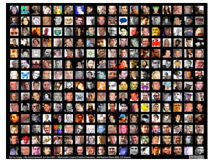

Social and Technological Network Analysis

Lecture 6: Network Robustness and Applications

Dr. Cecilia Mascolo

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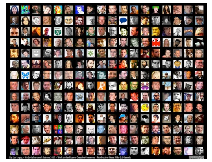
In This Lecture



- We revisit power-law networks and define the concept of robustness
- We show the effect of random and targeted attacks on power law networks versus random networks
- We discuss applications to various networks

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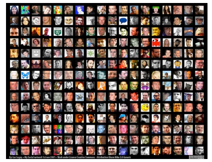
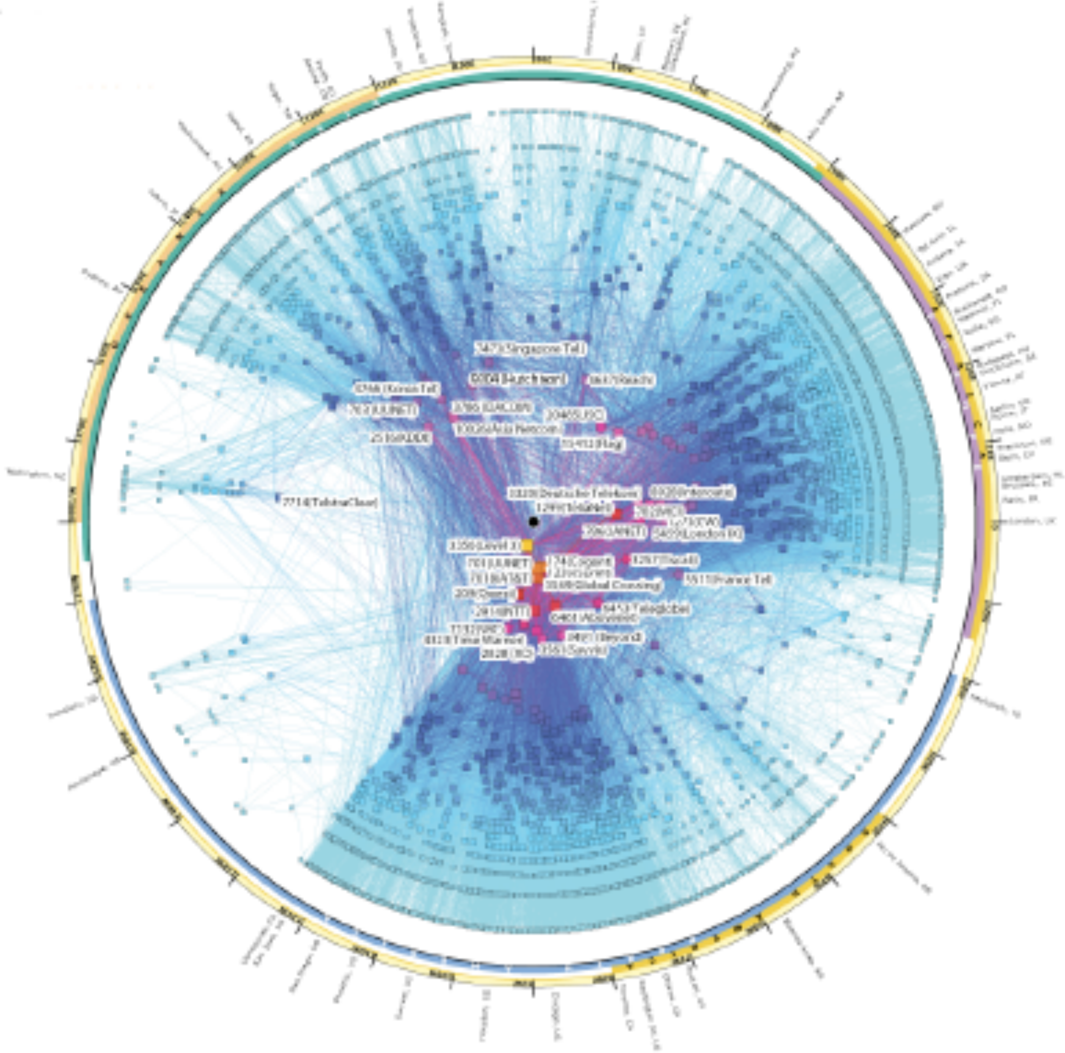


Internet AS topology

- Autonomous System (AS): a collection of networks under the same administration
- 2009: 25,000 ASs in the Internet



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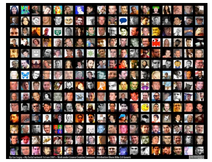
Topology Information



- By reading the routing tables of some gateways connected ASs, Internet topology information could be gathered
- October 08:
  – Over 30,000 ASs (including repeated entries)
  – Over 100,000 edges

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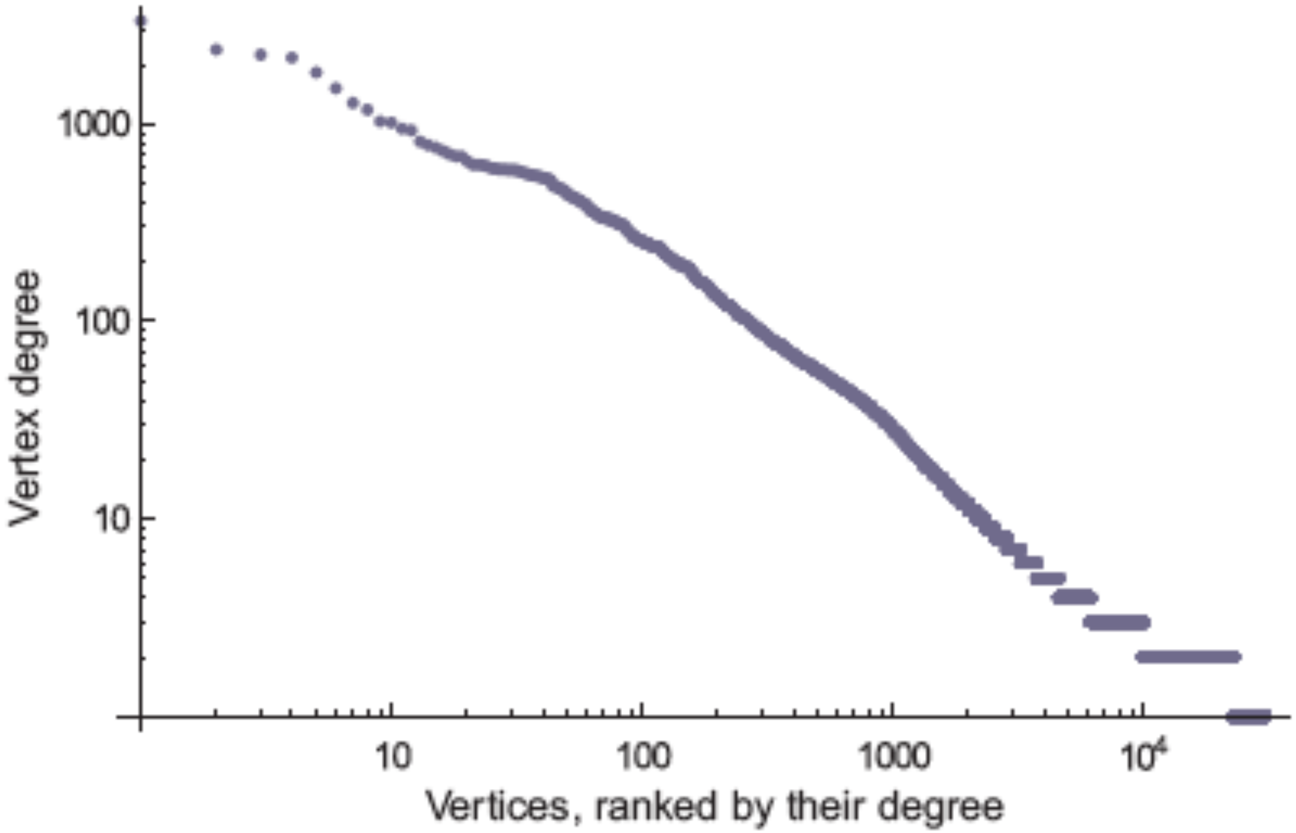
Degree distribution of ASs: A scale free network!



