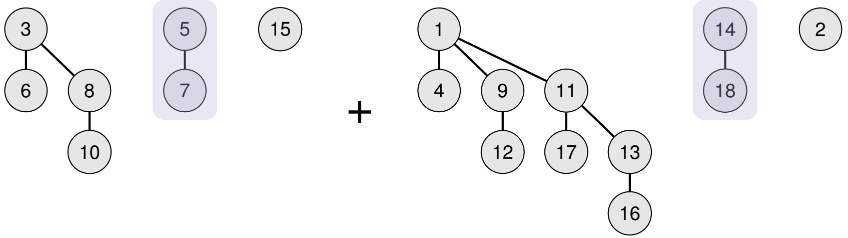
Merging two Binomial Heaps

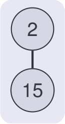




$$\begin{array}{cccccl} 0 & 0 & 1 & 1 & 1 & = 7 \\ 0 & 1 & 0 & 1 & 1 & = 11 \\ {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & \\ \hline 1 & 0 & 0 & 1 & 0 & = 18 \end{array}$$

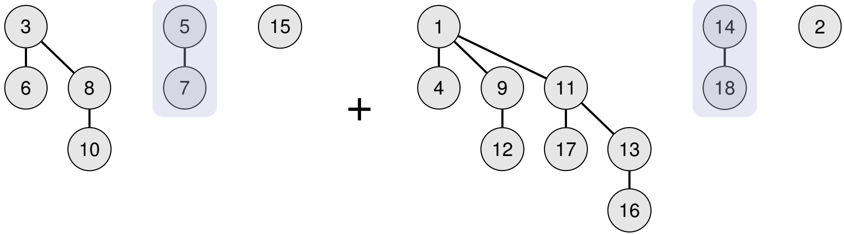
Merging two Binomial Heaps

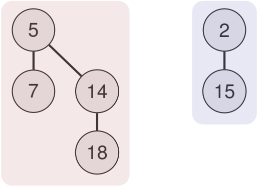




```
0  0  1  1  1   = 7
0  1  0  1  1   = 11
 1  1  1  1
----------------------
1  0  0  1  0   = 18
```

Merging two Binomial Heaps





0	0	1	1	1	= 7
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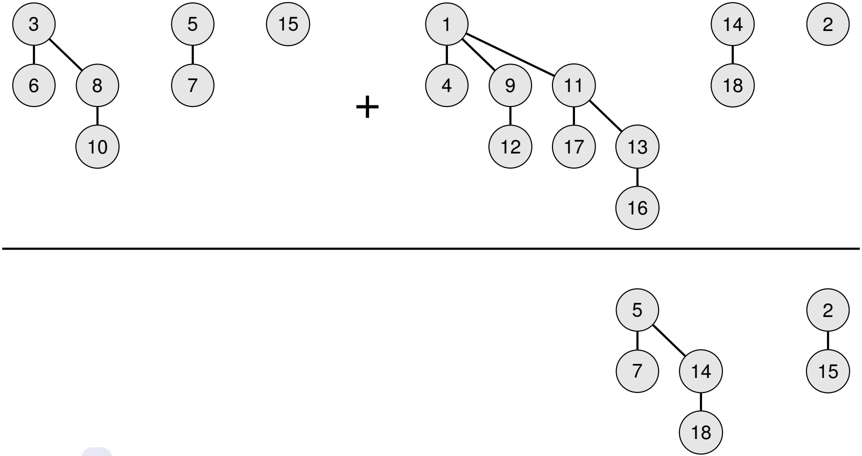
1	1	1	1		
1	0	0	1	0	= 18

Merging two Binomial Heaps



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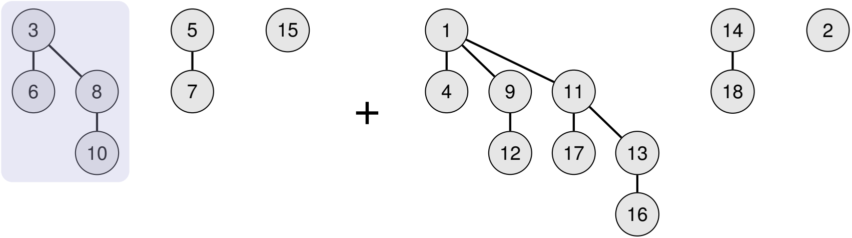
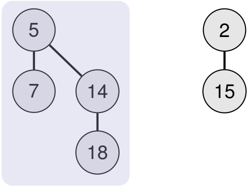
 0  1  0  1  1   = 11
  1  1  1  1
---------------------
 1  0  0  1  0   = 18
```

Merging two Binomial Heaps





$$\begin{array}{ccccccc}
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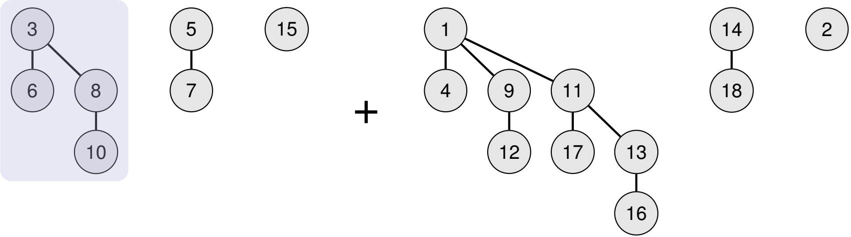
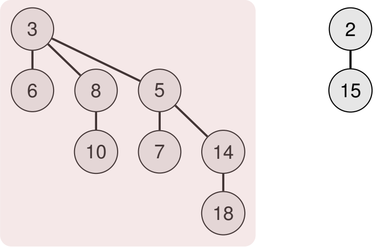
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & \\
\hline
 & 1 & 0 & 0 & 1 & 0 & = 18
\end{array}$$

Merging two Binomial Heaps





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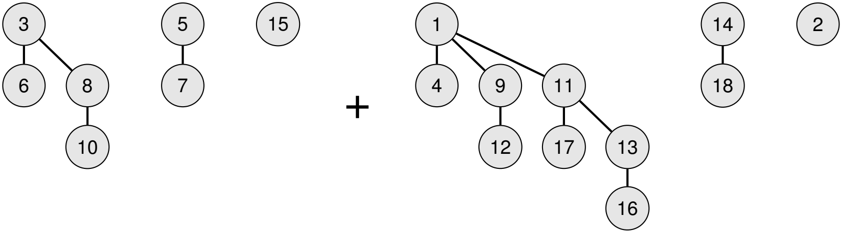
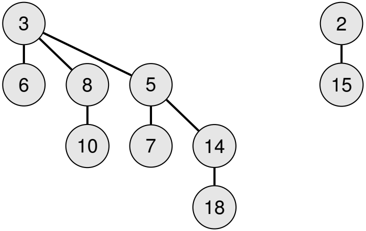
Merging two Binomial Heaps





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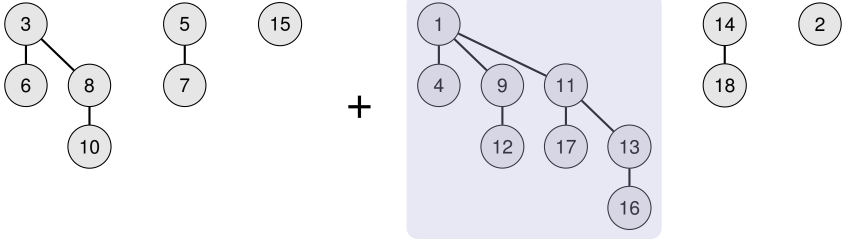
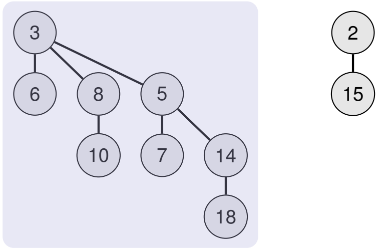
Merging two Binomial Heaps





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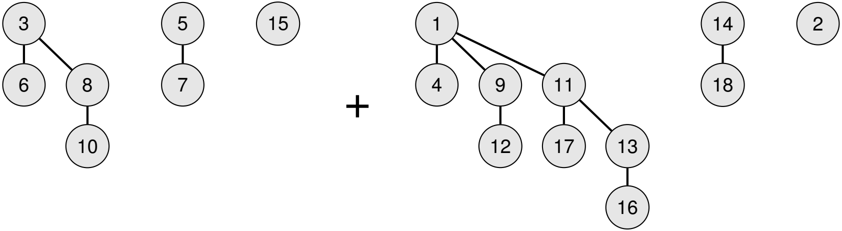
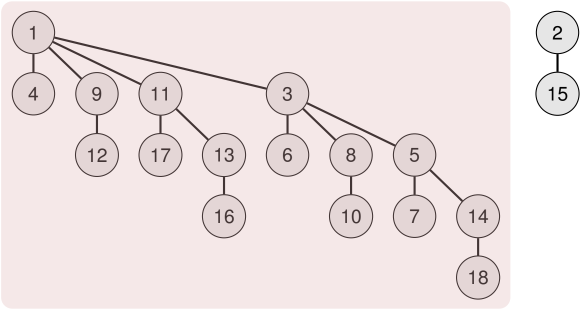
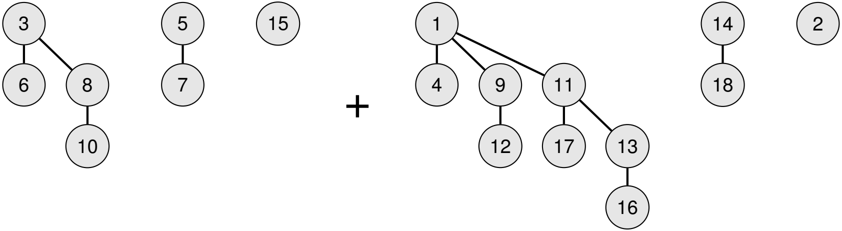
Merging two Binomial Heaps

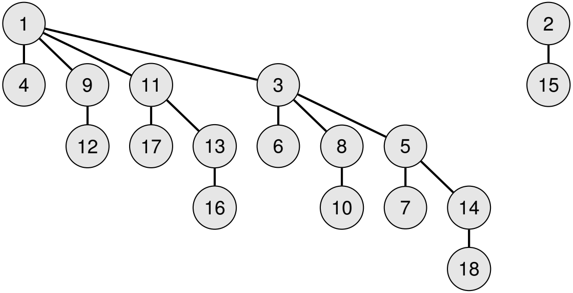


```
0  0  1  1  1   = 7
0  1  0  1  1   = 11
 1  1  1  1
---------------------
1  0  0  1  0   = 18
```



Merging two Binomial Heaps





0	0	1	1	1	= 7
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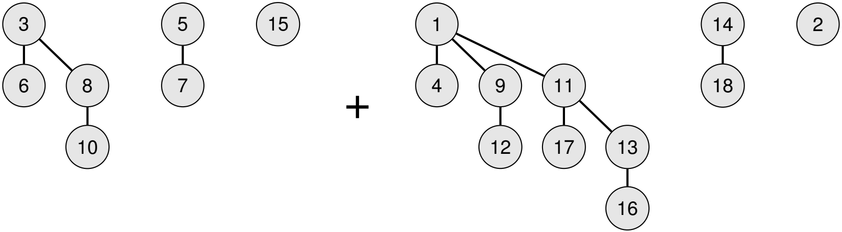
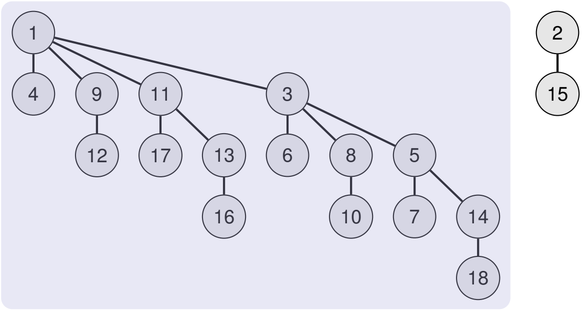
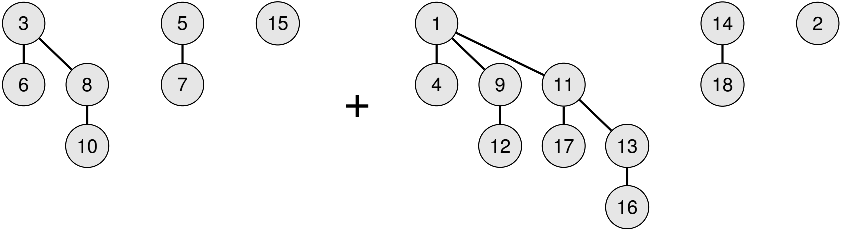
Merging two Binomial Heaps

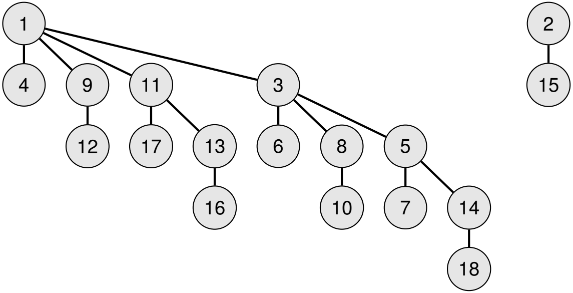




```
0  0  1  1  1   = 7
0  1  0  1  1   = 11
 1  1  1  1
1  0  0  1  0   = 18
```

Merging two Binomial Heaps





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1  0  0  1  0   = 18
```
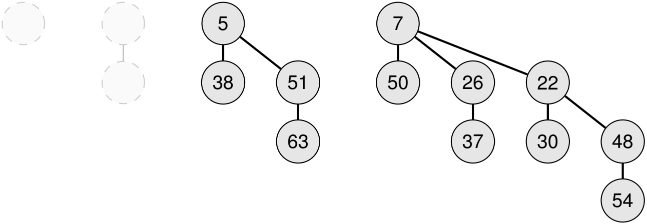

