Chapter 6:

Petri nets

Interleaving models (e.g. transition systems)

$$\alpha.\mathrm{nil} \parallel \beta.\mathrm{nil} \sim \alpha.\beta.\mathrm{nil} + \beta.\alpha.\mathrm{nil}$$

Petri nets: Transitions $\longmapsto$ Events

Global state $\longmapsto$ Conditions

A wide range of applications (+ growing!)

* Fairness - competition for time -

$\rightsquigarrow$ must α always occur?

* Partial-order model checking
* Security models + event-based reasoning [next lecture]
* Hardware models
* Biology

⋮

∞-multisets

$$\omega^\infty = \omega \cup \{\infty\}$$

Extend addition:

$$n + \infty = \infty, \quad \text{for } n \in \omega^\infty$$

Extend subtraction:

$$\infty - n = \infty, \quad \text{for } n \in \omega$$

Extend order: $n \leq \infty$, for $n \in \omega^\infty$

An ∞-multiset over a set X is a function $f : X \to \omega^\infty$. Called a multiset if $f : X \to \omega$.

Suppose f, g ∞-multisets over X.

$f \leq g$ iff $\forall x \in X.\ f(x) \leq g(x)$

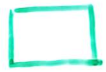
$f + g$ is the ∞-multiset s.t.
$\forall x \in X.\ (f+g)(x) = f(x) + g(x)$

If $f \leq g$ and f a multiset,

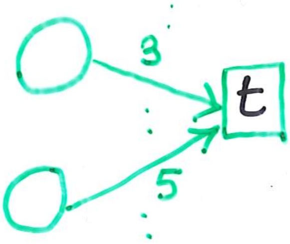
$g - f$ is the ∞-multiset s.t.
$\forall x \in X.\ (g-f)(x) = g(x) - f(x).$

General Petri net consists of

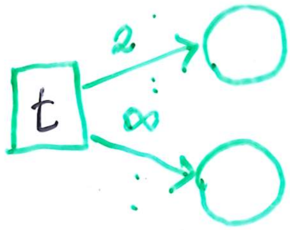
- set of conditions P 

- set of events T 

- a pre-condition map assigning a multiset of conditions ${}^\bullet t$ to each event t



- a post-condition map assigning a ∞-multiset of conditions $t^\bullet$ to each event t



- a capacity map an ∞-multiset Cap of conditions, assigning a capacity in $\omega \cup \{\omega\}$ to each condition.

A marking is an ∞-multiset

$$\mathcal{M} \leq Cap$$

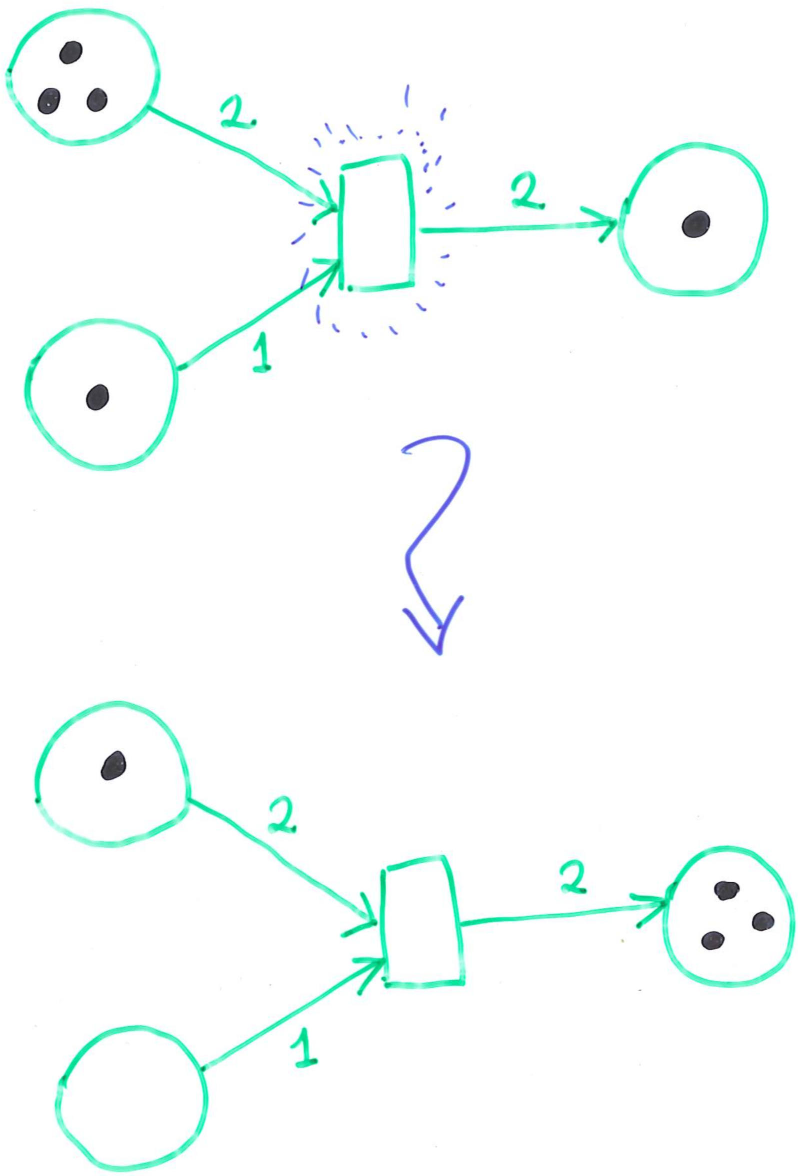
The "token game":