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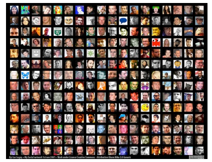
Properties



- The top AS is connected to almost 10% of all ASs
- This connectedness drops rapidly
- Very high clustering coefficient for top 1000 hubs: an almost complete graph
- Most paths no longer than 3-4 hops
- Most ASs separated by shortest paths of maximum length of 6

Rank:	1	2	3	4	5	6	7	8	9	10
Degree:	3309	2371	2232	2162	1816	1512	1273	1180	1029	1012

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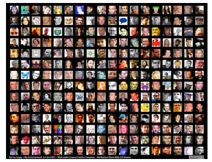
The Internet Now [Sigcomm10]



- They monitored inter-domain traffic for **2 years**
  - 3095 Routers
  - 110 ISPs
    - 18 Global
    - 38 Regional
    - 42 Consumer
  - 12 Terabits per second
  - 200 Exabytes total (200,000,000,000,000,000,000)
  - ~25% all inter-domain traffic.

- Inspect packets and classify them.

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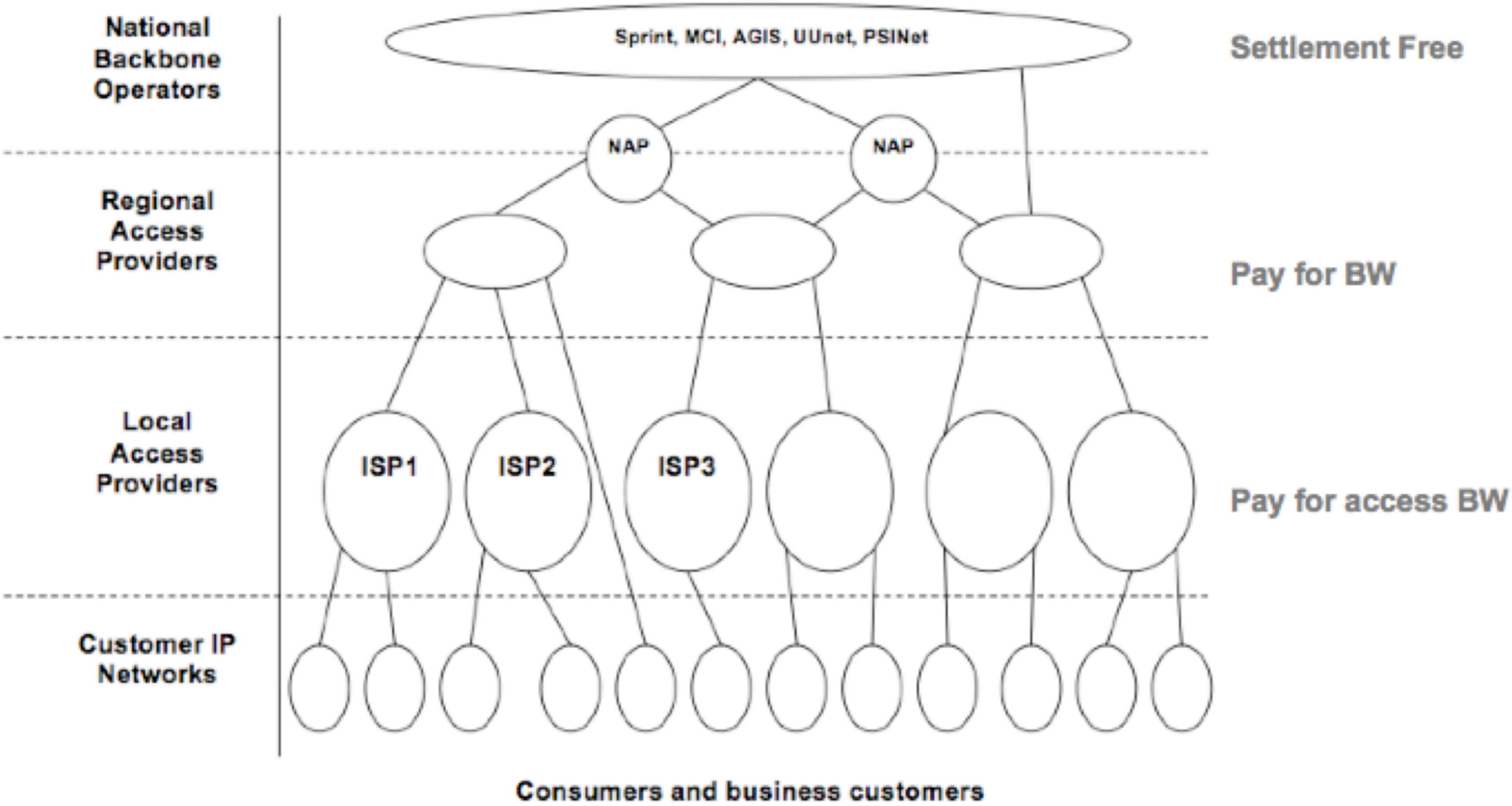
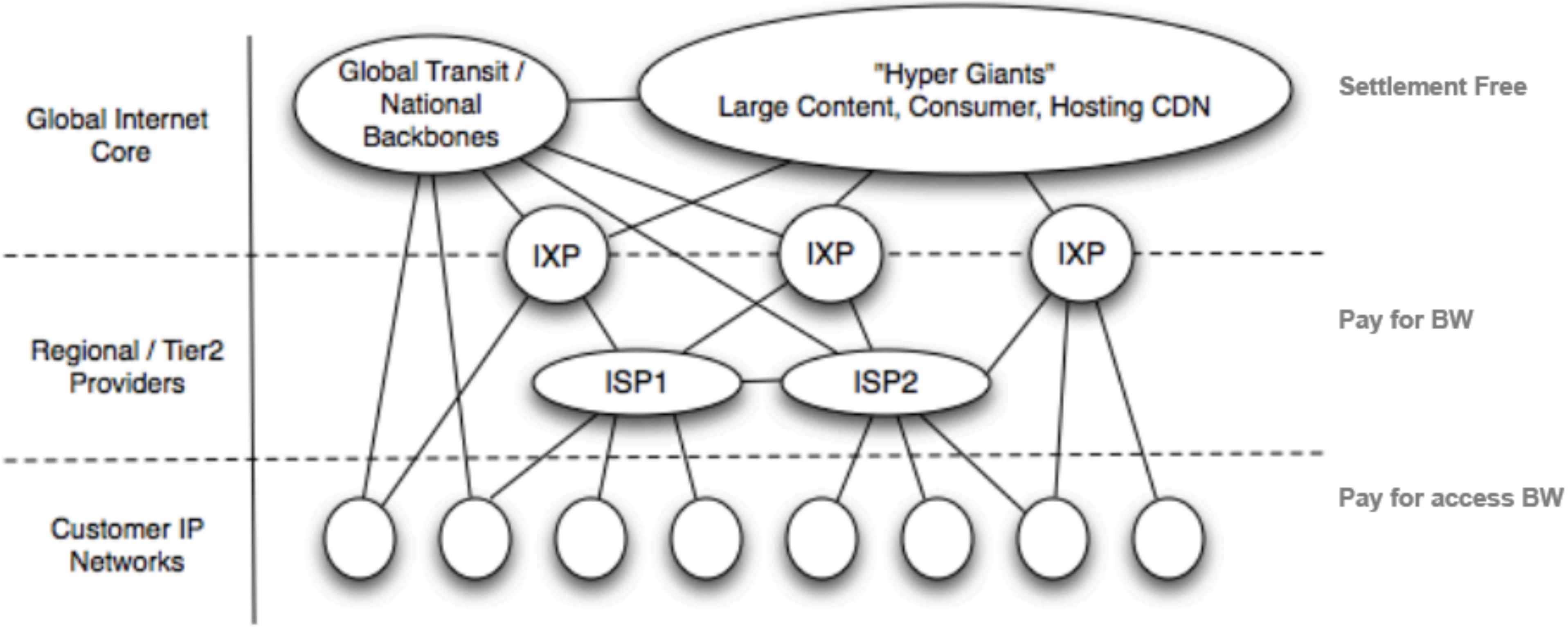
Internet 2007