Binomial Heap vs. Fibonacci Heap: Structure

Binomial Heap:

- consists of binomial trees, and every order appears at most once
- immediately tidy up after INSERT or MERGE



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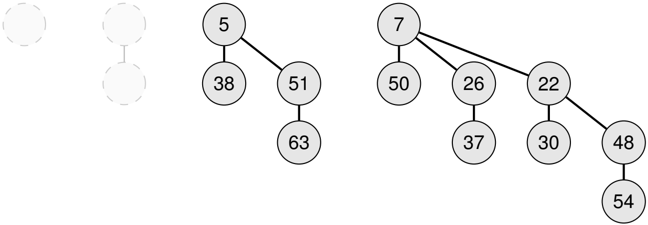
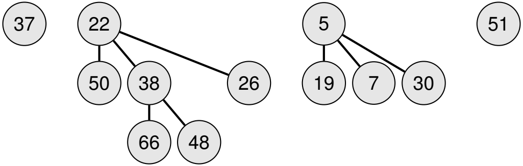


Fibonacci Heap:

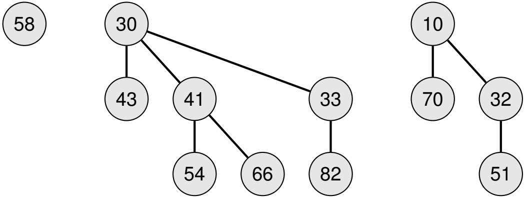
- forest of MIN-HEAPs
- lazily defer tidying up; do it on-the-fly when search for the MIN



Structure of Fibonacci Heaps

Fibonacci Heap

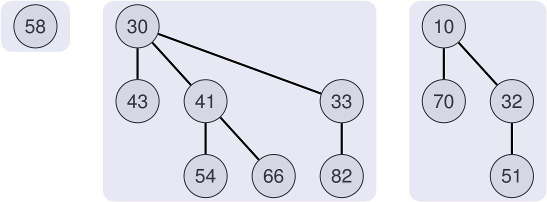
- Forest of MIN-HEAPs



Structure of Fibonacci Heaps

Fibonacci Heap

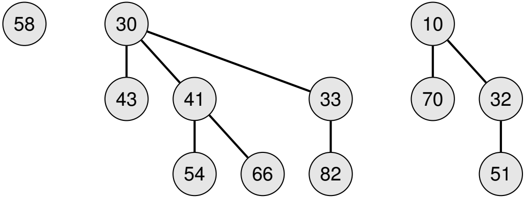
- Forest of MIN-HEAPs



Structure of Fibonacci Heaps

Fibonacci Heap

- Forest of MIN-HEAPs
- Nodes can be marked (roots are always unmarked)



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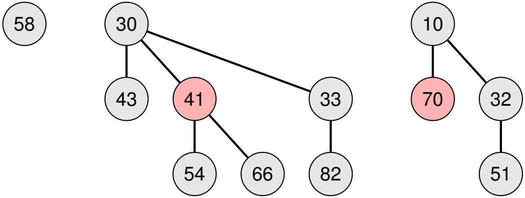
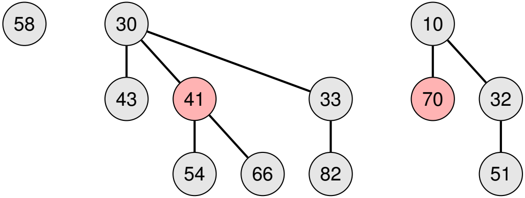


Structure of Fibonacci Heaps

Fibonacci Heap

- Forest of MIN-HEAPs
- Nodes can be marked (roots are always unmarked)
- Tree roots are stored in a circular, doubly-linked list



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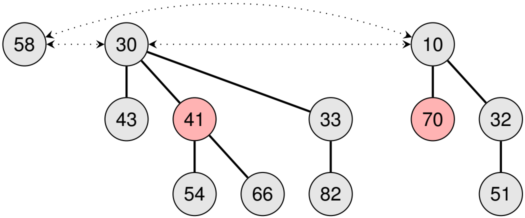
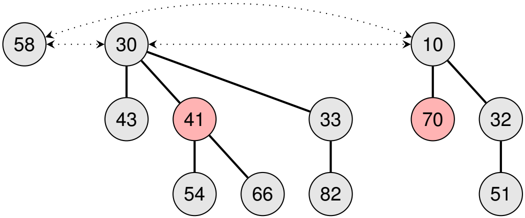


Structure of Fibonacci Heaps

Fibonacci Heap

- Forest of MIN-HEAPs
- Nodes can be marked (roots are always unmarked)
- Tree roots are stored in a circular, doubly-linked list
- Min-Pointer pointing to the smallest element



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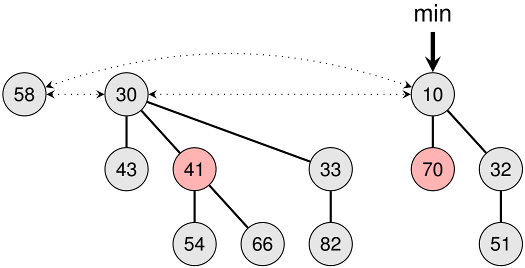

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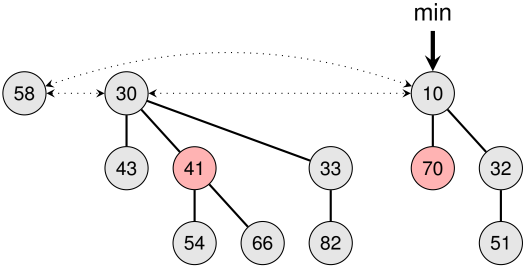
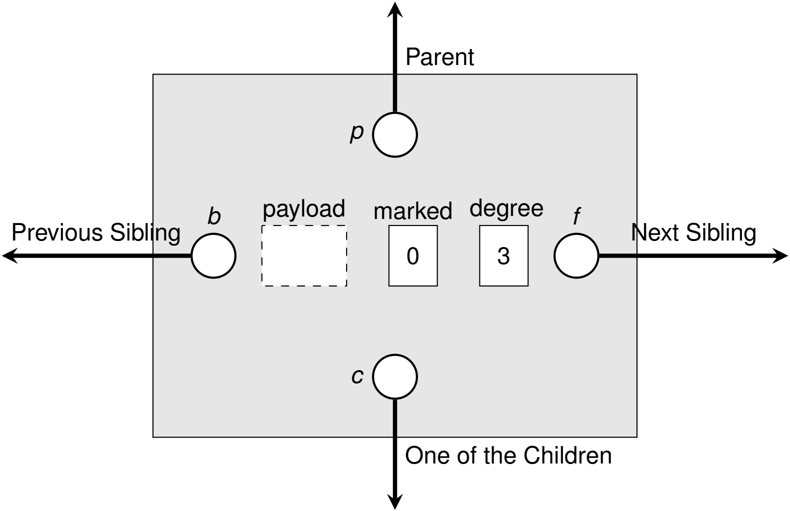
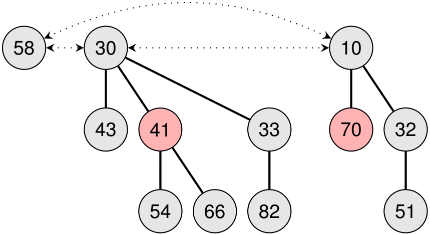


How do we implement a Fibonacci Heap?

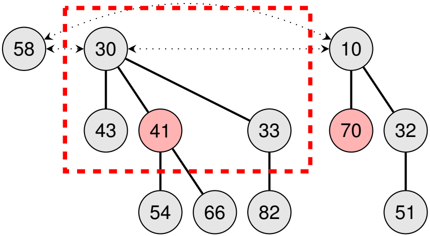
A single Node



Magnifying a Four-Node Portion



Magnifying a Four-Node Portion



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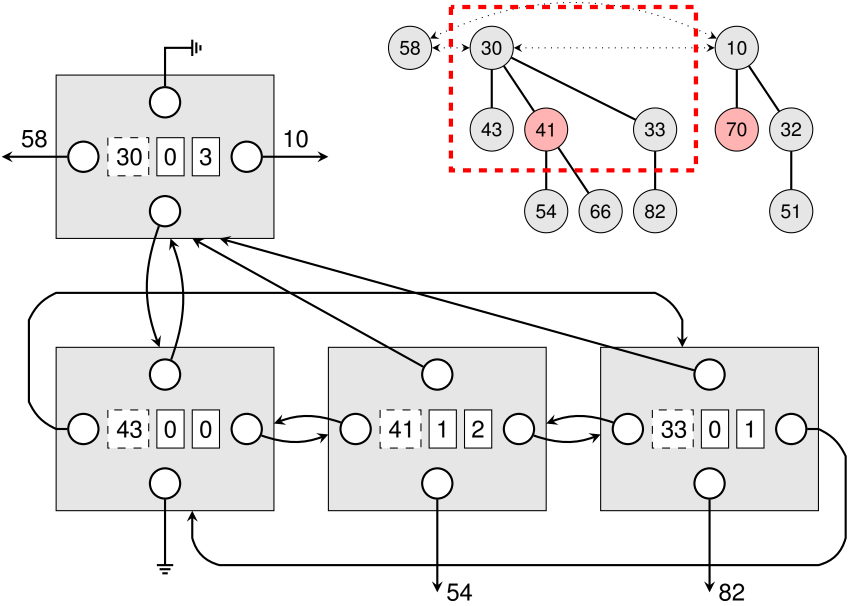


Outline

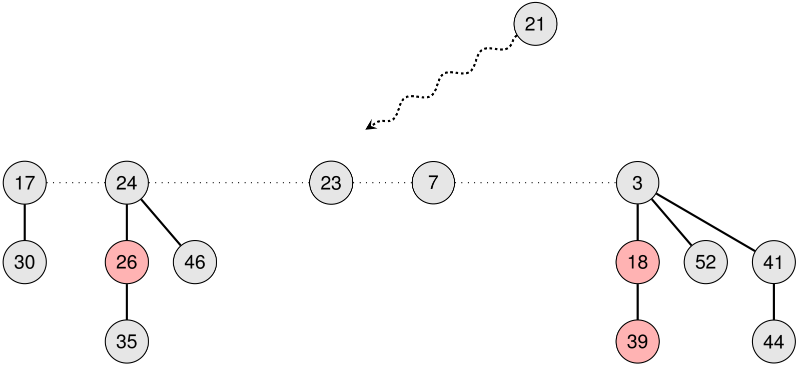
Structure

Operations

Glimpse at the Analysis

Fibonacci Heap: INSERT

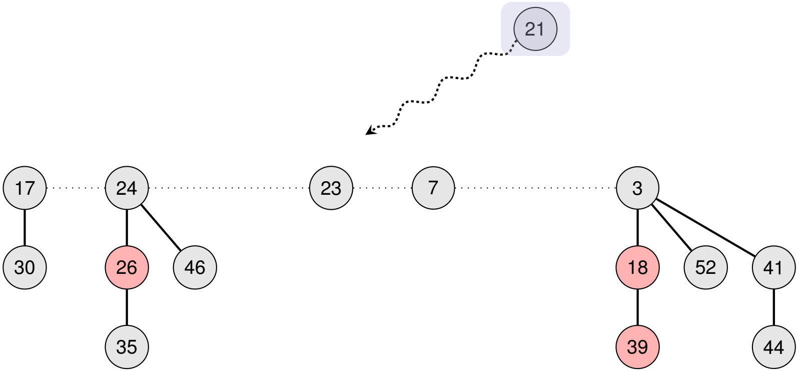
INSERT



Fibonacci Heap: INSERT

INSERT

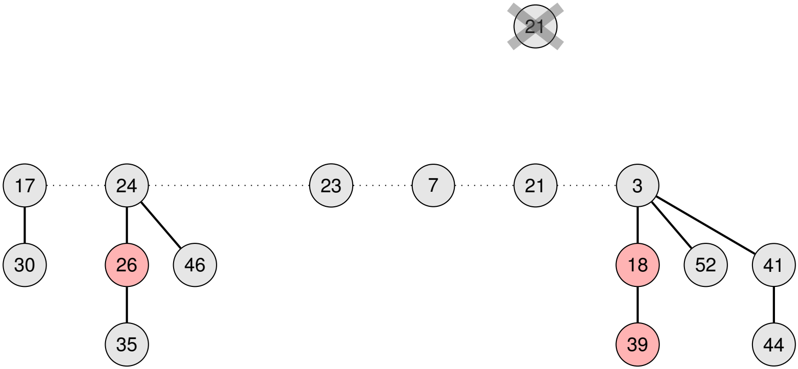
- Create a singleton tree



Fibonacci Heap: INSERT

INSERT

- Create a singleton tree
- Add to root list



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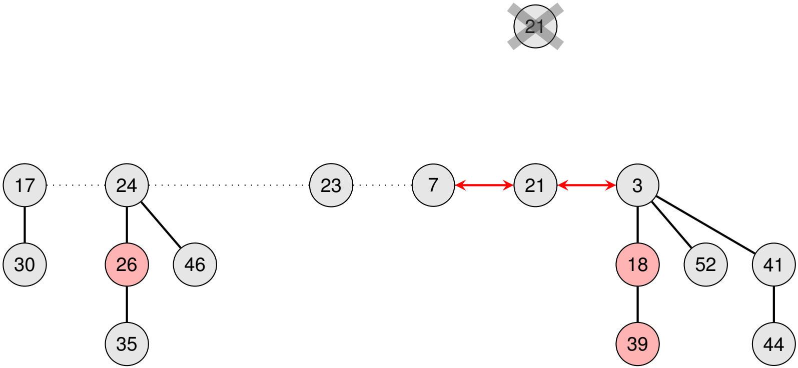

