

Social and Technological Network Analysis

Lecture 1: Networks, Random Graphs and Metrics

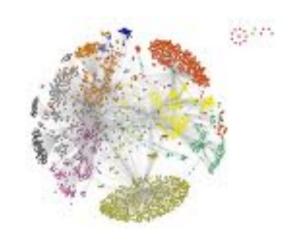
Dr. Cecilia Mascolo



About Me



- Reader in Mobile Systems
 - NetOS Research Group
- Research on Mobile, Social and Sensor Systems
- More specifically,
 - Human Mobility and Social Network modelling
 - Geo-social recommendation systems
 - Mobile Sensor Systems and Networks







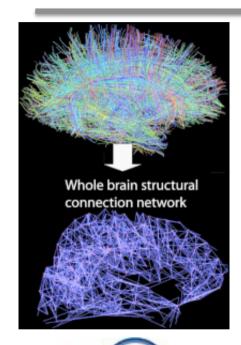




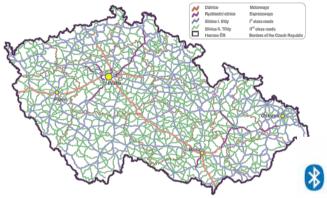


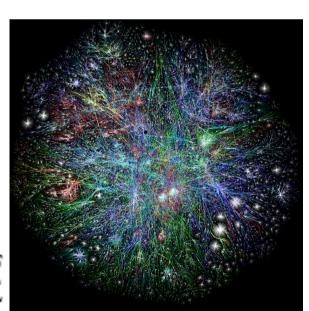


















Facebook Friendship Network

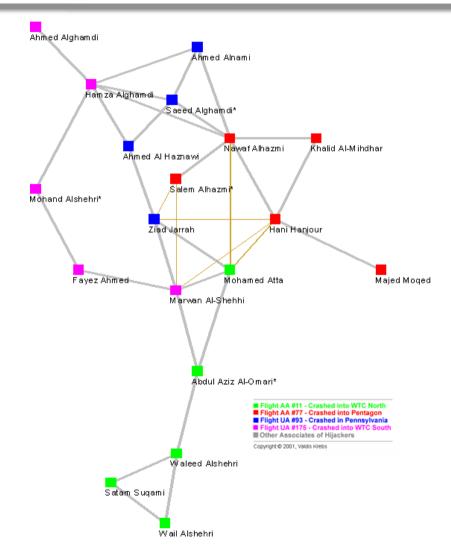








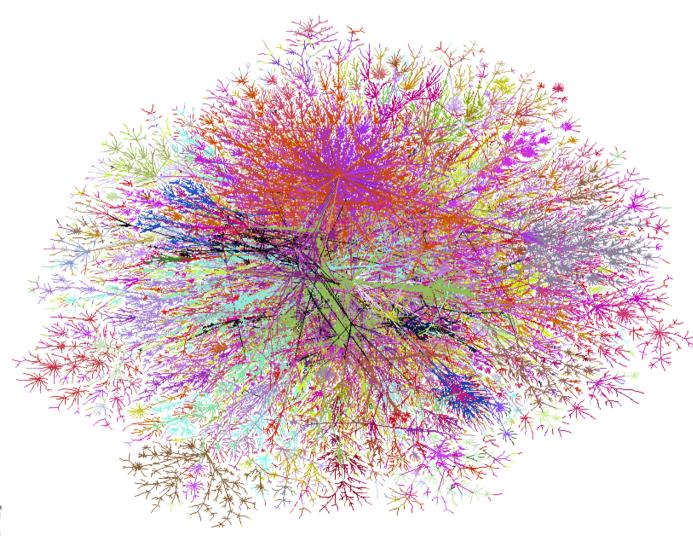
"Six degrees of Mohammed Atta"