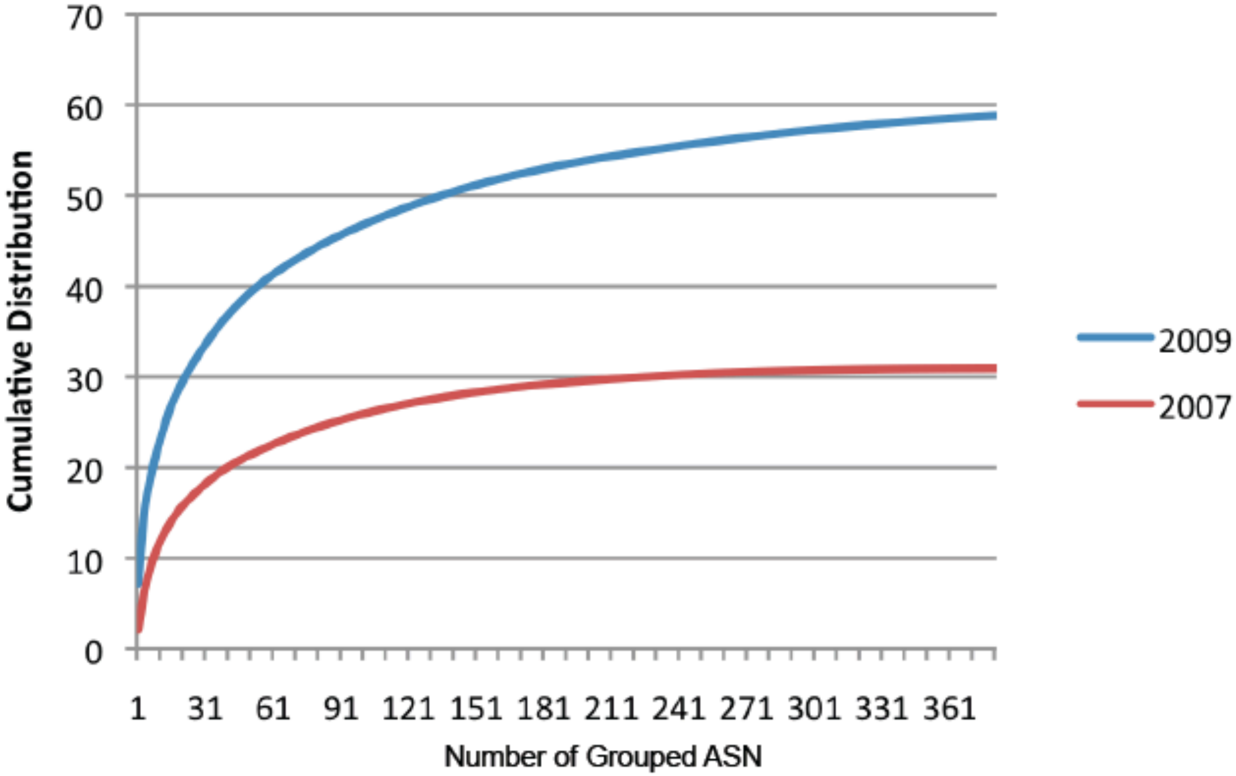


Internet 2009



- Flatter and much more densely interconnected Internet
- Disintermediation between content and "eyeball" networks
- New commercial models between content, consumer and transit

Internet traffic: responsibility to few



- In 2007, thousands of ASNs contributed 50% of content
- In 2009, 150 ASNs contribute 50% of all Internet traffic

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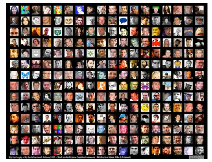
Robustness



- If a fraction of nodes or edges are removed:
  - How large are connected components?
  - What is the average distance between nodes in the components?
- This is related to *Percolation*
  - each edge/node is removed with probability (1-p)
    - Corresponds to random failure
  - Targeted attacks: remove nodes with high degree, or edges with high betweenness.
- The formation or dissolution of a giant component defines the percolation threshold

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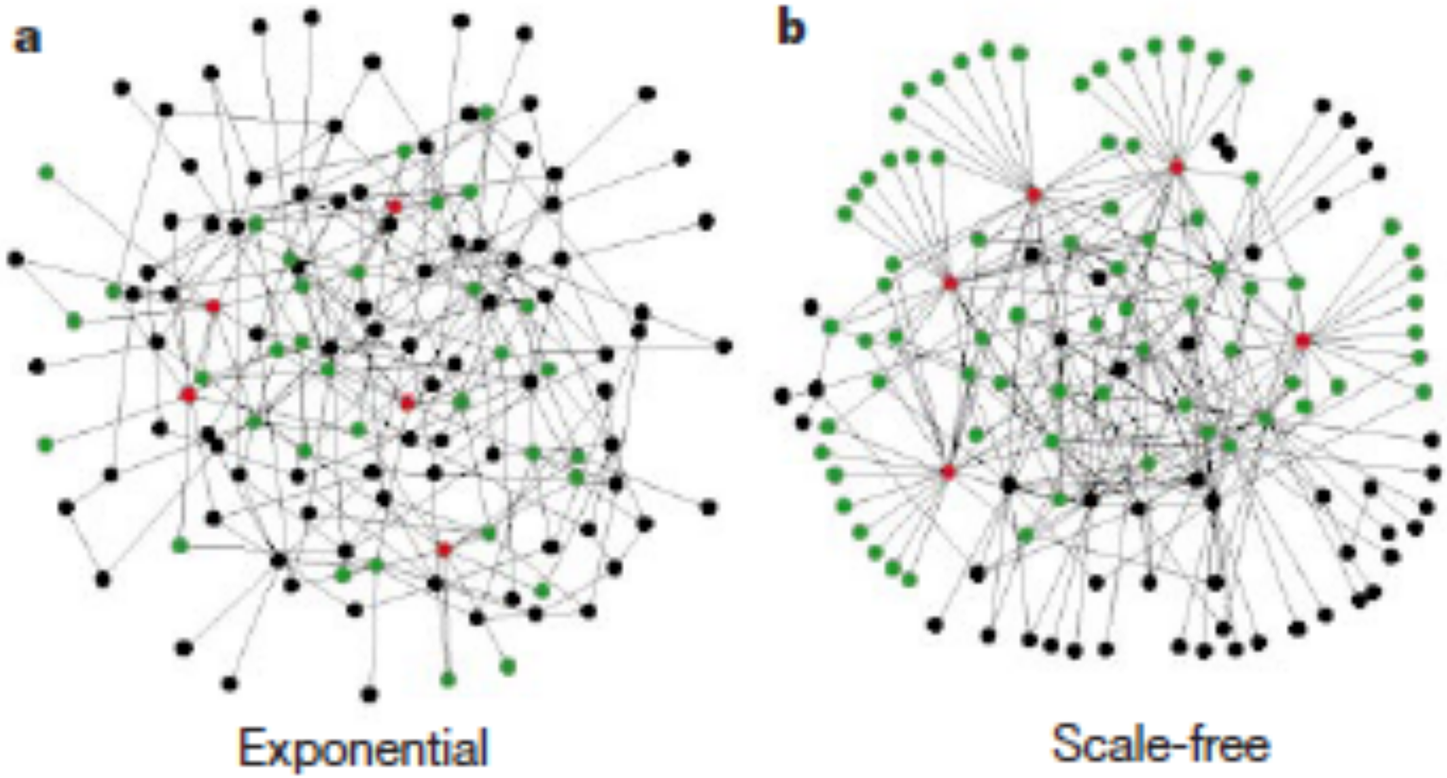
How Robust are These?



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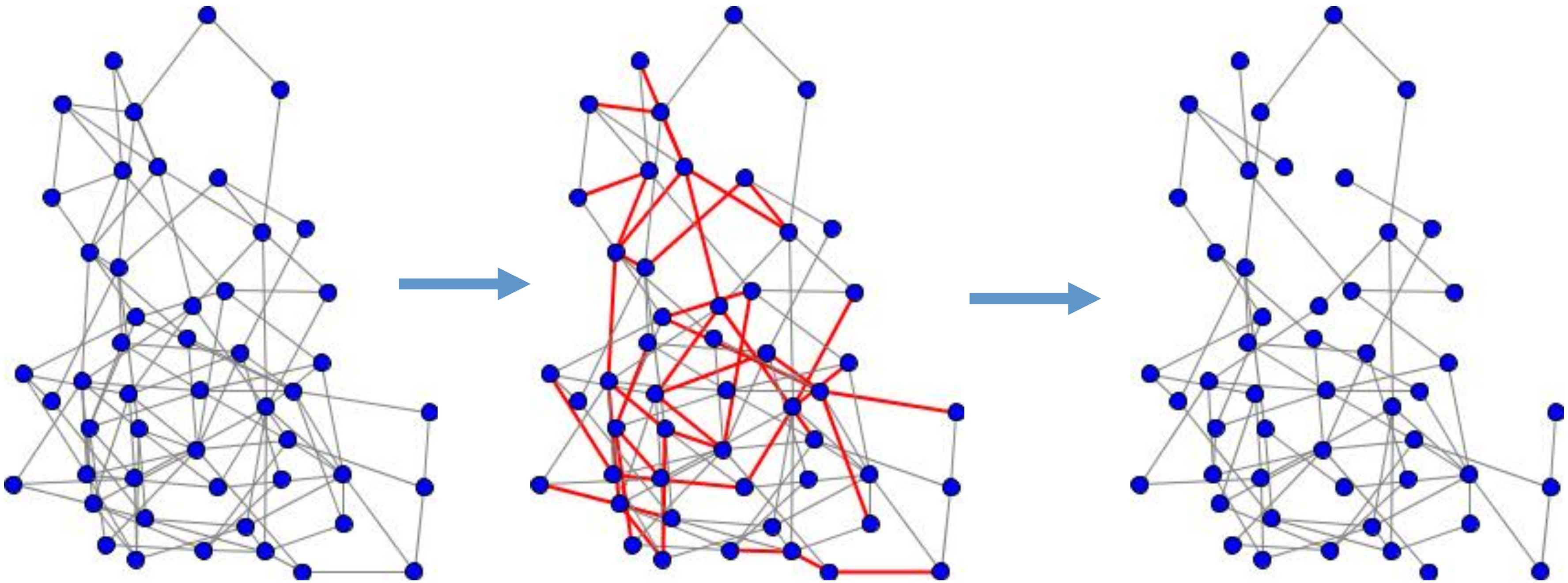
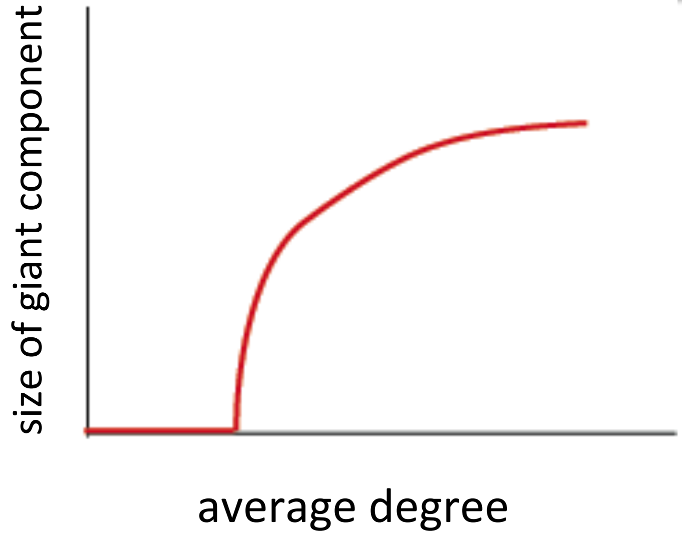
Edge (or Bond) Percolation