Fibonacci Heap: INSERT

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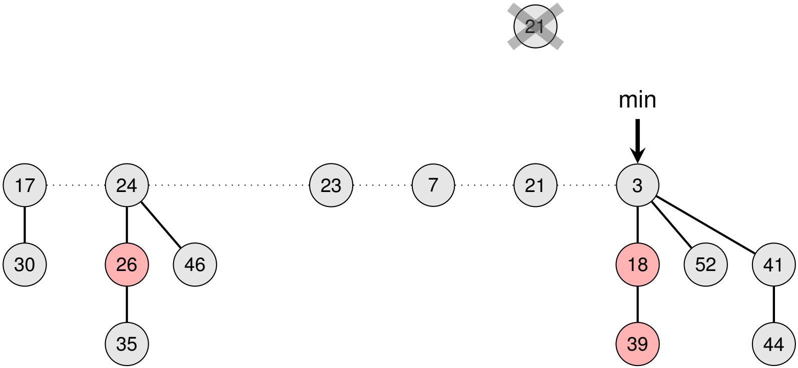
- Create a singleton tree
- Add to root list and update min-pointer (if necessary)



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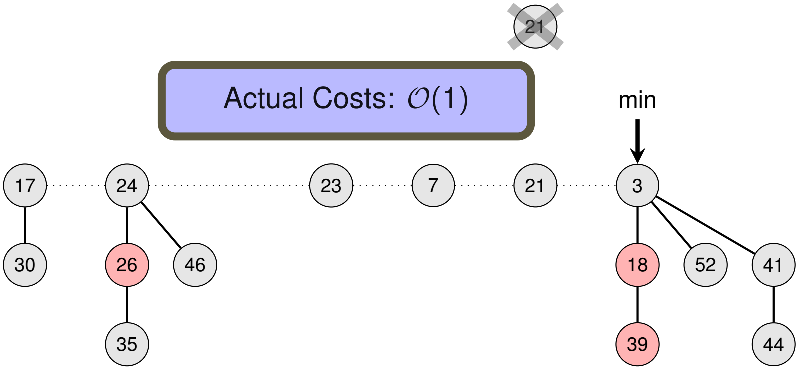
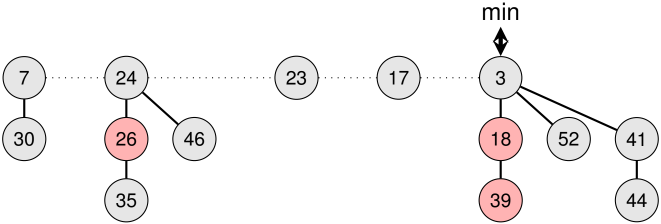


Fibonacci Heap: EXTRACT-MIN

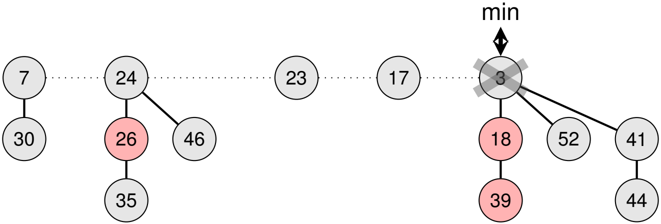
EXTRACT-MIN



Fibonacci Heap: EXTRACT-MIN

EXTRACT-MIN

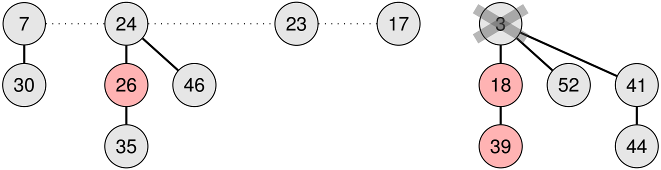
- Delete min



Fibonacci Heap: EXTRACT-MIN

EXTRACT-MIN

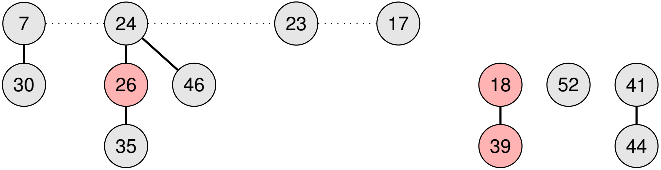
- Delete min ✓



Fibonacci Heap: EXTRACT-MIN

EXTRACT-MIN

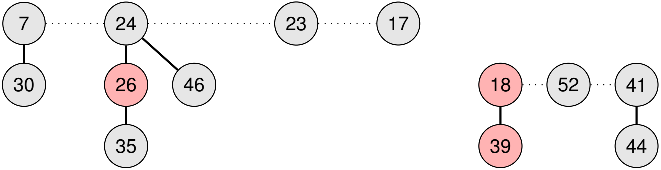
- Delete min ✓
- Meld childen into root list and unmark them



Fibonacci Heap: Extract-Min

Extract-Min

- Delete min ✓
- Meld childen into root list and unmark them



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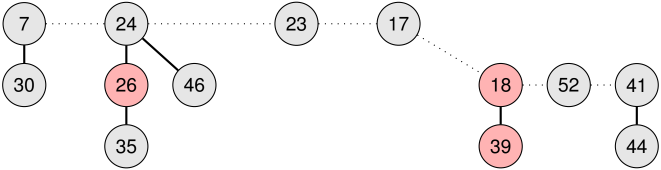

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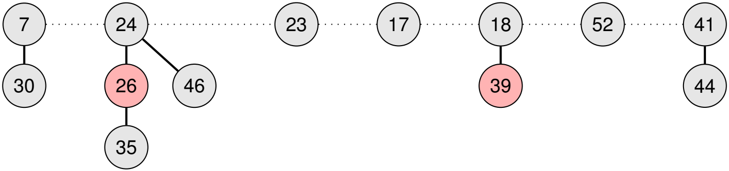

Fibonacci Heap: EXTRACT-MIN

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- Delete min ✓
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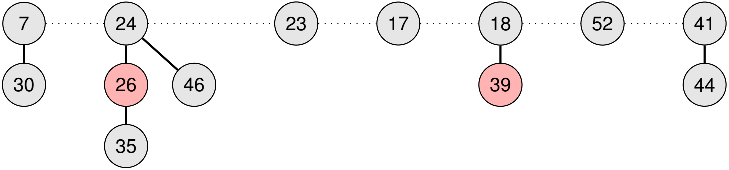
- Consolidate so that no roots have the same degree



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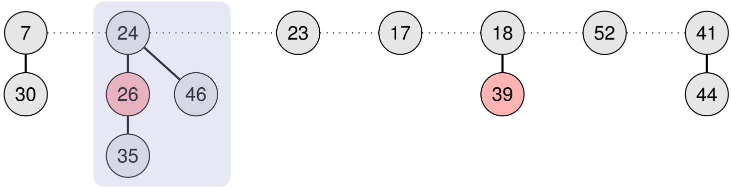

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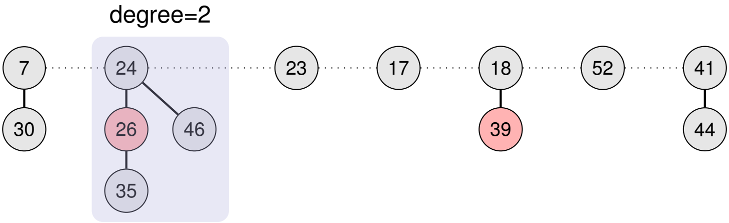

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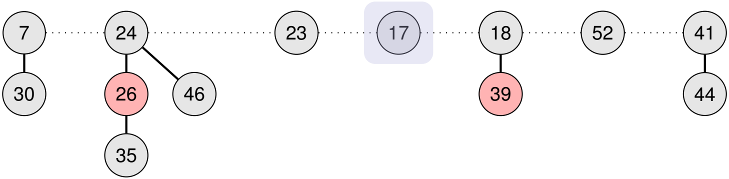

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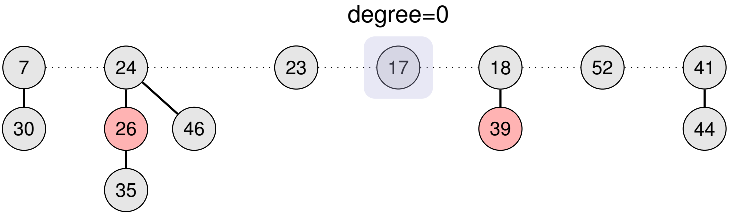

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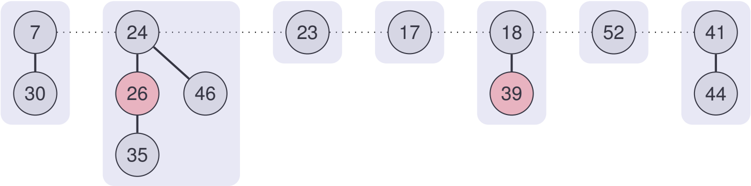

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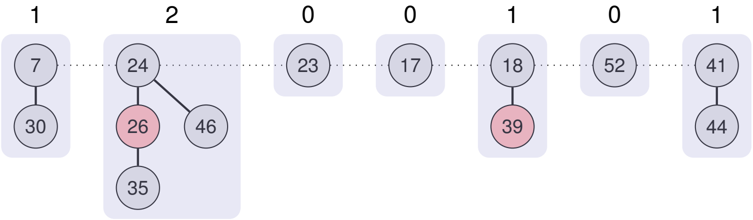

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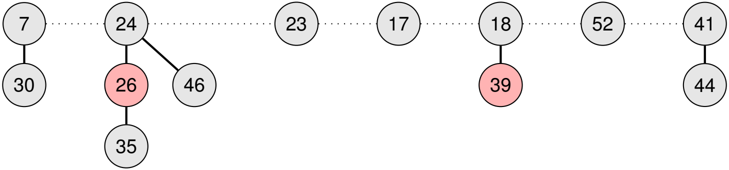
degree

0	1	2	3



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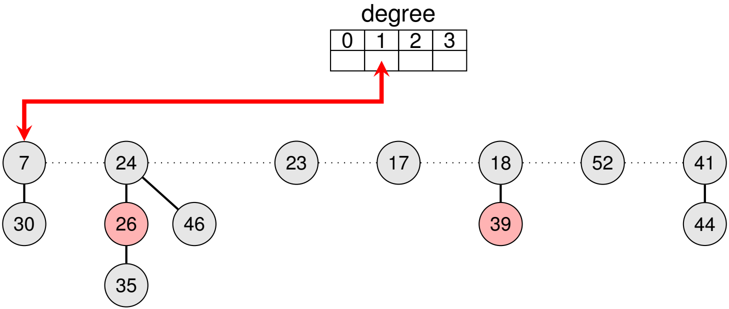

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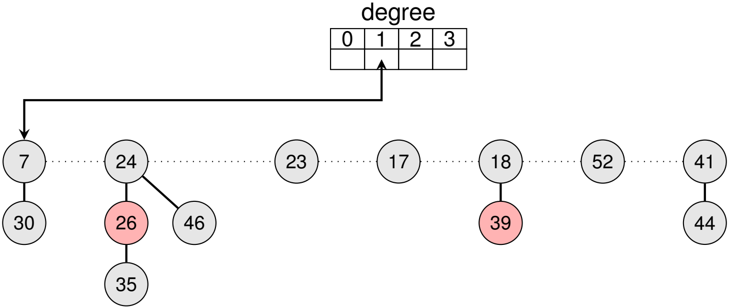

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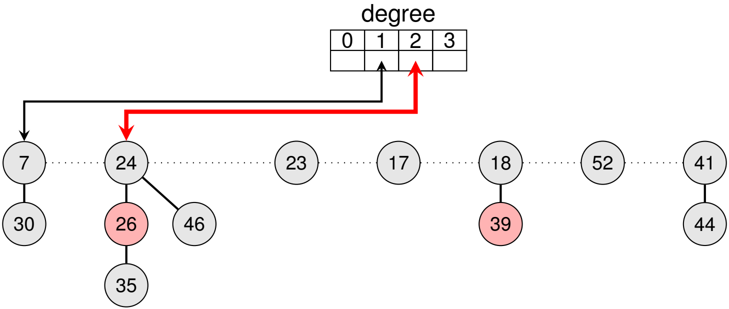

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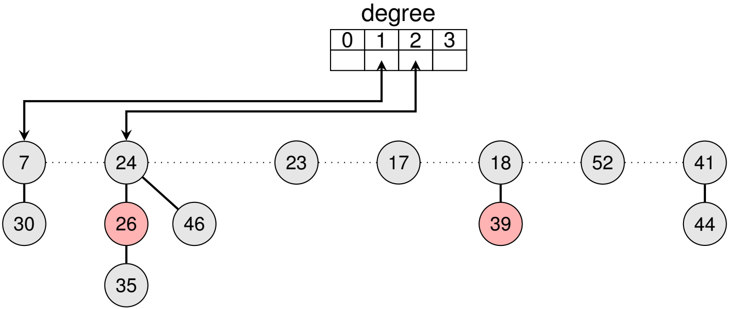

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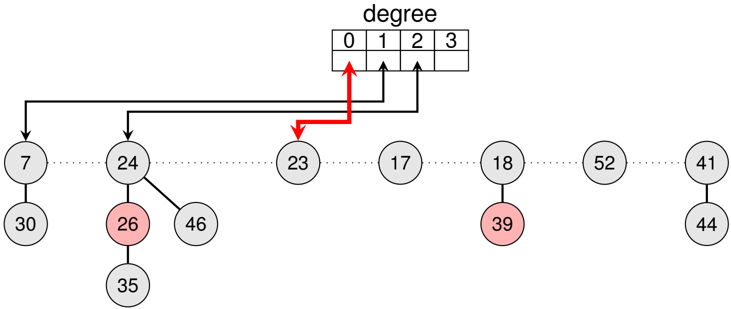