The Internet



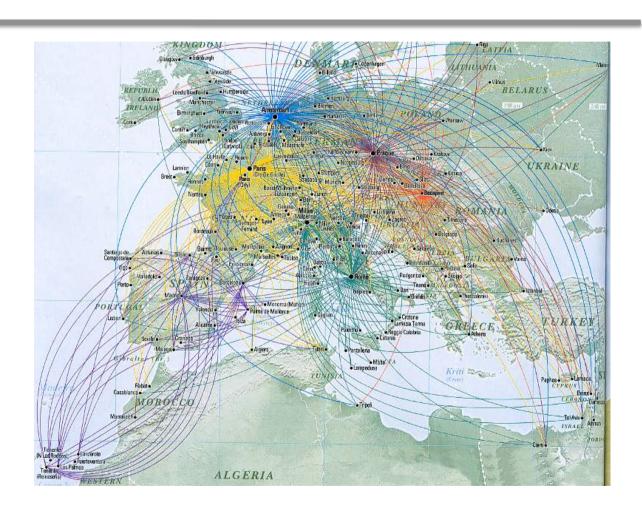




Source: Bill Cheswick http://www.cheswick.com/ches/map/gallery/index.html

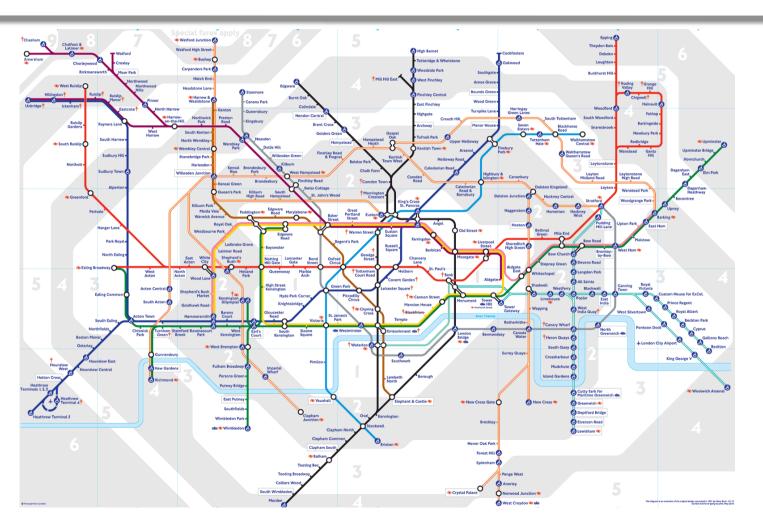


Airline Network













Geo-social Networks





What Kind of Networks?

- Who talks to whom?
- Who is friend with whom?
- What leads to what?
- Who is a relative of whom?
- Who eats whom?
- Who sends messages to whom?





In This Course

- We will study the models and metrics which allow us to understand these phenomena.
- We will show analysis over large datasets of real social and technological networks.



List of Lectures

- Lecture 1: Networks and Random Graphs
- Lecture 2: Small World and Weak Ties
- Lecture 3: Centrality and Applications
- Lecture 4: Community Detection, Modularity, Overlapping Communities
- Lecture 5: Structure of the Web, Search and Power laws
- Lecture 6: Network Robustness and Applications
- Lecture 7: Cascade and Behaviour Influence
- Lecture 8 and 9: 2h Practical Tutorial
- Lecture 10: Epidemic Spreading and Examples
- Lecture 11: Spatial Networks
- Lecture 12: Cascades and Epidemics Applications
- Lecture 13: Time Varying Social Networks
- Lecture 14: Industrial Presentation (Facebook)
- Lecture 15-16: Student Presentations





Assessment

- All information on the course page: http://www.cl.cam.ac.uk/teaching/1213/L109/materials.html
- One report (of approximately 1,500 words) on one assigned research paper. The report is due on 14th February (noon) and it is worth 30% of the final mark.
- The second assignment will consist of analysis of an assigned dataset according to some indicated network measures: the analysis should be reported in a document of about 1,500 words where the results are commented and justified. This should be handed in by 11th March and will be worth 50% of the final mark.
- Presentation of the findings of 2nd assignment on 12th March.
 The presentation is worth 20% of the final mark.
 UNIVERSITY OF



Structure of First Report

- The first report should be approximately 1,500 words. The report will contain two parts of about 750 words each:
 - Critical analysis of the papers including, possibly, comparisons and references to other material presented in the course or found by the student and comments on how solid the result obtained are (e.g., comments on the evaluation methods or on the analysis applied can be included);
 - Discussion of possible future research ideas in the area.



Choices

- First assignment: send me email immediately with a choice of paper (and a backup choice) from the website [assignment is first come first served: you will receive an email confirming which paper you are assigned to].
 Do this by 25th January.
- Second assignment: same. At that point you will receive an email with a link to the dataset.
 - Do this by 14th February.







- We will introduce:
 - Networks/graphs
 - Basic network measures
 - Random Graphs
 - Examples



A Network is a Graph

A **graph** G is a tuple (V,E) of a set of vertices V and edges E. An edge in E connects two vertices in V.