- 50 nodes, 116 edges, average degree 4.64
- after 25% edge removal
- 76 edges, average degree 3.04 – still well above percolation threshold

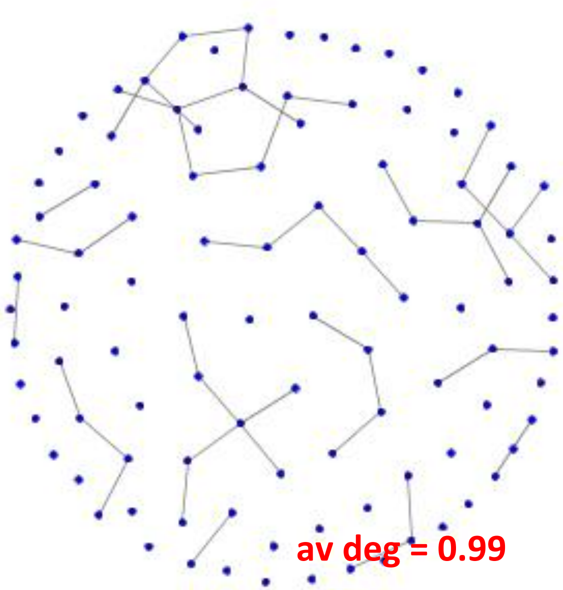
Percolation threshold in Random Graphs

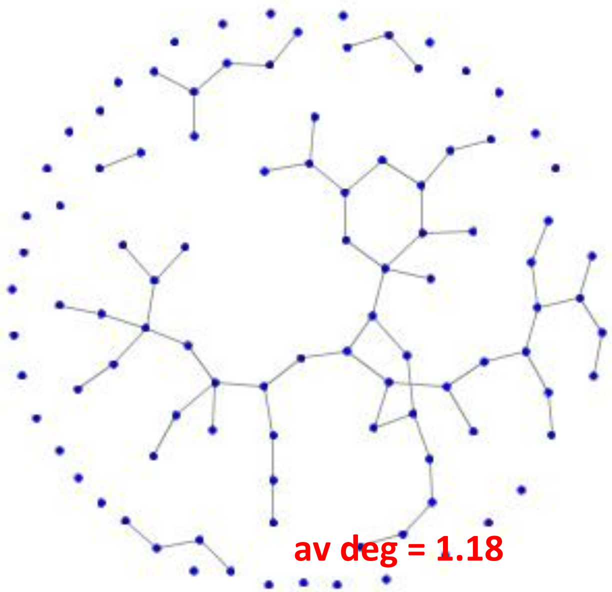


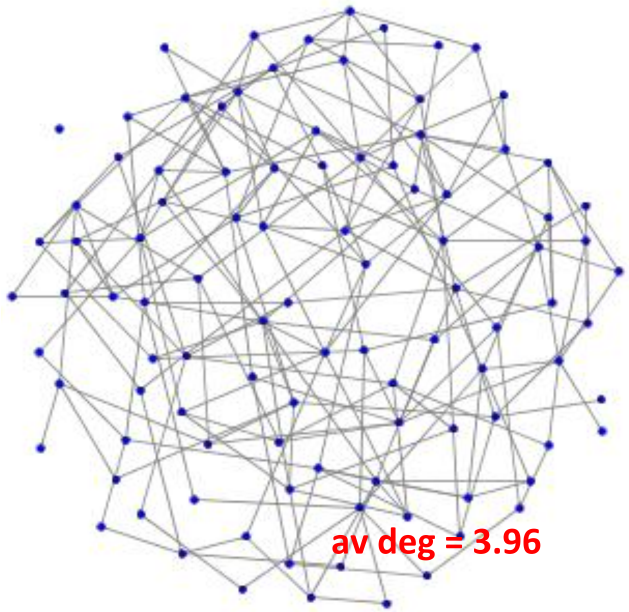
Percolation threshold: how many edges have to be removed before the giant component disappears?

As the average degree increases to 1, a giant component suddenly appears

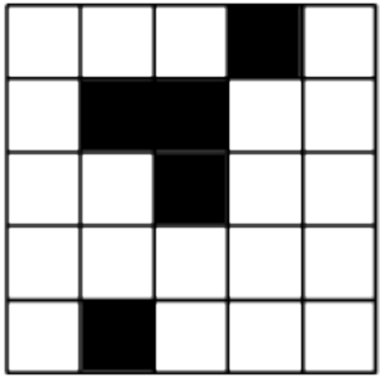
Edge removal is the opposite process – at some point the average degree drops below 1 and the network becomes disconnected







Site Percolation



site percolation

Ordinary Site Percolation on Lattices:
Fill in each site (site percolation) with probability p

- **low p**: small islands of connected components.
- **p critical**: giant component forms, occupying finite fraction of infinite lattice. Other component sizes are power-law distributed
- **p above critical value**: giant component occupies an increasingly large fraction of the system.

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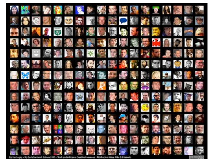
Barabasi-Yeong-Albert's study (2000)



- Given 2 networks (one exponential one scale free) with same number of nodes and links
- Remove a small number of nodes and study changes in average shortest path to see if information communication has been disrupted and how much.

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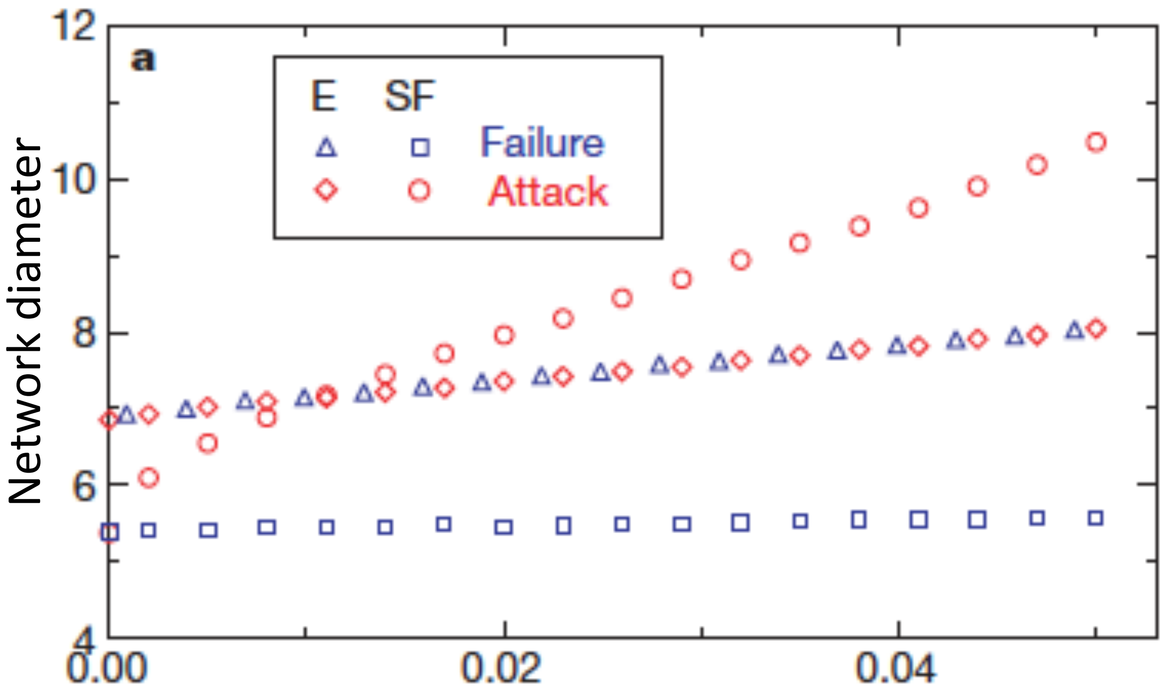
Let’s look at the blue lines