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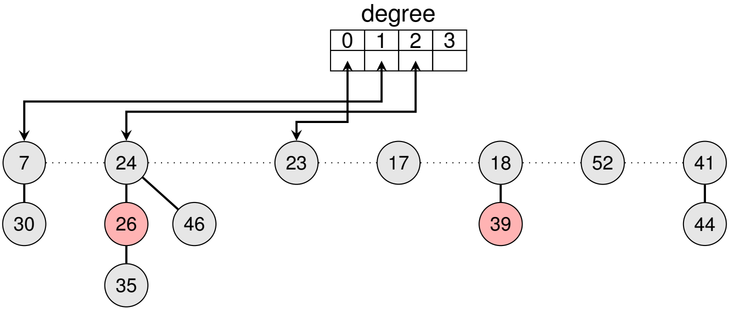

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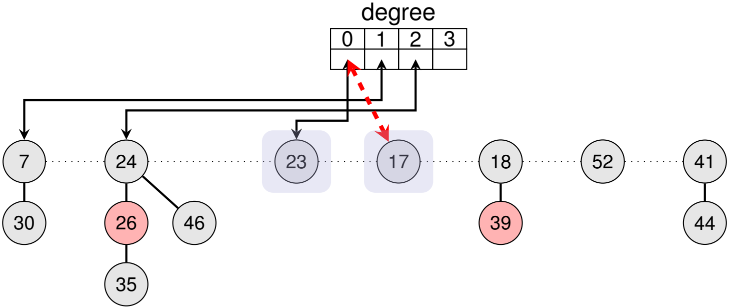

Fibonacci Heap: Extract-Min

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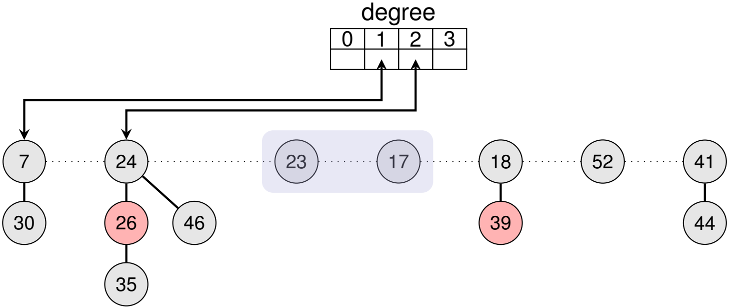

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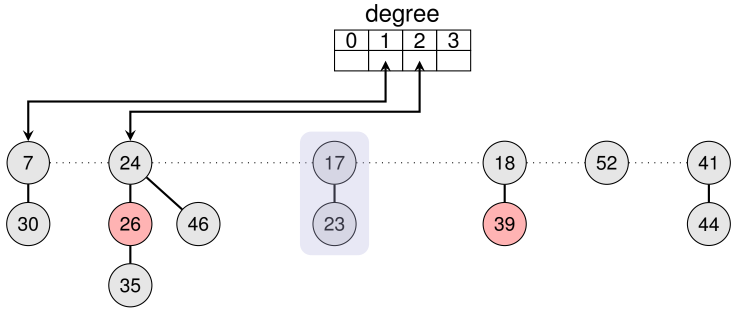

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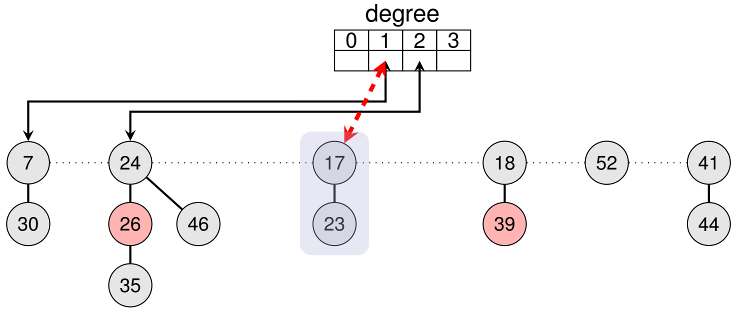

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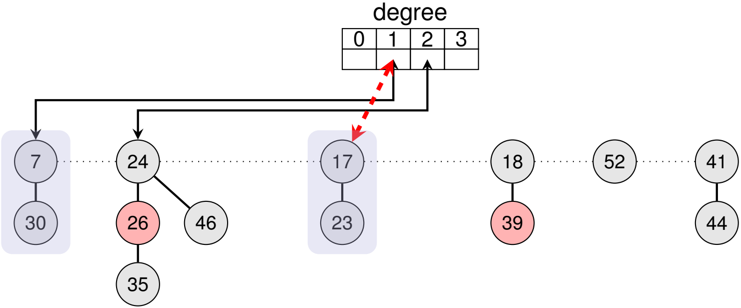

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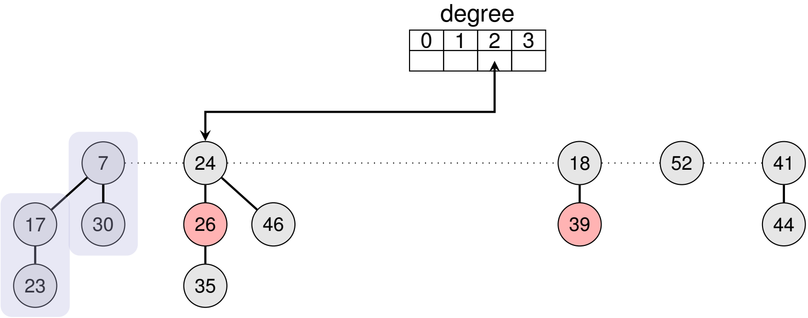

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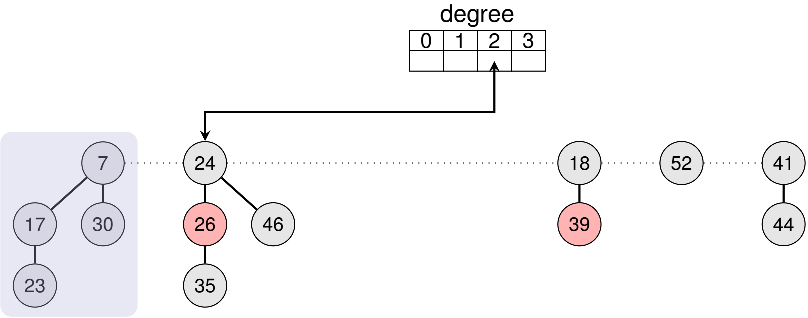

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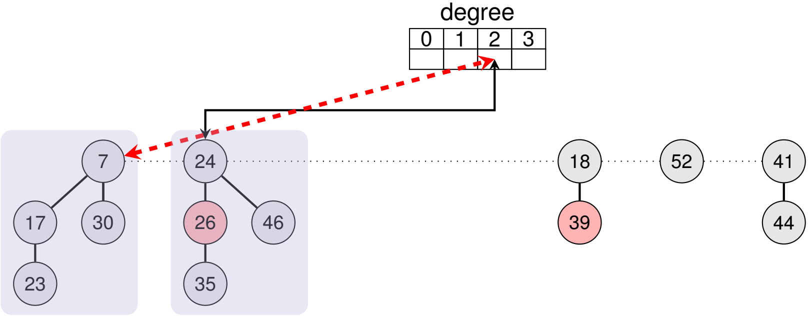

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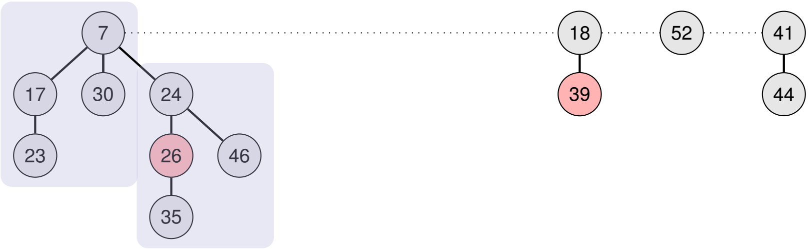
degree

0	1	2	3



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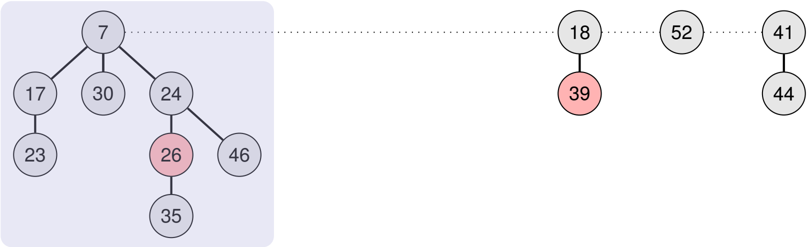

Fibonacci Heap: Extract-Min

Extract-Min

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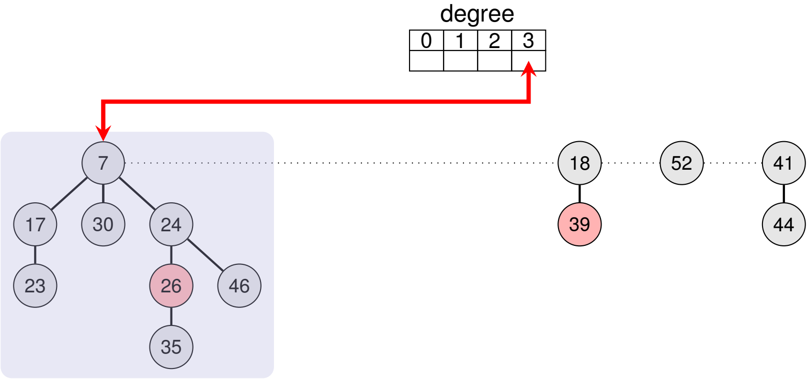

Fibonacci Heap: EXTRACT-MIN

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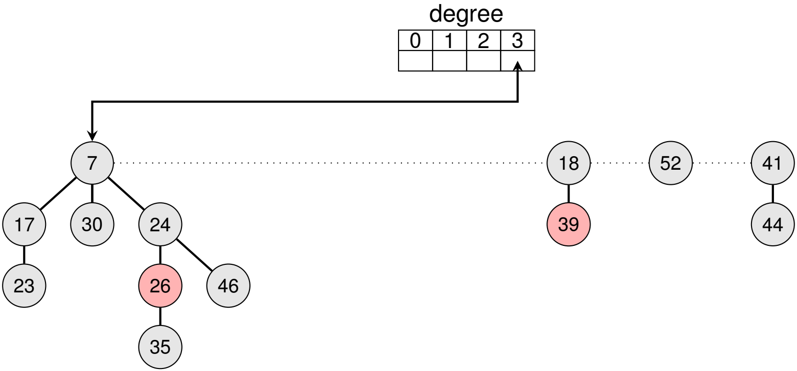

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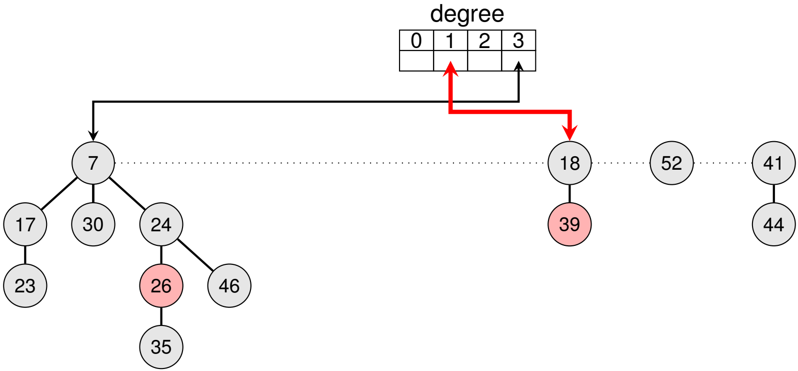

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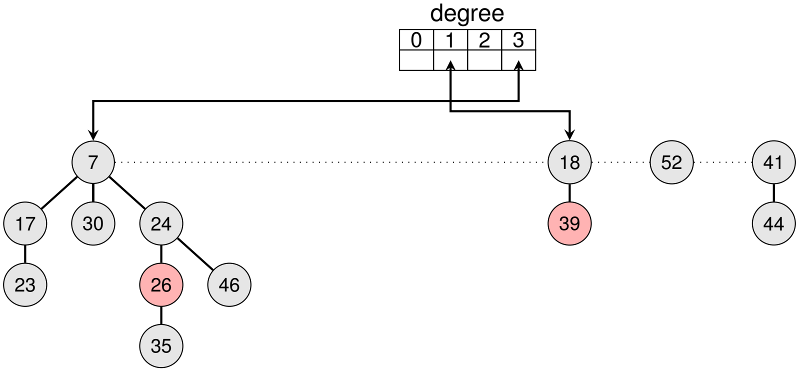

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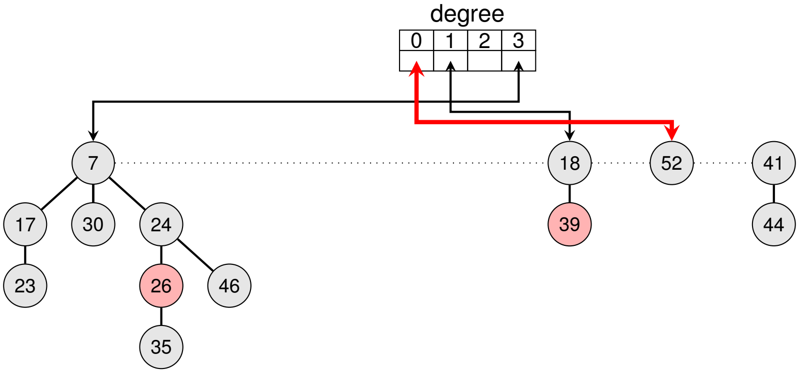

