Addendum to Lecture 3: Edit Distance (with animation)

Information Retrieval
Computer Science Tripos Part II

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Edit distance

- **Edit distance** between two strings s_1 and s_2 is the minimum number of basic operations that transform s_1 into s_2.
- **Levenshtein distance:** Admissible operations are insert, delete and replace

Levenshtein distance

dog	–	do	1 (delete)
cat	–	cart	1 (insert)
cat	–	cut	1 (replace)
cat	–	act	2 (delete+insert)

Levenshtein distance: Distance matrix

		s	n	o	w
	0	1	2	3	4
o	1	1	2	2	3
s	2	1	2	3	3
l	3	2	2	3	4
o	4	3	3	2	3

Edit Distance: Four cells

			s		n		o		w	
		0	**1**	**1**	**2**	**2**	**3**	**3**	**4**	**4**
o		**1** **1**	**1** 2	2 **1**	**2** **2**	3 **2**	**2** 3	4 **2**	4 **3**	5 **3**
s		**2** **2**	**1** 3	2 **1**	**2** **2**	3 **2**	**3** **3**	**3** **3**	**3** 4	4 **3**
l		**3** **3**	3 4	**2** **2**	**2** 3	3 **2**	**3** **3**	4 **3**	**4** **4**	**4** **4**
o		**4** **4**	4 5	**3** **3**	**3** 4	**3** **3**	**2** 4	4 **2**	4 **3**	5 **3**

Dynamic Programming

Cormen et al:

- Optimal substructure: The optimal solution contains within it subsolutions, i.e, optimal solutions to subproblems
- Overlapping subsolutions: The subsolutions overlap and would be computed over and over again by a brute-force algorithm.

For edit distance:

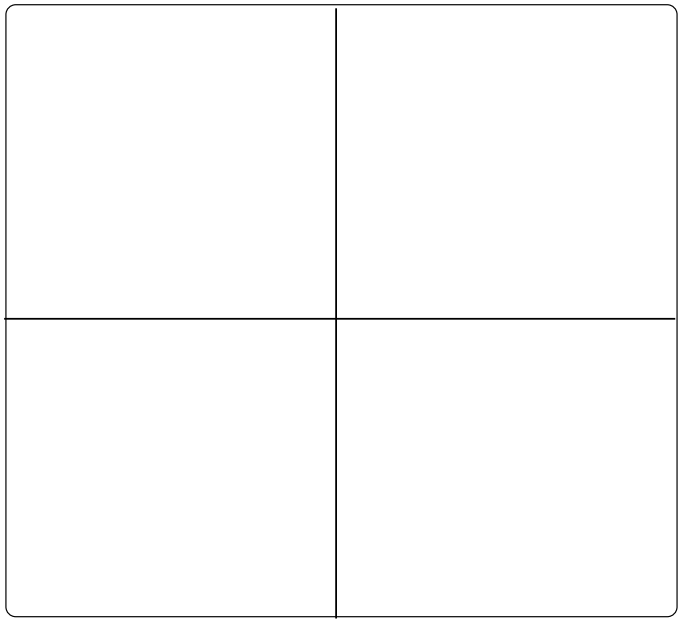
- Subproblem: edit distance of two prefixes
- Overlap: most distances of prefixes are needed 3 times (when moving right, diagonally, down in the matrix)

Levenshtein distance: Algorithm

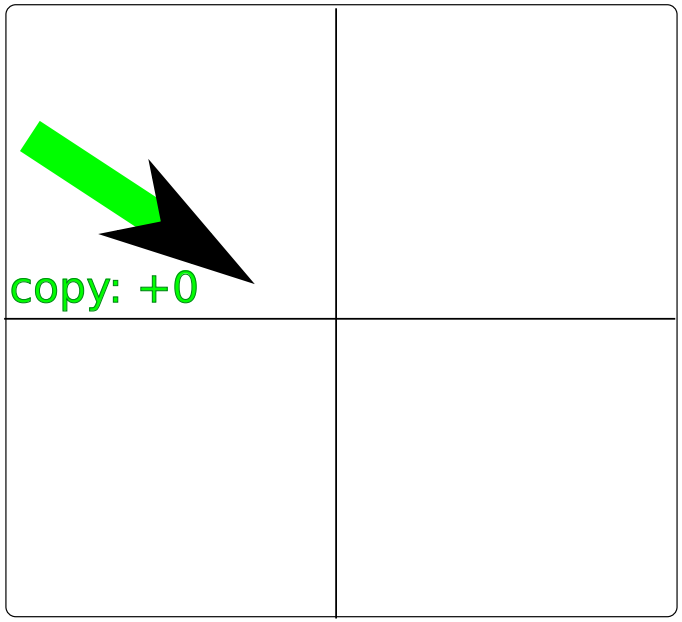
LevenshteinDistance(s_1, s_2)

```
 1  for i ← 0 to |s1|
 2  do m[i, 0] = i
 3  for j ← 0 to |s2|
 4  do m[0, j] = j
 5  for i ← 1 to |s1|
 6  do for j ← 1 to |s2|
 7       do if s1[i] = s2[j]
 8            then m[i, j] = min{m[i-1, j]+1, m[i, j-1]+1, m[i-1, j-1]}
 9            else  m[i, j] = min{m[i-1, j]+1, m[i, j-1]+1, m[i-1, j-1]+1}
10  return m[|s1|, |s2|]
```