- Random graph: increasing monotonically
- SF: remains unchanged until at least 5%



Fraction of deleted nodes

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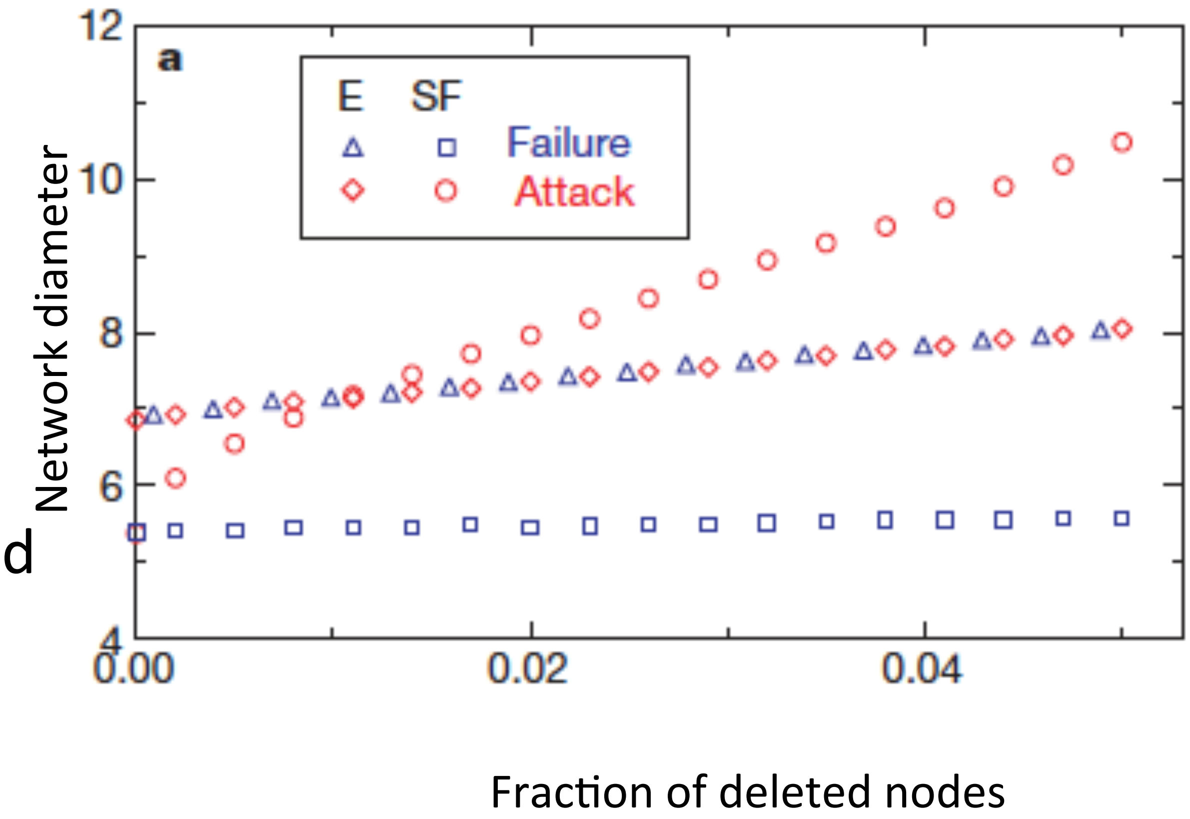
Let's look at the red lines

- Random graph: same behaviour if nodes with most links are chosen first
- SF: with 5% nodes removed the diameter is doubled



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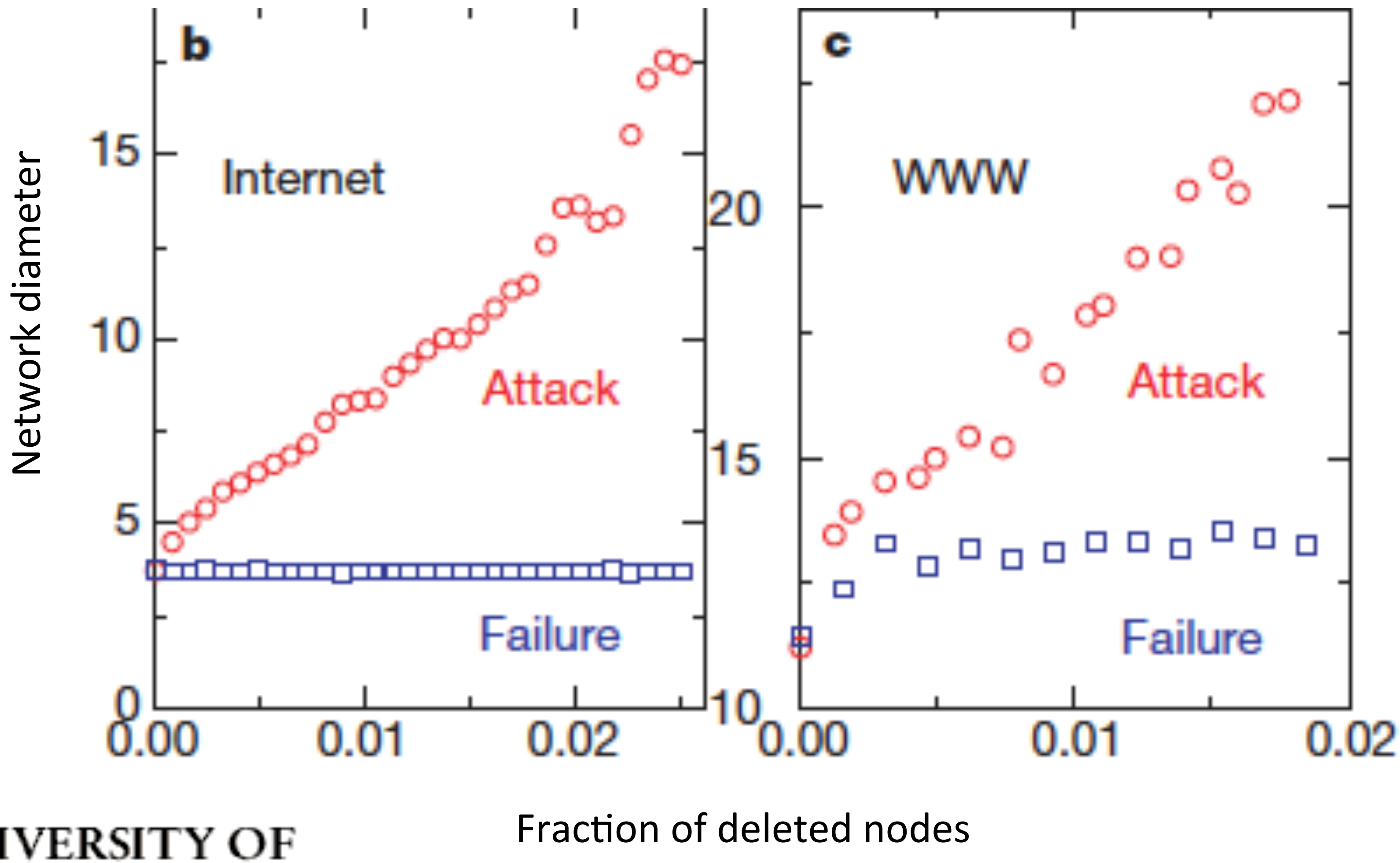
Effect of attacks and failure on WWW and Internet