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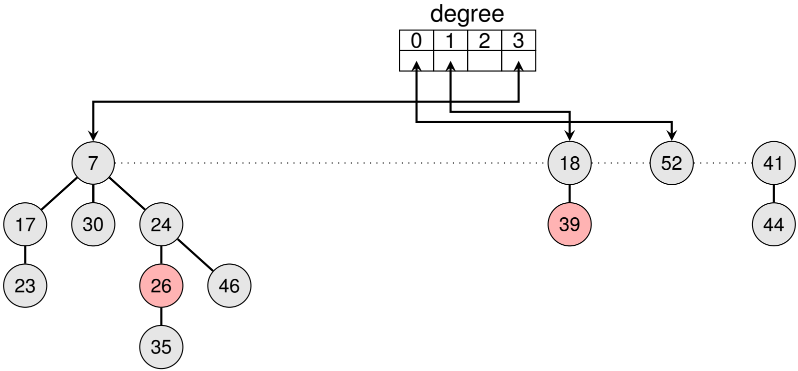

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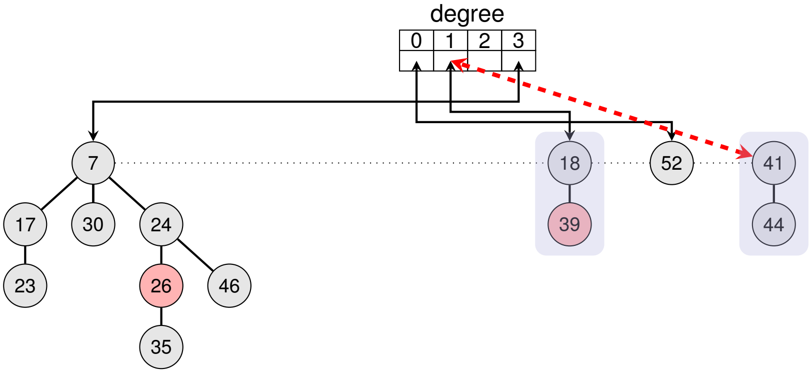

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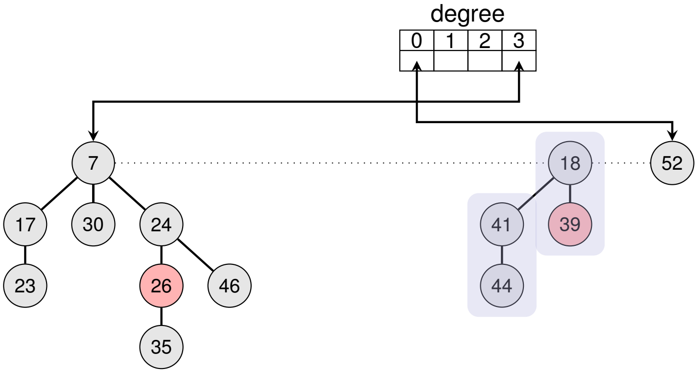

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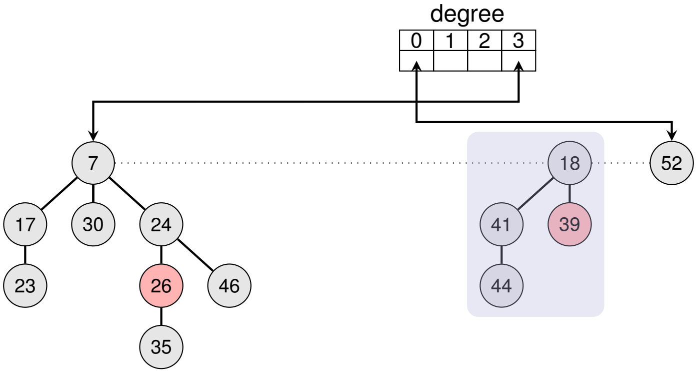

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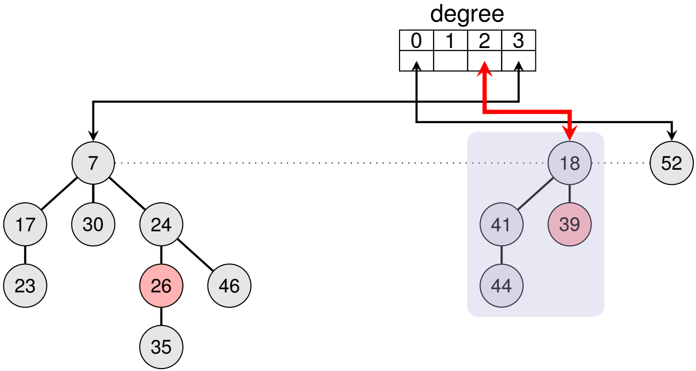

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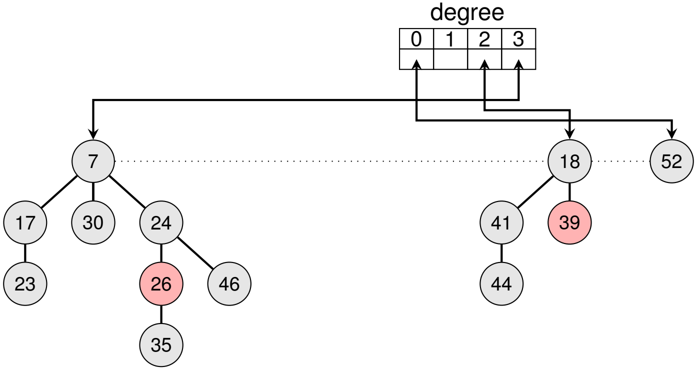

Fibonacci Heap: EXTRACT-MIN

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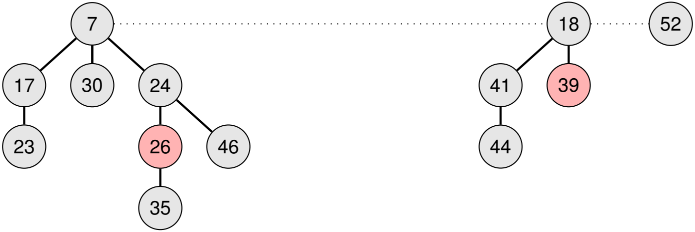
- Consolidate so that no roots have the same degree (# children) ✓
- Update minimum



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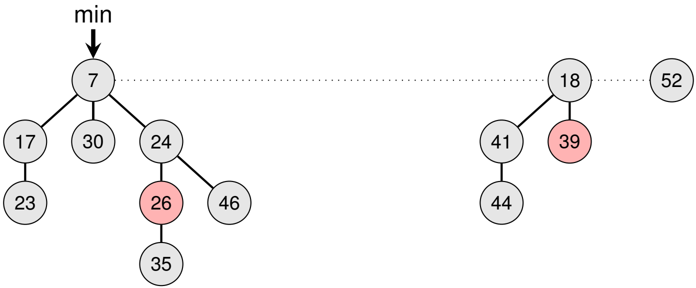

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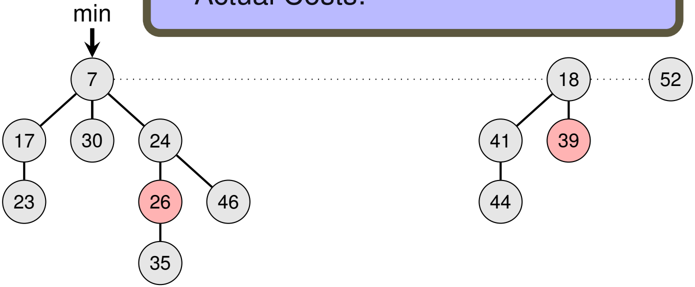

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Every root becomes child of another root at most once!

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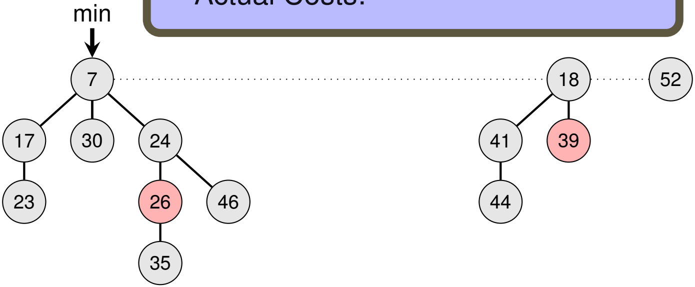

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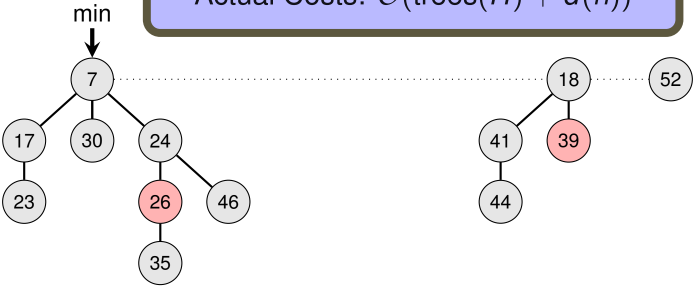
$d(n)$ is the maximum degree of a root in any Fibonacci heap of size n

Actual Costs: $\mathcal{O}(\text{trees}(H) + d(n))$



Fibonacci Heap: DECREASE-KEY (First Attempt)

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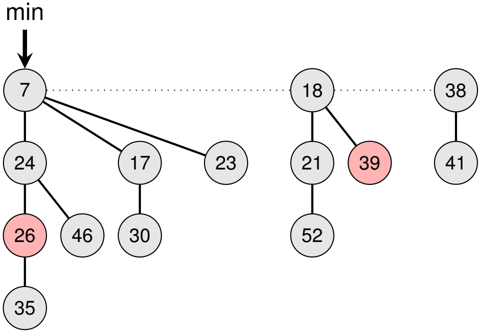

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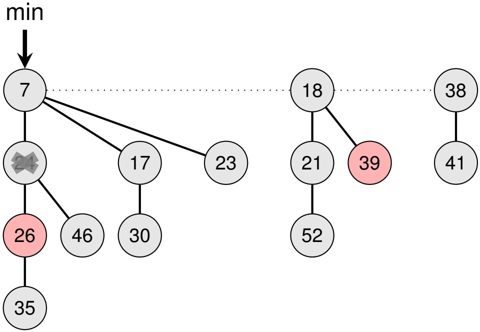

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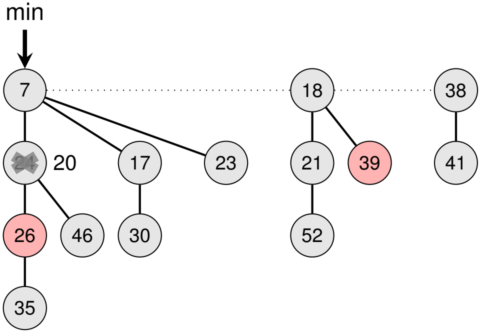

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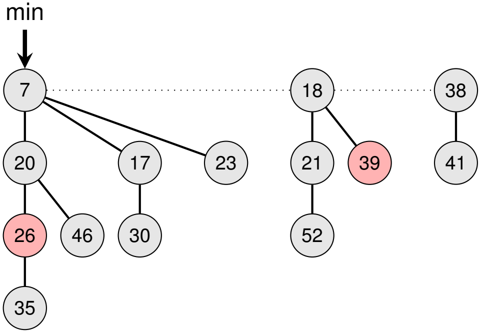

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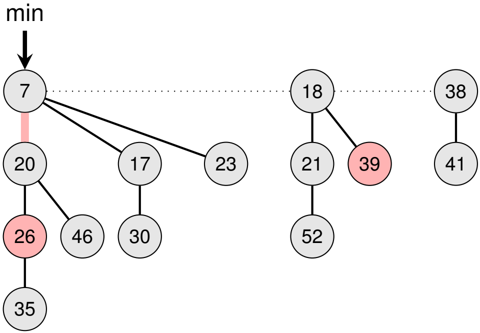

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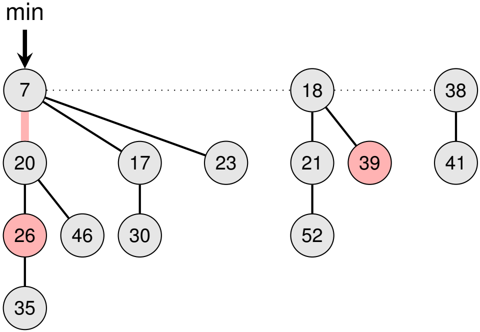

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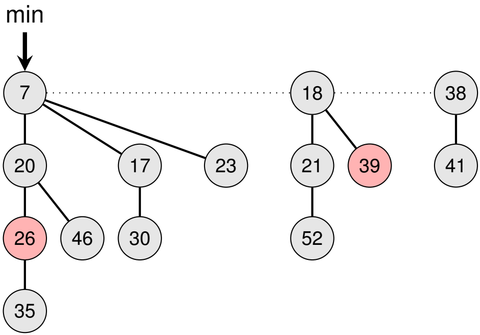

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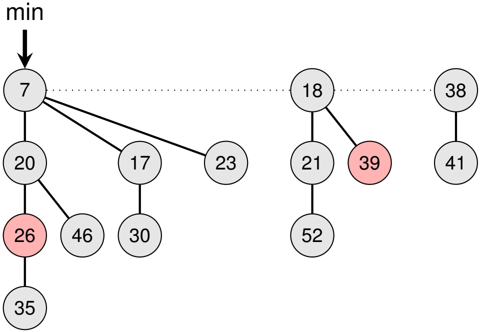