A **neighbour set** N(v) is the set of vertices adjacent to v:

$$N(v) = \{ u \in V \mid u \neq v, (v, u) \in E \}$$

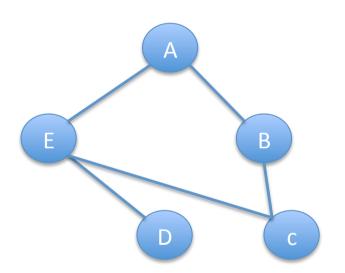






- The node degree is the number of neighbours of a node
- E.g., Degree of A is 2

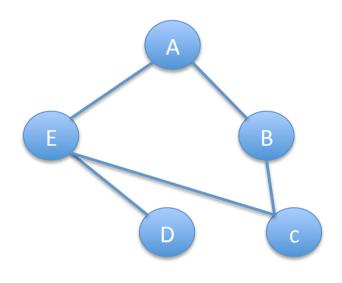
The study of the degree distribution of networks allows the classification of networks in different categories

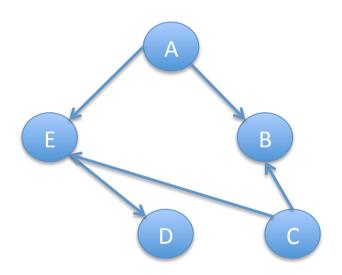






Directed & Undirected Graphs





Undirected Graph

Directed Graph



Example of Undirected Graphs: Facebook, Co-presence

Examples of Directed: Twitter, Email, Phone Calls



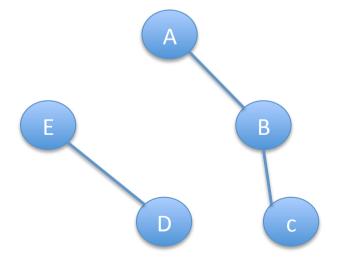
Paths and Cycles

- A path is a sequence of nodes in which each pair of consecutive nodes is connected by an edge.
 - If a graph is directed the edge needs to be in the right direction.
 - E.g. A-E-D is a path in both previous graphs
- A cycle is a path where the start node is also the end node
 - E.g. E-A-B-C is a cycle in the undirected graph



Connectivity

- A graph is connected if there is a path between each pair of nodes.
- Example of disconnected graph:







Components

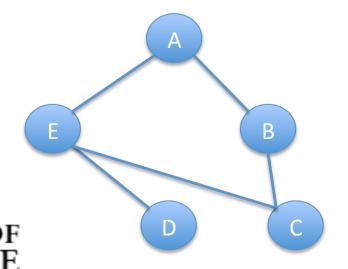
- A **connected component** of a graph is the subset of nodes for which each of them has a path to all others (and the subset is not part of a larger subset with this property).
 - Connected components: A-B-C and E-D
- A giant component is a connected component containing a significant fraction of nodes in the network.
 - Real networks often have one unique giant component.





Path Length/Distance

- The distance (d) between two nodes in a graph is the length of the shortest path linking the two graphs.
- The diameter of the graph is the maximum distance between any pair of its nodes.



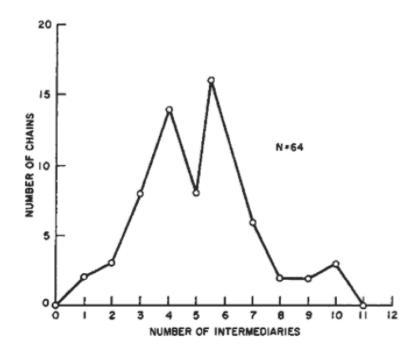
What's the diameter here?

Small-world Phenomenon Milgram's Experiment



- Two random people are connected through only a few (6) intermediate acquaintances.
- Milgram's experiment (1967) shows the known "six degrees of separation":
 - Choose 300 people at random
 - Ask them to send a letter through friends to a stockbroker near Boston.
 - 64 successful chains.







Recent Remake

- In 2003 the experiment was redone.
- Choice of 18 targets over the world. Choice of senders (60K) from a commercially obtained email list.
- Website to control the "email" contact from one participant to the next (two weeks to choose next hop)
- Verification of relationship by receiver (to avoid cheating by web search).



Findings



- Use of "weak ties" and professional relationships
- Median of 5-7 steps
- "Network structure alone is not everything"
- Some different incentives had a high impact on completion rate of chains
 - If the target was in a prominent place (eg professor)







facebook

Search for people, places and things

Q.