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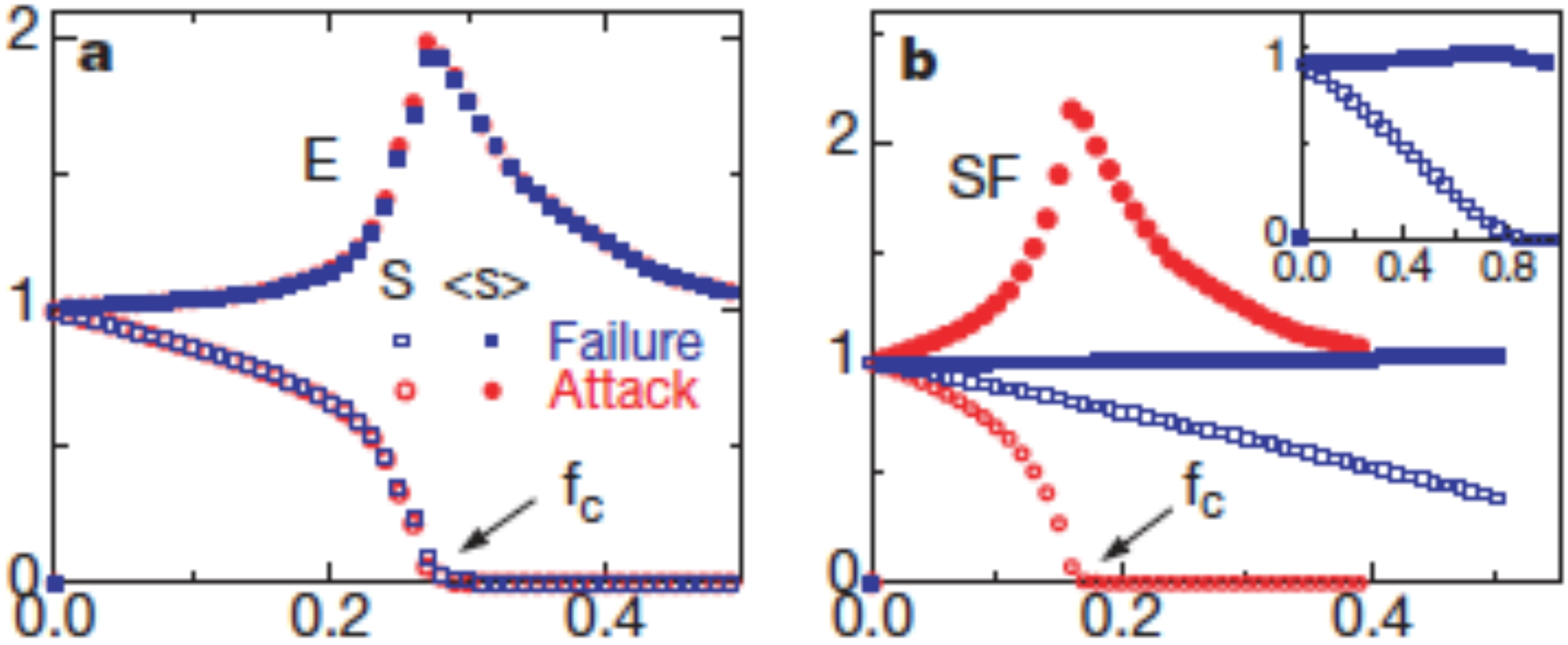
Effect on Giant Component



Fraction of deleted nodes

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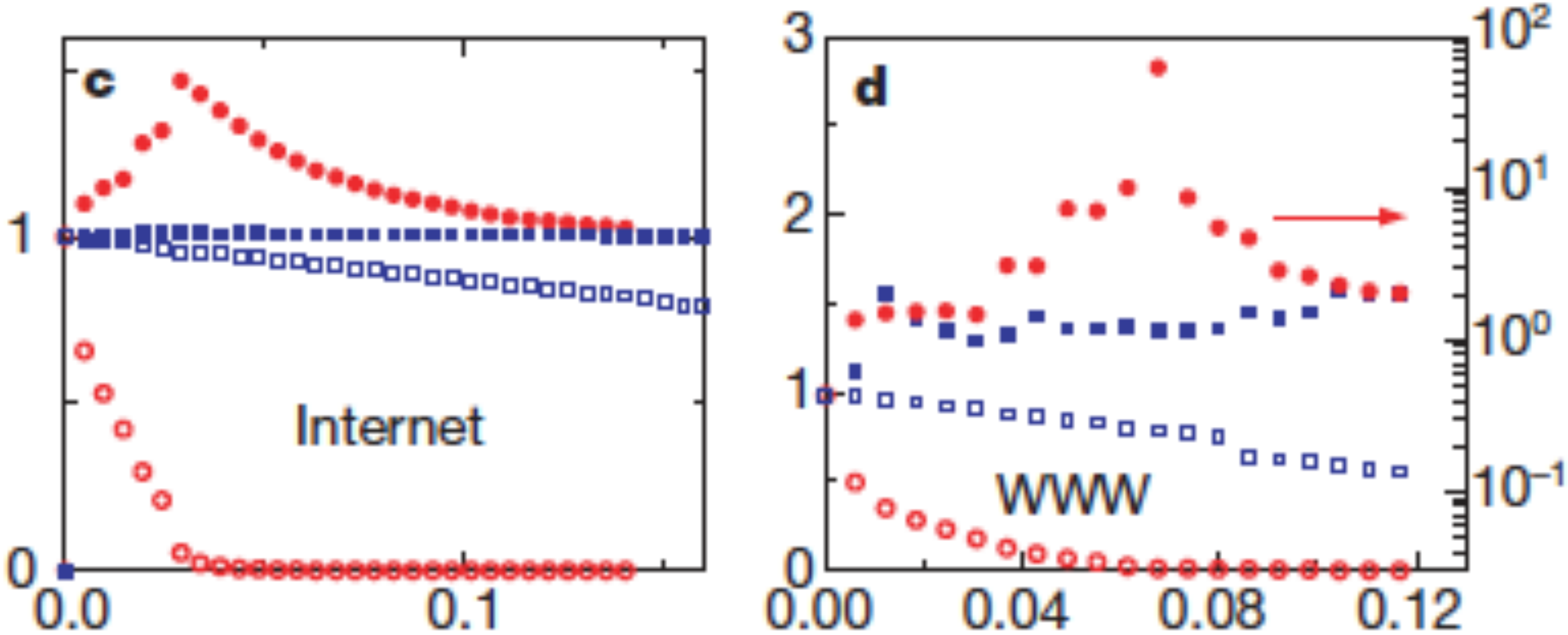
Internet and WWW: Effect on Giant Component



Fraction of deleted nodes

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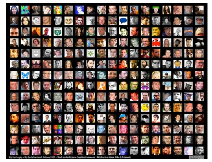
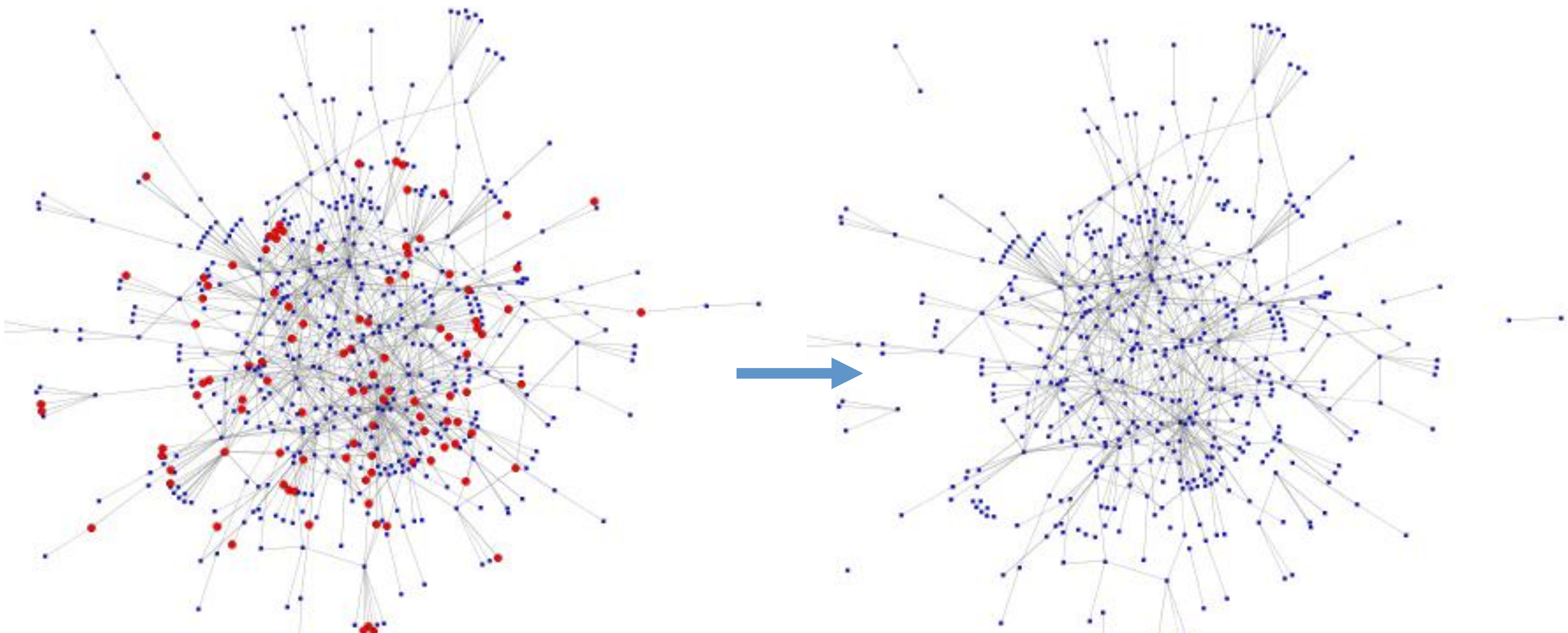
Scale-free networks are resilient with respect to random attack