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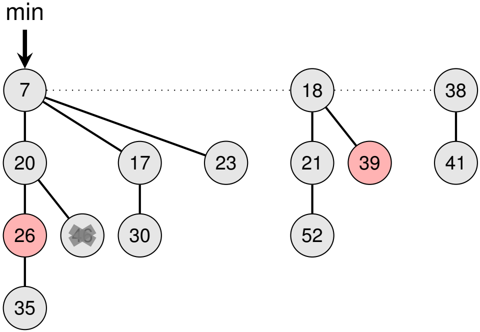

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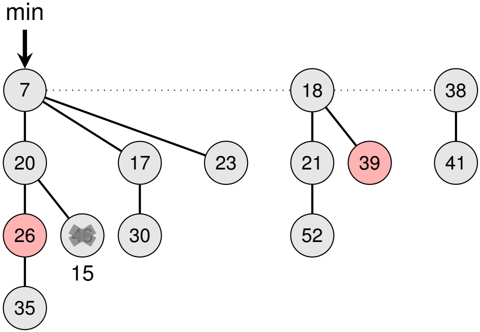

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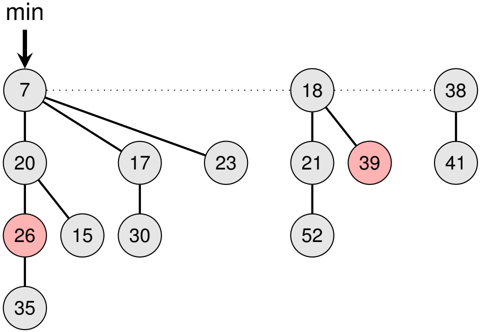

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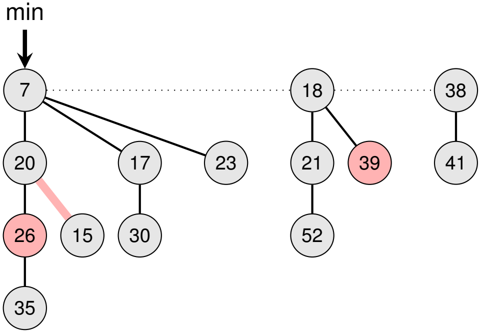

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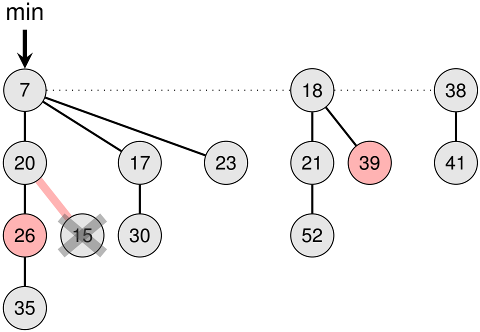

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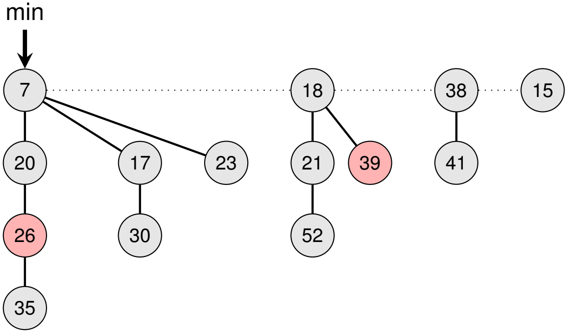

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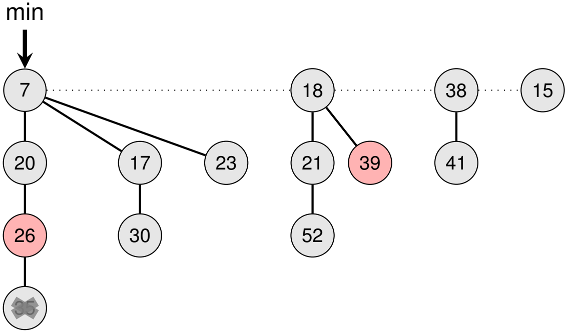

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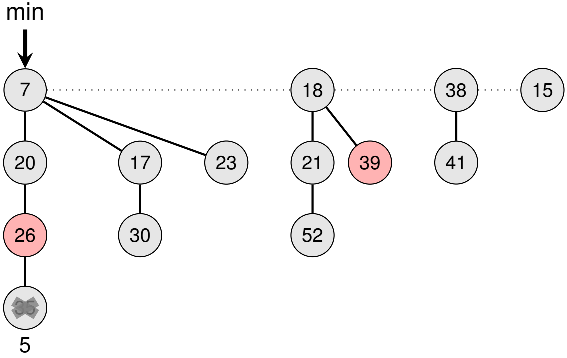

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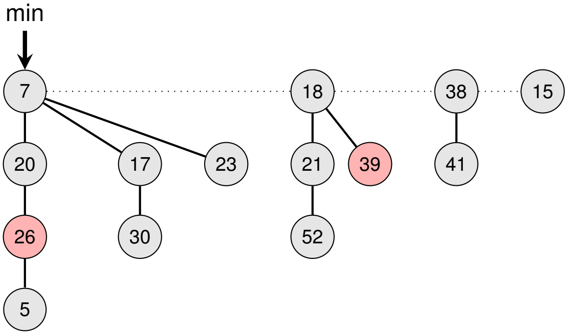

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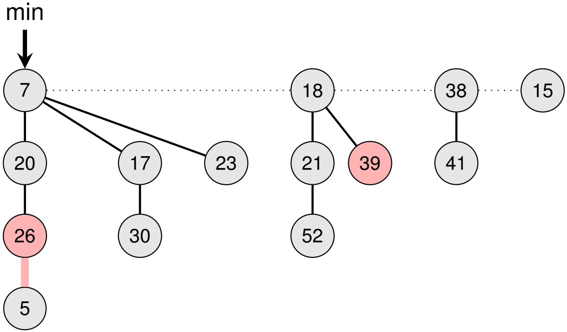

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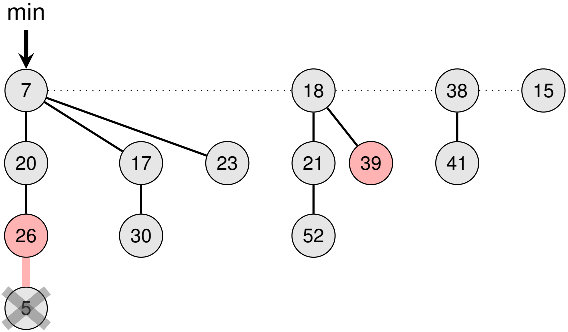

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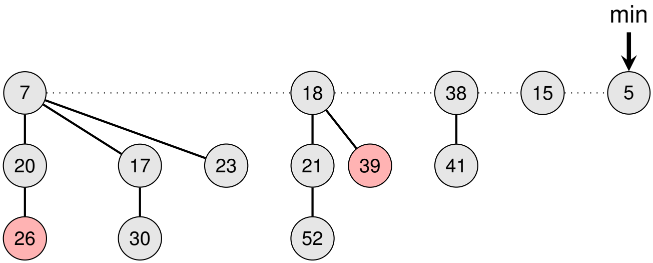

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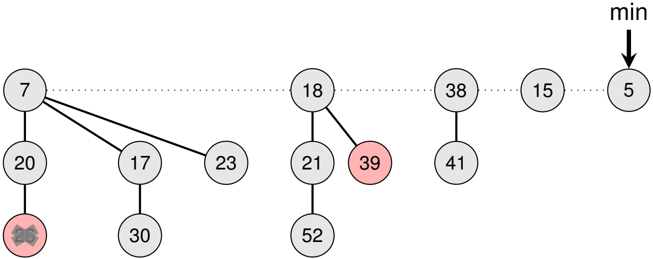

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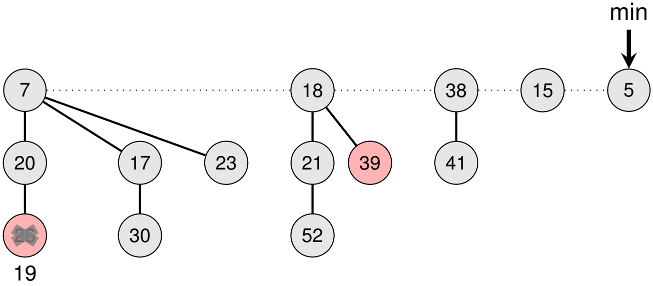

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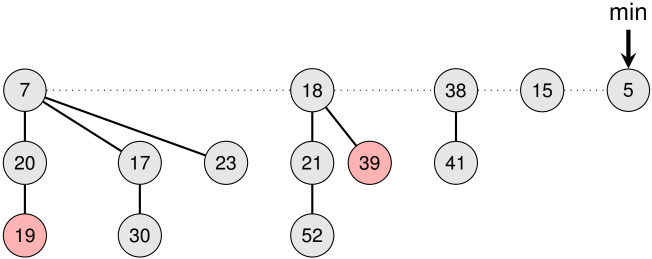

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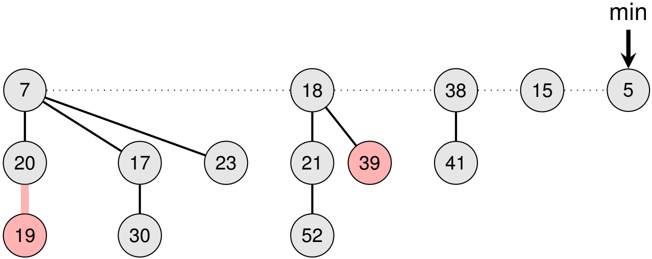

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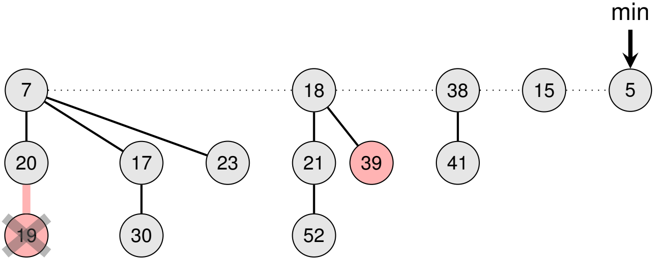

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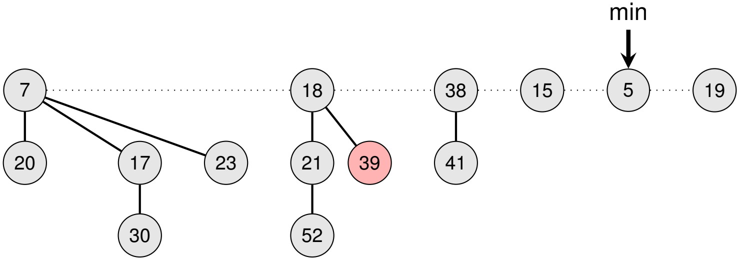

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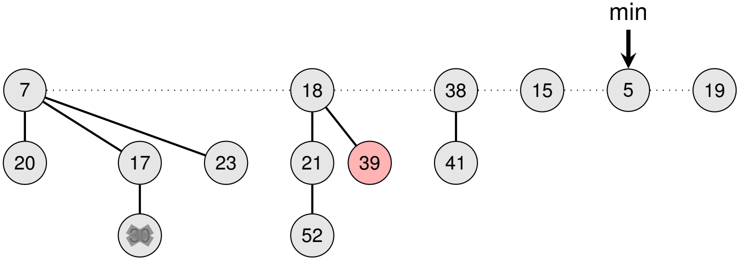

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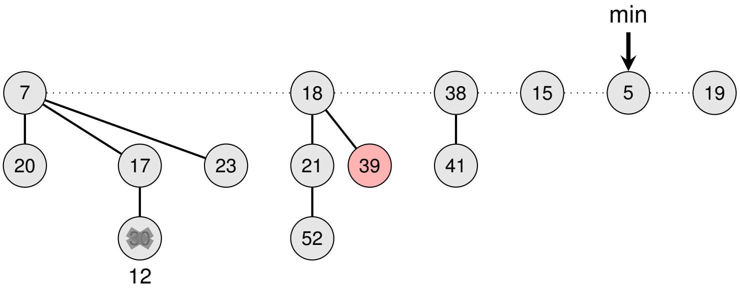

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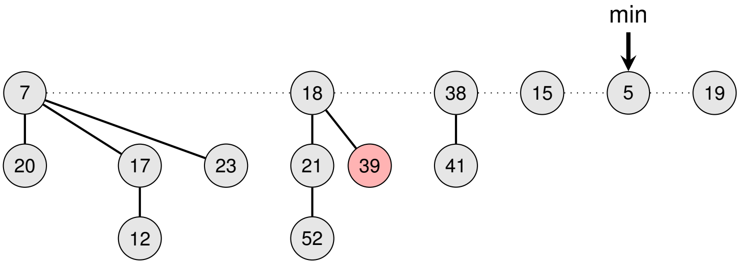

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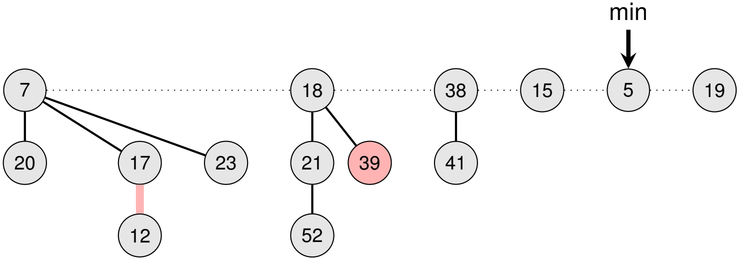