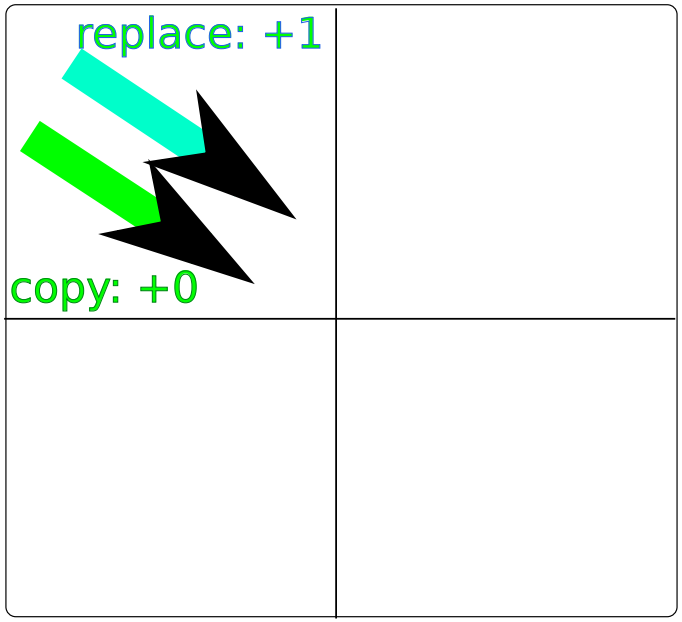
Each cell of Levenshtein matrix



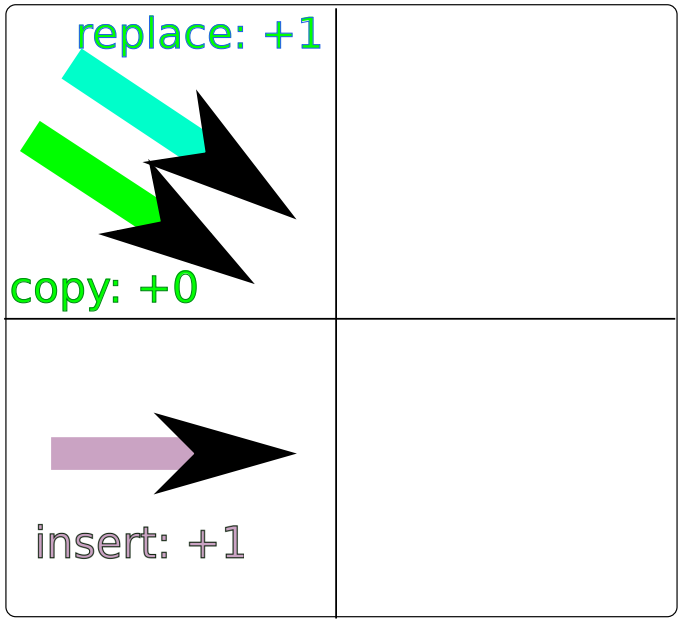
Each cell of Levenshtein matrix



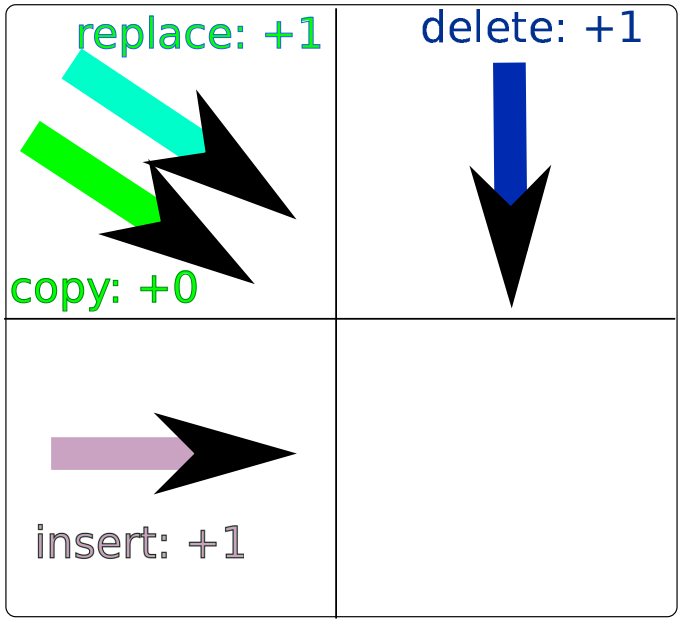
Each cell of Levenshtein matrix



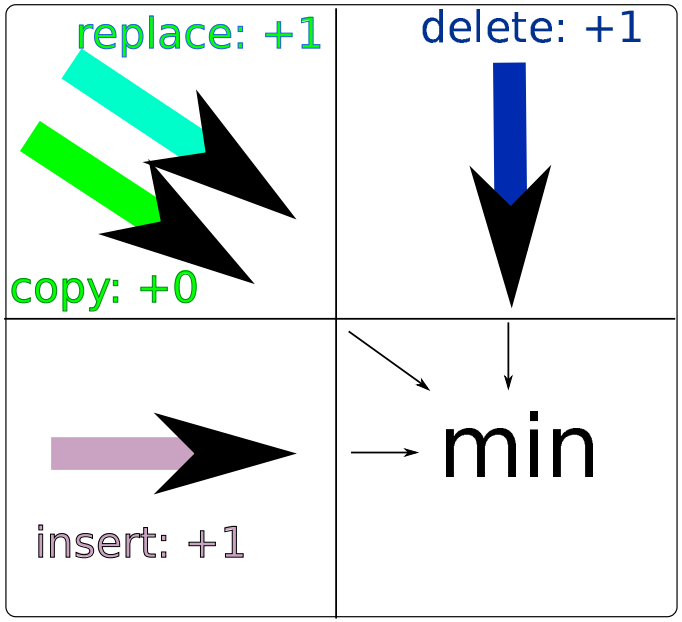
Each cell of Levenshtein matrix



Each cell of Levenshtein matrix



Each cell of Levenshtein matrix



Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1				
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2			
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1			
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2		
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2		
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3	
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2				
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3			
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1			
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2		
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2		
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3	
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3				
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4			
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2			
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3		
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	**1** 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	**2** 2	1 2 / 3 1	2 3 / 2 2	3 **3** / 3 3	3 4 / 4 3
l	**3** 3	3 **2** / 4 2	2 3 / 3 2		
o	**4** 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3	
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4 4
o	4 / 4				

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4 4
o	4 / 4	4 3 / 5			