- Example: Gnutella network, 20% of nodes

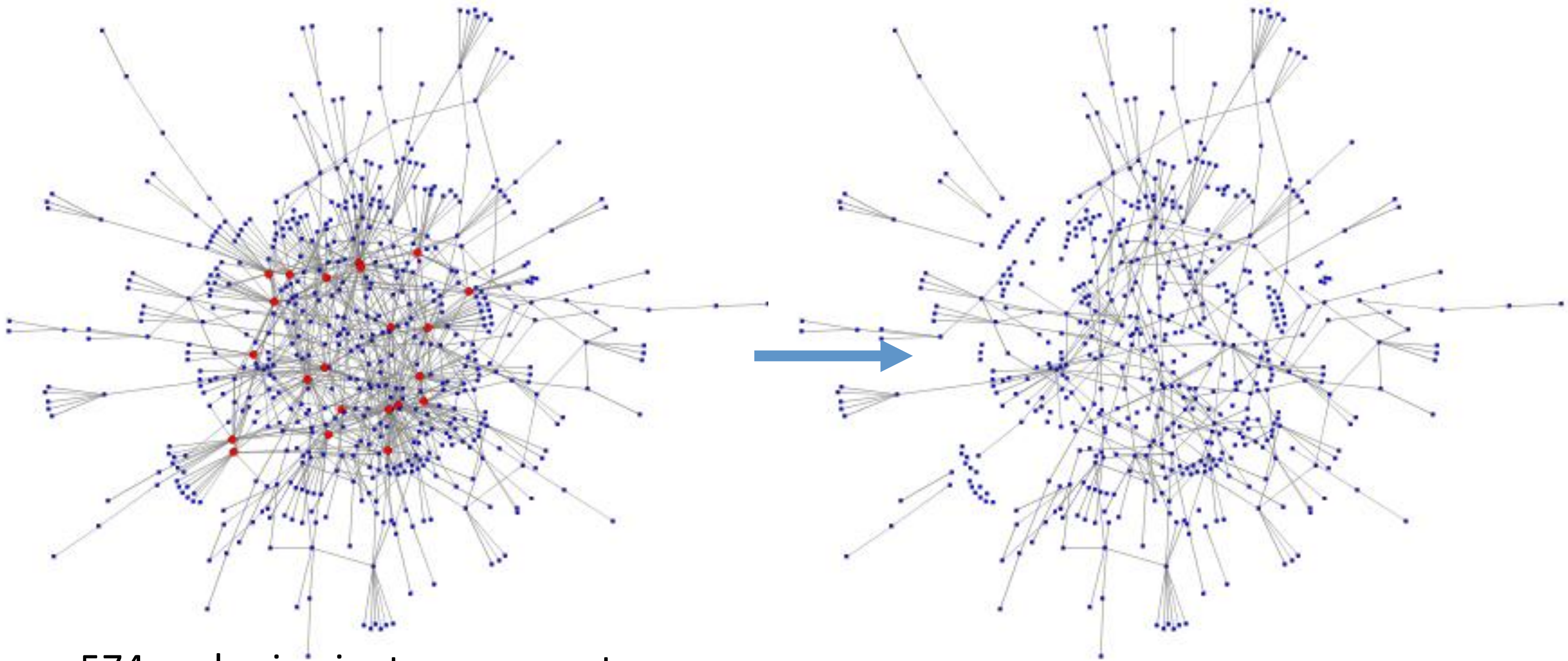


574 nodes in giant component

427 nodes in giant component

Targeted attacks are affective against scale-free networks

- Example: same Gnutella network, 22 most connected nodes removed (2.8% of the nodes)



574 nodes in giant component

301 nodes in giant component

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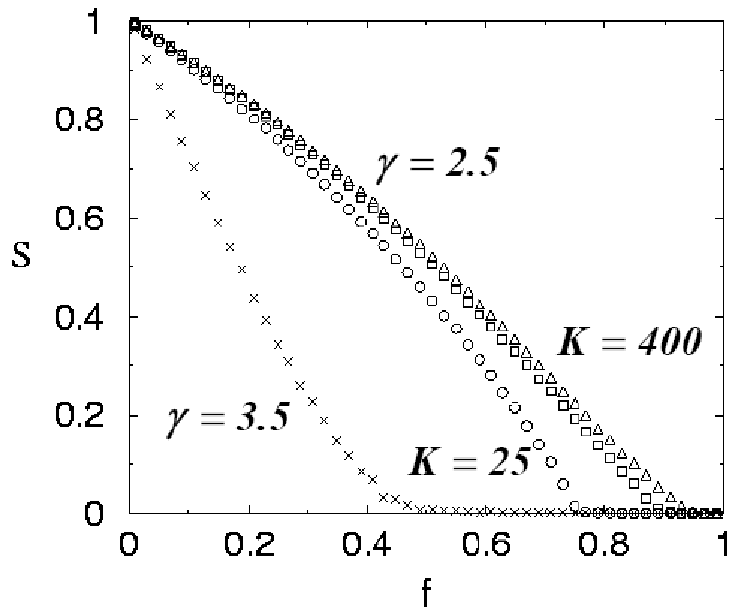
Another study of power-laws

- Graph shows fraction of GC size over fraction of nodes randomly removed
- Robustness of the Internet (γ is the exponent of PL).
  - γ =2.5 Virtually no threshold exists which means a GC is always present
  - For γ=3.5 there is a threshold around .0.4
- K indicates the connectivity network considered



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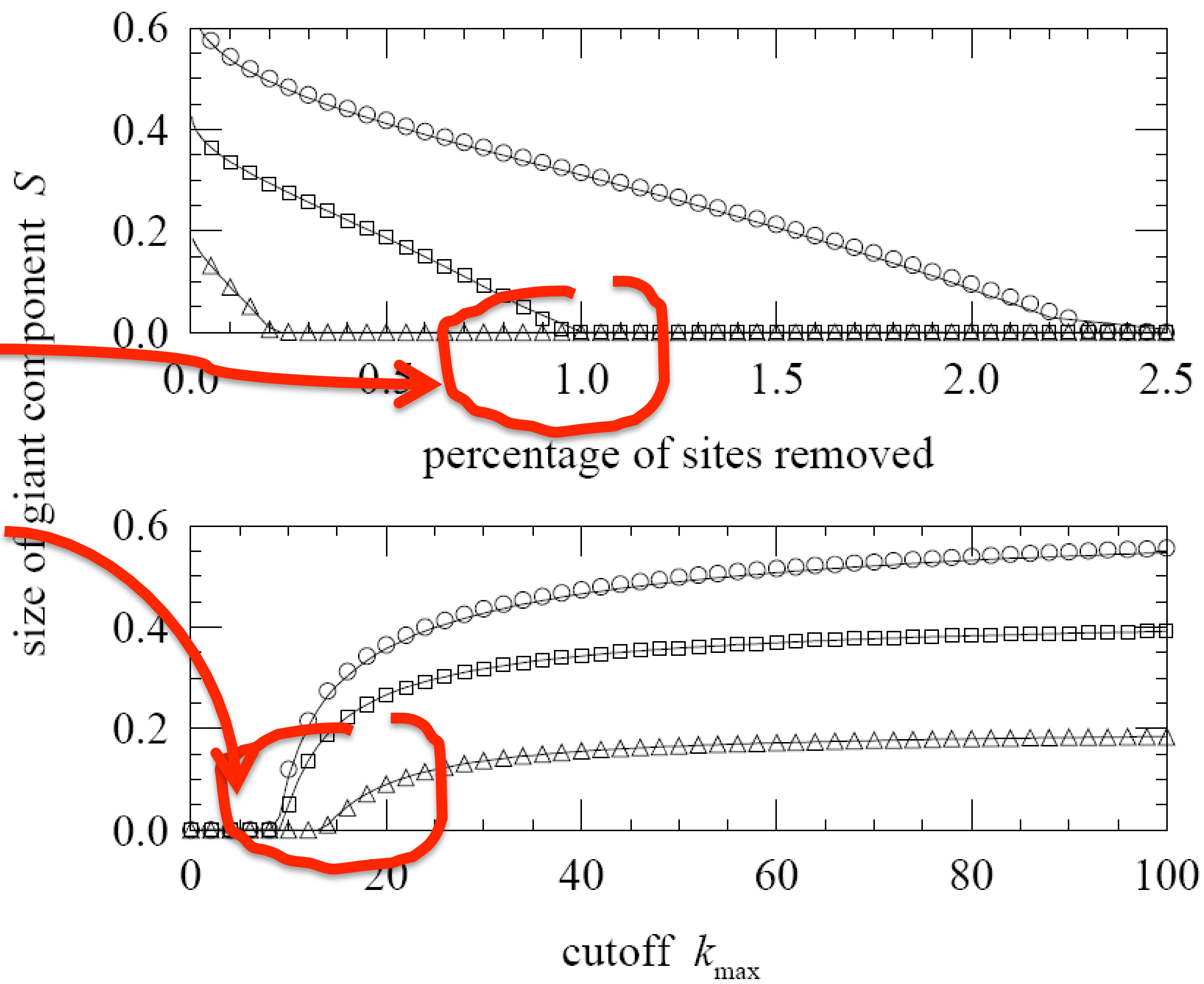
Skewness of power-law networks and effects and targeted attack