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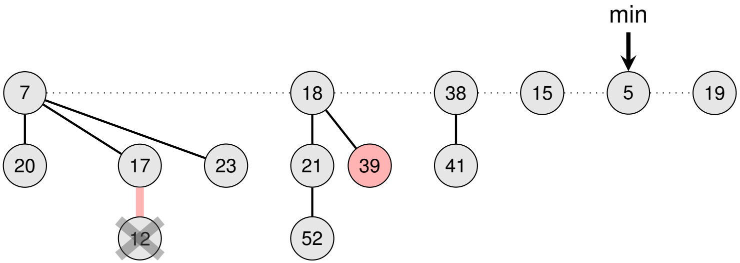

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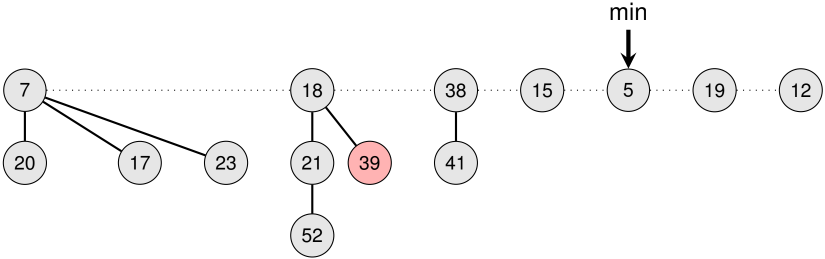

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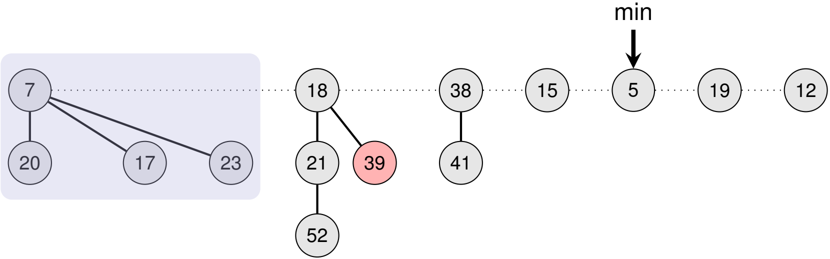

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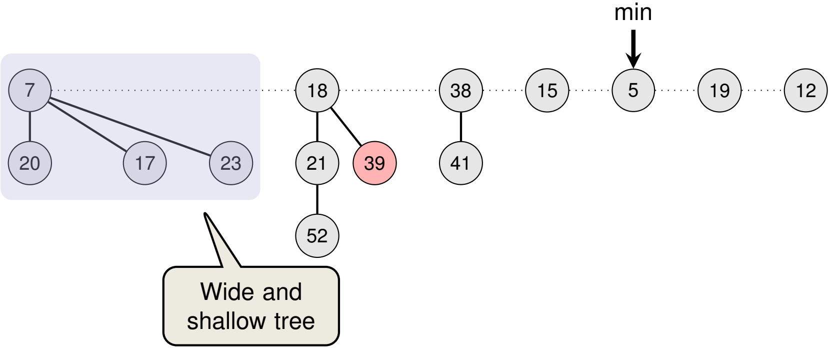

Wide and shallow tree

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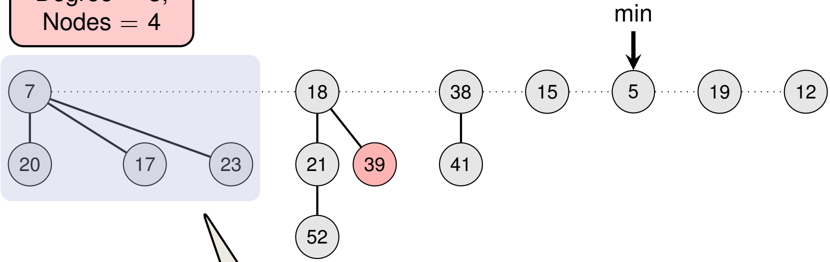
Degree = 3, Nodes = 4



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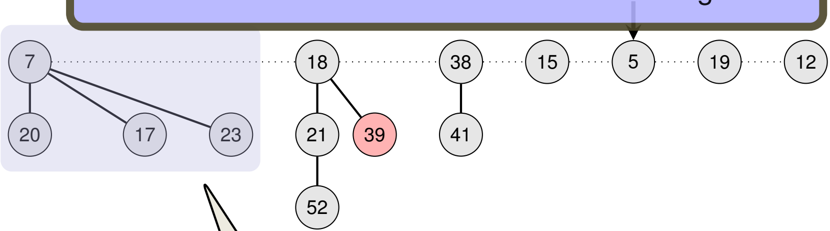
Peculiar Constraint: Make sure that each non-root node loses at most one child before becoming root



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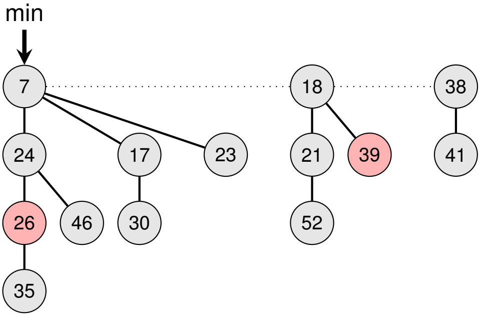

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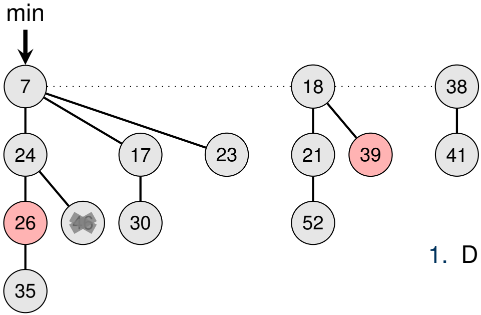

1. Decrease-Key 46 ⇝ 15

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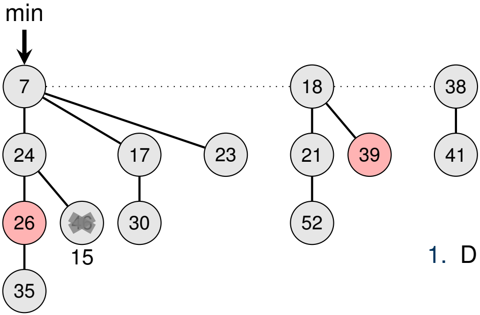

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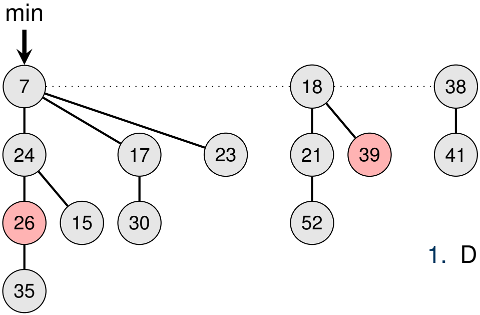

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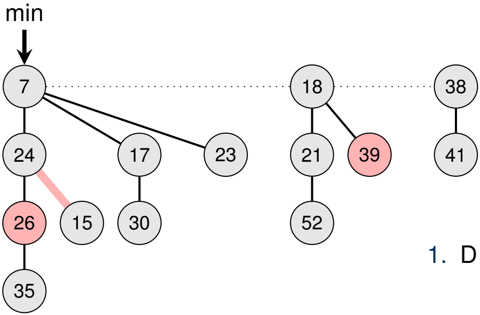

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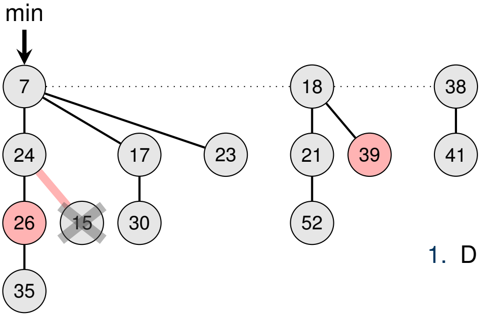

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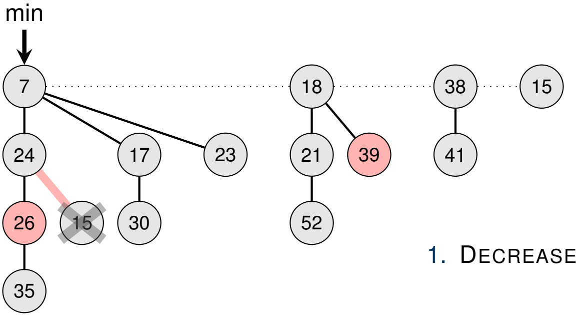

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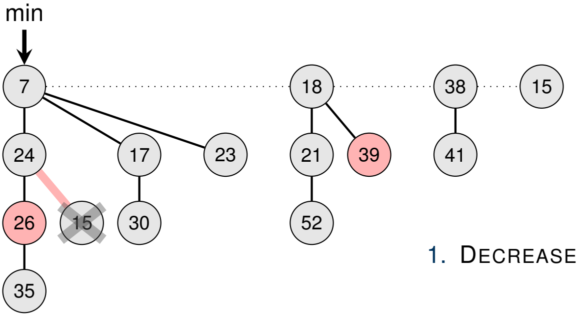

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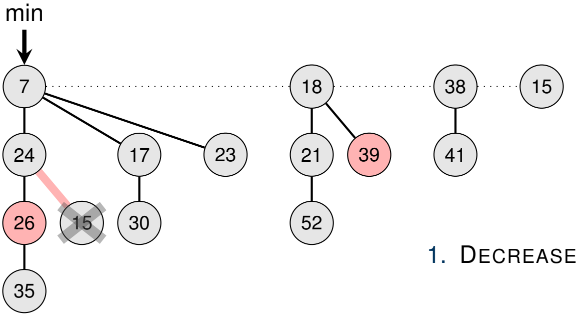

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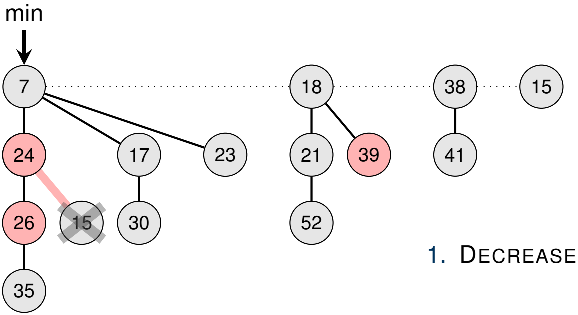

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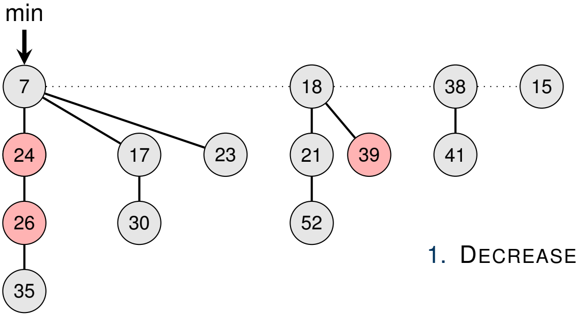

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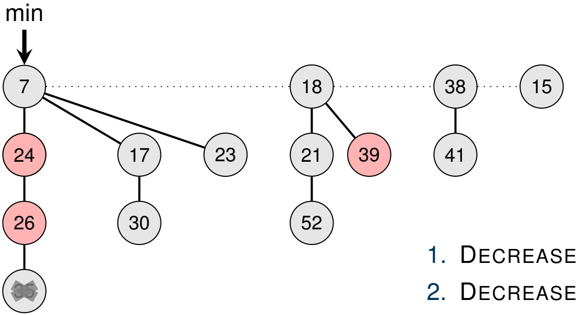

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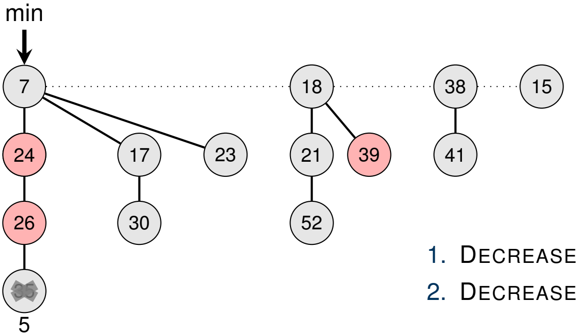

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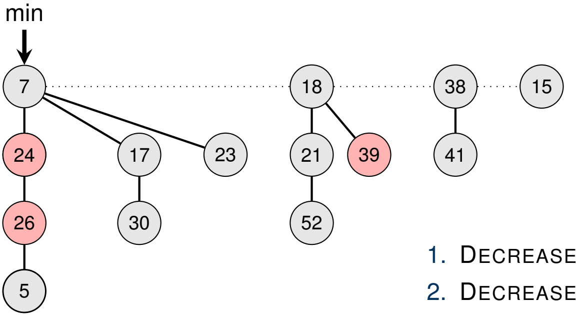

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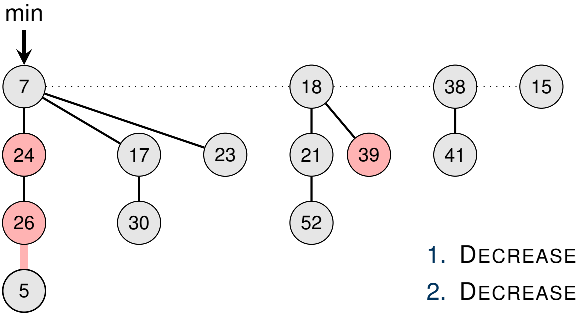