Cecilia Mascolo

Home

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Anatomy of Facebook

By Lars Backstrom on Tuesday, 22 November 2011 at 01:04 🚷

Think back to the last time you were in a crowded airport or bus terminal far from home. Did you consider that the person sitting next to you probably knew a friend of a friend of a friend of yours? In the 1960s, social psychologist Stanley Milgram's "small world experiment" famously tested the idea that any two people in the world are separated by only a small number of intermediate connections, arguably the first experimental study to reveal the surprising structure of social networks.

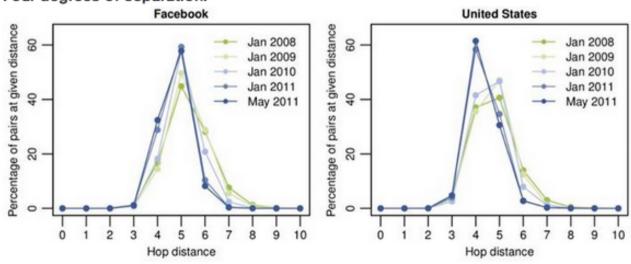
With the rise of modern computing, social networks are now being mapped in digital form,

Facebook Data Science

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Four degrees of separation.

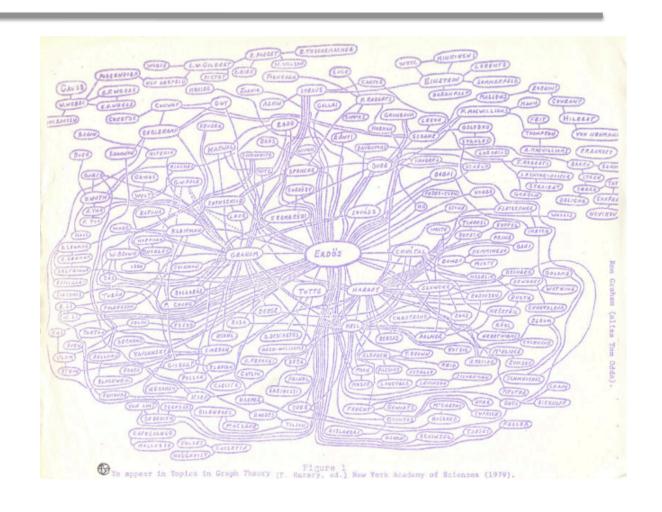


While 99.6% of all pairs of users are connected by paths with 5 degrees (6 hops), 92% are connected by only four degrees (5 hops)





Erdos Number:
 distance from
 the
 mathematician
 (most people
 are 4-5 hops
 away) based on
 collaboration.









- A network of actors who costarred in a movie.
- Most actors are no more than ~3 hops from Kevin Bacon.
- One very obscure movie was at distance 8.





Random Graphs



- First way to model these networks:
- Erdos-Renyi Random Graph [Erdos-Renyi '59]:

G(n,p): graph with n vertices where an edge exists with independent random probability 0<p<1 for each edge.



Random Graph Model

- For each node n1, an edge to node n2 exists with probability p.
- Degree distribution is binomial.
- The probability of a node to have degree k:

$$P(k_i = k) = C_{N-1}^k p^k (1-p)^{N-1-k}$$

• Where
$$C_{N-1}^k = {N-1 \choose k}$$

• Expected Degree of a node: $(N-1)p \approx Np$





Random Graphs Properties

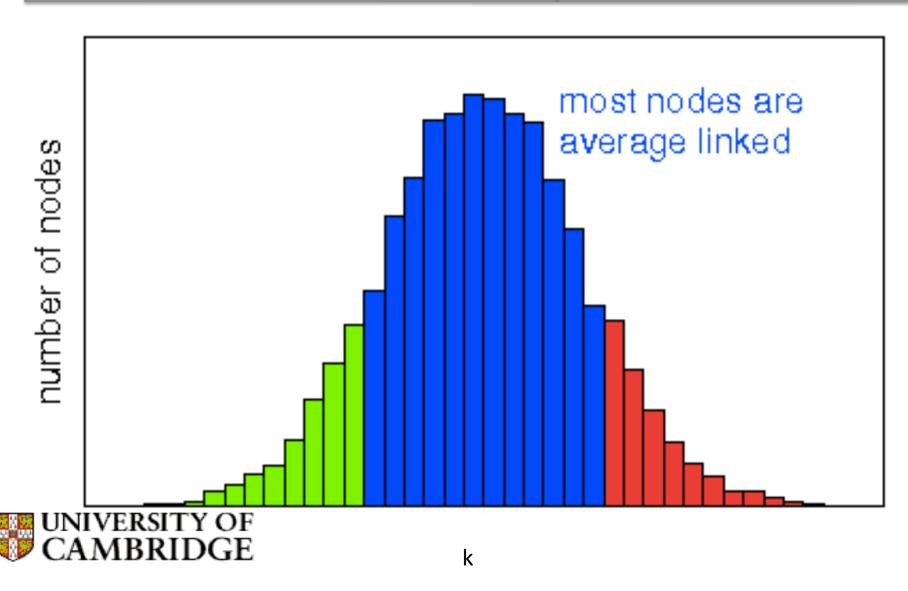
 For large N this is approximated by the Poisson distribution with