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4 4
o	4 / 4	4 3 / 5 3			

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4 4
o	4 / 4	4 3 / 5 3	3 3 / 4		

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4 4
o	4 / 4	4 3 / 5 3	3 3 / 4 3		

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4	

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	4 5 3

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	4 5 3 3

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	4 5 3 3

Edit distance OSLO–SNOW is 3!

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	4 5 3 3

How do I read out the editing operations that transform OSLO into SNOW?

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	4 5 3 3

cost	operation	input	output
1	insert	*	w

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4 4
o	4 4	4 3 / 5 3	3 3 / 4 3	2 4 / 4 2	4 5 / 3 3

cost	operation	input	output
0	(copy)	o	o
1	insert	*	w

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	4 5 3 3

cost	operation	input	output
1	replace	l	n
0	(copy)	o	o
1	insert	*	w

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 / 1	1 2 / 2 1	2 3 / 2 2	2 4 / 3 2	4 5 / 3 3
s	2 / 2	1 2 / 3 1	2 3 / 2 2	3 3 / 3 3	3 4 / 4 3
l	3 / 3	3 2 / 4 2	2 3 / 3 2	3 4 / 3 3	4 4 / 4 4
o	4 / 4	4 3 / 5 3	3 3 / 4 3	2 4 / 4 2	4 5 / 3 3

cost	operation	input	output
0	(copy)	s	s
1	replace	l	n
0	(copy)	o	o
1	insert	*	w

Example: Edit Distance OSLO – SNOW

		s	n	o	w
	0	1 1	2 2	3 3	4 4
o	1 1	1 2 2 1	2 3 2 2	2 4 3 2	4 5 3 3
s	2 2	1 2 3 1	2 3 2 2	3 3 3 3	3 4 4 3
l	3 3	3 2 4 2	2 3 3 2	3 4 3 3	4 4 4 4
o	4 4	4 3 5 3	3 3 4 3	2 4 4 2	4 5 3 3

cost	operation	input	output
1	delete	o	*
0	(copy)	s	s
1	replace	l	n
0	(copy)	o	o
1	insert	*	w