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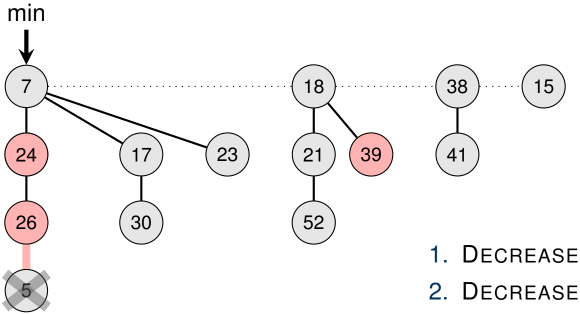

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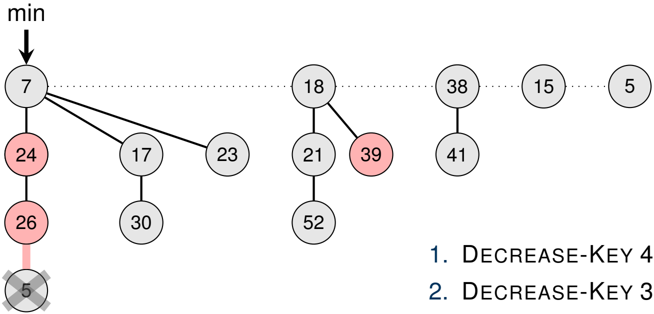

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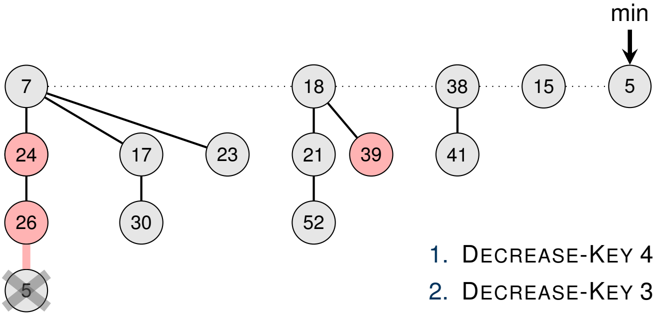

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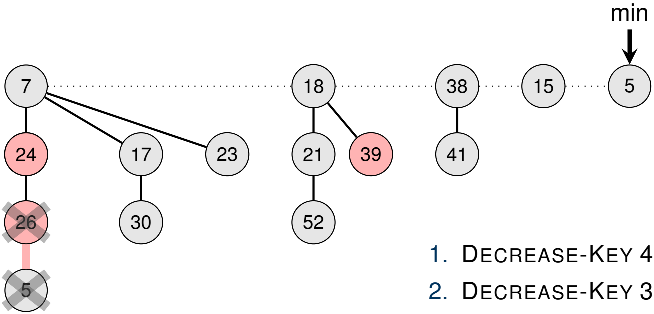

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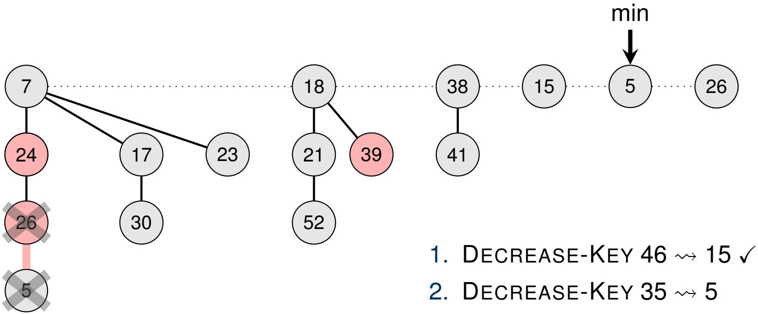

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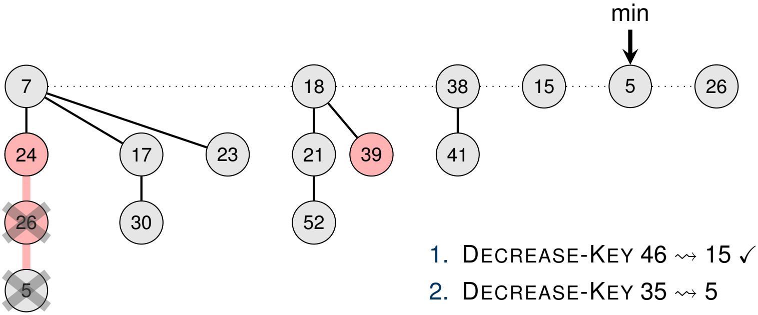

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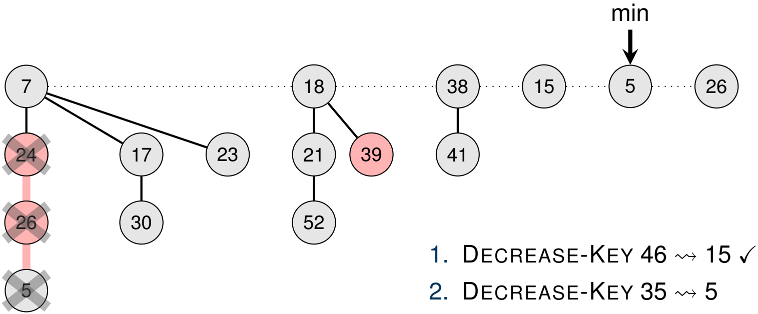

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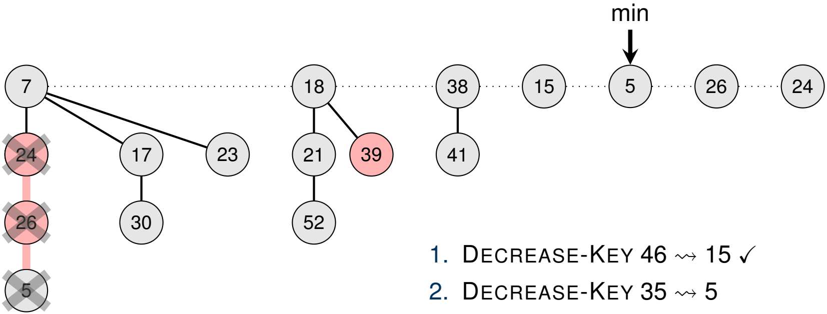

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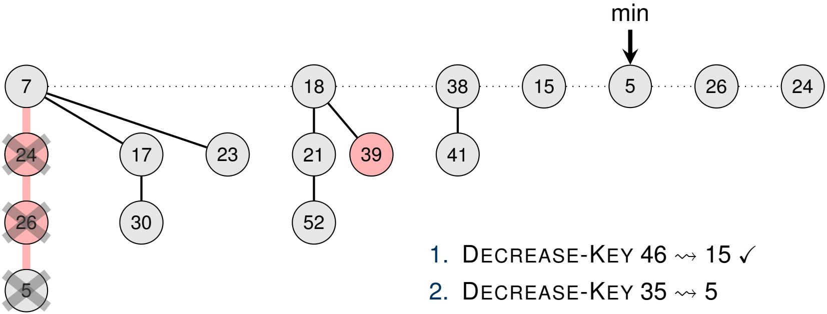

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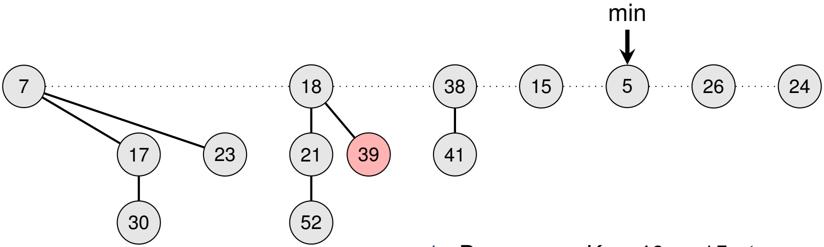

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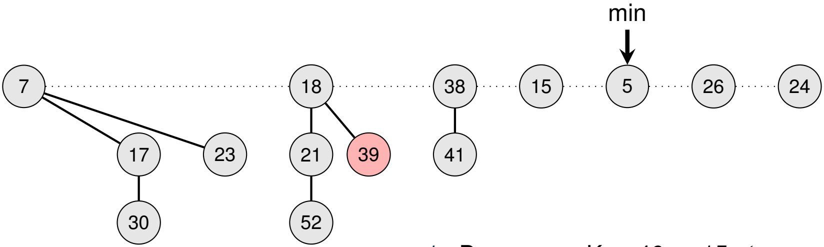

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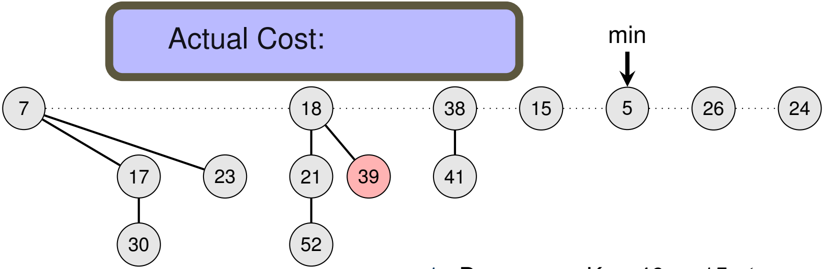

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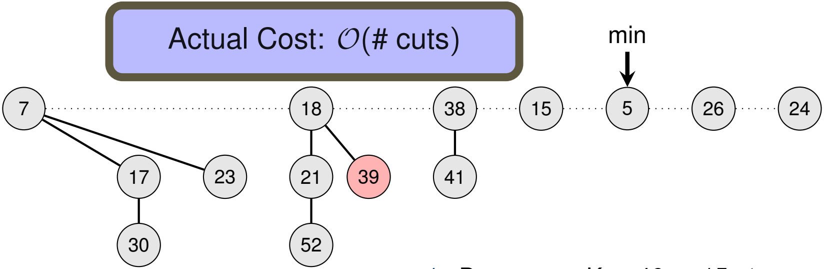
Actual Cost: $\mathcal{O}(\text{\# cuts})$



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Outline

Structure

Operations

Glimpse at the Analysis

Amortized Analysis via Potential Method

- INSERT: actual $\mathcal{O}(1)$
- EXTRACT-MIN: actual $\mathcal{O}(\text{trees}(H) + d(n))$
- DECREASE-KEY: actual $\mathcal{O}(\#\text{ cuts}) \leq \mathcal{O}(\text{marks}(H))$

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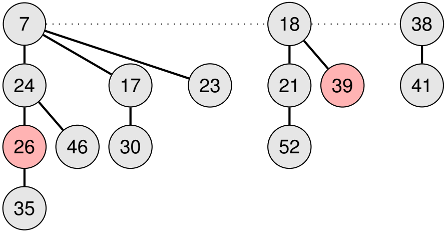
- DECREASE-KEY: actual $\mathcal{O}(\#\text{ cuts}) \leq \mathcal{O}(\text{marks}(H))$

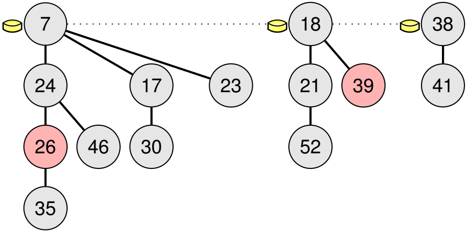
$$\Phi(H) = \text{trees}(H) + 2 \cdot \text{marks}(H)$$



Amortized Analysis via Potential Method

- INSERT: actual $\mathcal{O}(1)$
- EXTRACT-MIN: actual $\mathcal{O}(\text{trees}(H) + d(n))$
- DECREASE-KEY: actual $\mathcal{O}(\#\text{ cuts}) \leq \mathcal{O}(\text{marks}(H))$

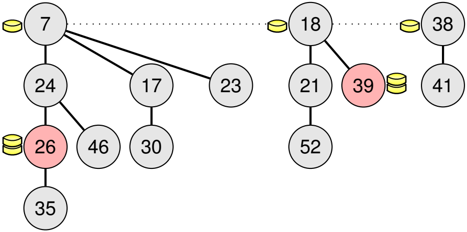
$$\boxed{\Phi(H) = \text{trees}(H) + 2 \cdot \text{marks}(H)}$$



Amortized Analysis via Potential Method

- INSERT: actual $\mathcal{O}(1)$
- EXTRACT-MIN: actual $\mathcal{O}(\text{trees}(H) + d(n))$
- DECREASE-KEY: actual $\mathcal{O}(\#\text{ cuts}) \leq \mathcal{O}(\text{marks}(H))$

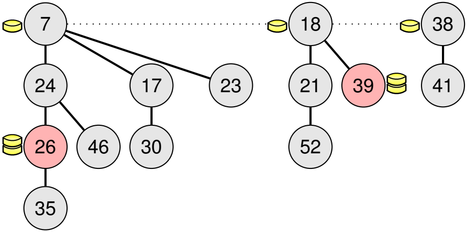
$$\Phi(H) = \text{trees}(H) + 2 \cdot \text{marks}(H)$$



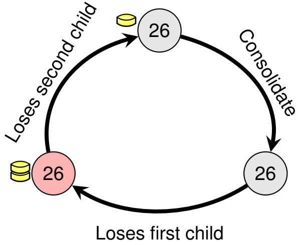
Amortized Analysis via Potential Method

- INSERT: actual $\mathcal{O}(1)$ amortized $\mathcal{O}(1)$
- EXTRACT-MIN: actual $\mathcal{O}(\text{trees}(H) + d(n))$ amortized $\mathcal{O}(d(n))$
- DECREASE-KEY: actual $\mathcal{O}(\#\text{ cuts}) \leq \mathcal{O}(\text{marks}(H))$ amortized $\mathcal{O}(1)$

$$\Phi(H) = \text{trees}(H) + 2 \cdot \text{marks}(H)$$



Lifecycle of a node



26
Consolidate
26
Loses first child
26
Loses second child