% of nodes removed, from highest to lowest degree

γ= 2.7 only 1% nodes removed leads to no GC

Kmax needs to be very low (10) to destroy the GC

k_{max} is the highest degree among the remaining nodes



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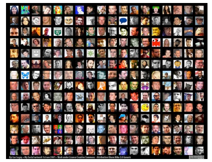
Percolation: let's get formal

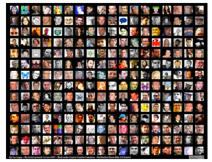


- Percolation process:
  - Occupation probability ϕ = number of nodes in the network [ie not removed]
- It can be proven that the critical threshold depends on the degree:

$$\phi_c = \frac{<k>}{<k^2> - <k>}$$

- This tells us the minimum fraction of nodes which must exist for a GC to exist.

Threshold for Random Graphs



- For Random networks $\varphi_{critical}$=1/c where c is the mean degree
  - If c is large the network can withstand the loss of many vertices
  - c=4 then ¼ of vertices are enough to have a GC [3/4 of the vertices need to be destroyed to destroy the GC]

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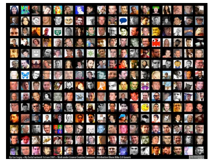
Threshold for Scale Free Networks



- For the Internet and Scale Free networks with $2<\alpha<3$
  - Finite mean $<k>$ however $<k^2>$ diverges (in theory)
  - Then $\varphi_{critical}$ **is zero: no matter how many vertices we remove there will always be a GC**
  - In practice $<k^2>$ is never infinite for a finite network, although it can be very large, resulting in very small $\varphi_{critical}$, so still highly robust networks

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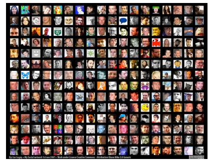
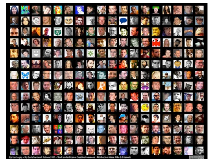
Non random removal



- The threshold models we have presented hold for random node removal but not for targeted attacks [ie removal of high degree nodes first]

- The equation for non random removal cannot be solved analytically

Robustness Study and Improvements



- A method to improve network resilience
- Percolation threshold q ignores situation when the network is very damaged but not collapsing.
- Robustness:

$$R = \frac{1}{N}\sum_{Q=1}^{N} s(Q)$$

S(Q)= nodes in the connected component after removing Q=qN nodes

- R ranges from 1/N to 0.5 (star and fully connected graph).

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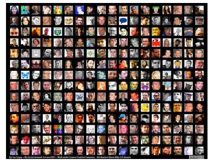
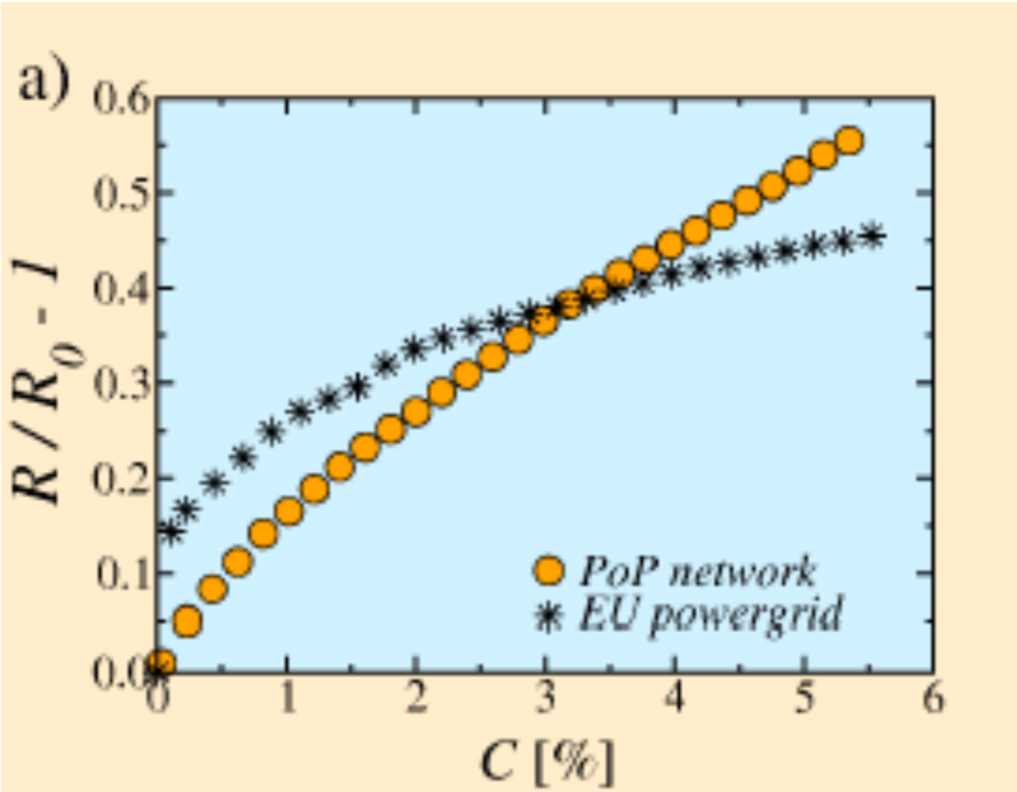
Improve Robustness



- Add links until network is fully connected: not practical.
- Swap edges of 2 random nodes so that R'>R
  - Repeat until no substantial improvement (a value delta);

  - Some additional constraints could be introduced (limit the geographical length of new edges for economic reasons).

Robustness Improvement over edge changes



Robustness improved by 55-45% with ~5% link change. Percolation threshold remains unchanged.

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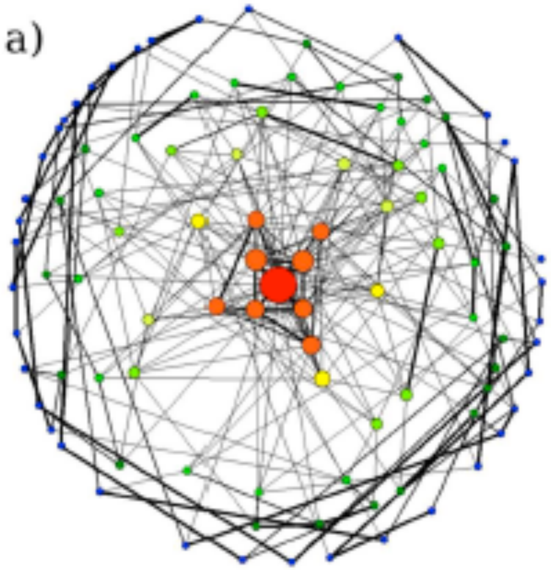
Best Network for Robustness

- How do we design a robust network with a fixed degree distribution?
- Scale free N=100 edges=300, exponent=2.5
- Onion-like structure!

a)



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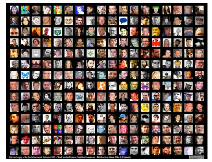
Robustness of Technological and Social Network



- Targeted attacks on high degree nodes are lethal to a technological and a biological as well as transport network.

- However as seen in Lecture 2, for social systems it is the bridges and weak ties which make a difference…

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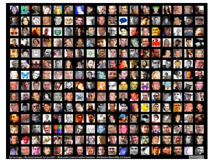
References



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