$$P(k) \approx e^{-Np} \frac{(Np)^k}{k!} = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$



Degree Distribution of Random Graphs





Random Graph Diameter

 The diameter of a random graph and the average path length of the graph have been demonstrated to be:

$$d = \frac{\ln(N)}{\ln(pN)} = \frac{\ln(N)}{\ln(\langle k \rangle)} \approx l_{rand}$$

The average distance between two nodes is quite small wrt to the size of the graph.



Relationship of <k> and connectivity

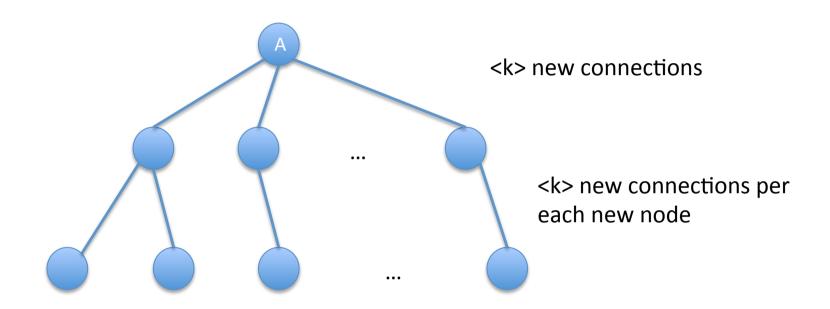


- <k> = average degree (np)
- If <k> < 1 disconnected network
- If <k>> 1 a giant component appears
- If <k> >= In(N) graph is totally connected



Random Graph Diameter: An Intuition





• The nodes at distance I from A will be ~ <k>|

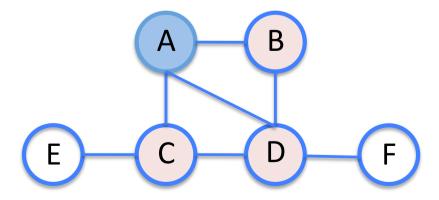


N=k^l log N= l log k l = log N/log k



Clustering Coefficient

- The clustering coefficient defines the proportion of A's neighbours (N(A)) which are connected by an edge (are friends).
- The number of triangles in which A is involved wrt to the ones it could be involved in.









Local Clustering Coefficient

$$C_{i} = \frac{2 |\{e_{jk}\}|}{k_{i}(k_{i}-1)} : v_{j,}v_{k} \in N_{i}, e_{j,k} \in E$$

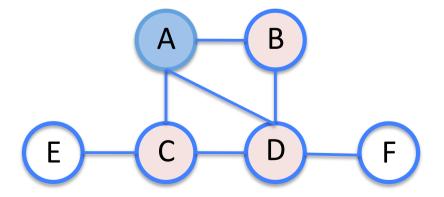
Network Clustering Coefficient

$$CG = \frac{1}{N} \sum_{i} C_{i}$$



Clustering Coefficient: Example

- Ki=3
- Nominator= 2*2=4
- Denominator=6
- Ci=4/6=2/3





Clustering Coefficient of a Random Graph



The clustering coefficient of a random graph is

$$C_{rand} = p = \frac{\langle k \rangle}{N}$$

$$p * \left(\frac{n_v}{2}\right) / \left(\frac{n_v}{2}\right) = p$$

- The probability that 2 neighbours of a node are connected is equal to the probability that 2 random nodes are connected
- Is this mirroring the clustering coefficient of real networks?



Question



 Are Random Graphs representatives of Real Networks?





Summary

- We have introduced graphs definitions and measures.
- Random graphs are a first examples of models for networks.



References



- Material from Chapter 1, 2 of
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