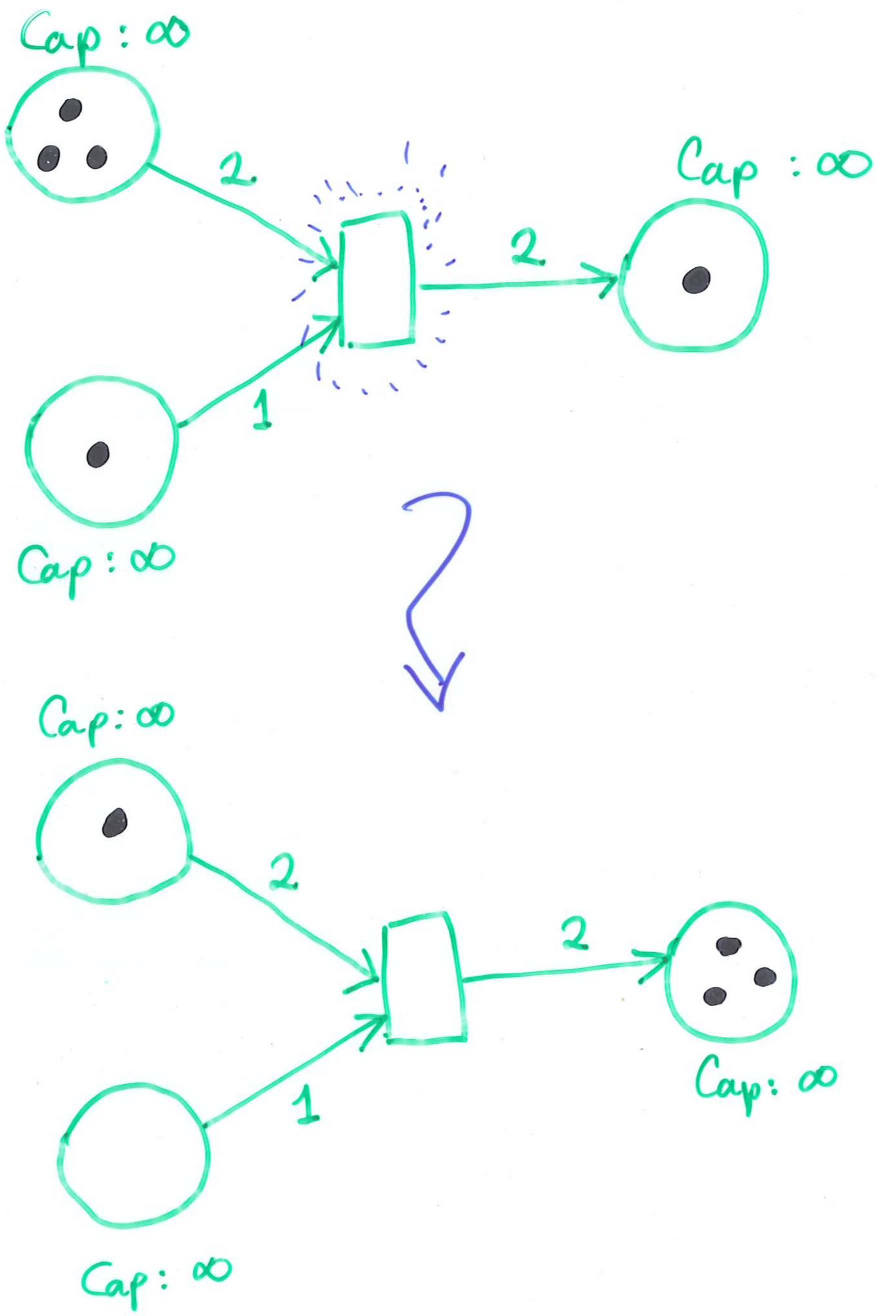
$\mathcal{M}, \mathcal{M}'$ markings, t an event.

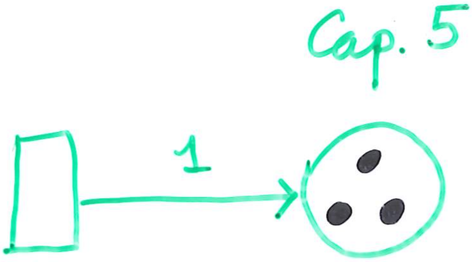
$$\mathcal{M} \xrightarrow{t} \mathcal{M}' \text{ iff}$$

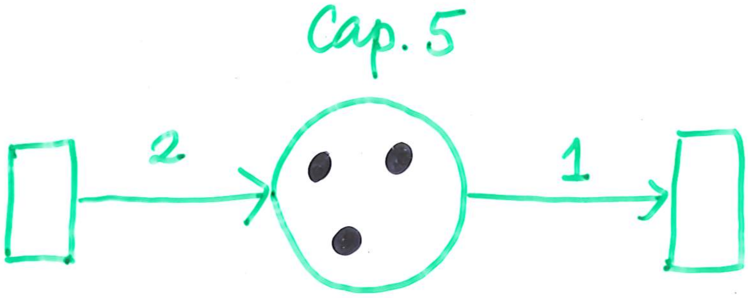
$${}^\bullet t \leq \mathcal{M} \quad \& \quad \mathcal{M}' = \mathcal{M} - {}^\bullet t + t^\bullet$$

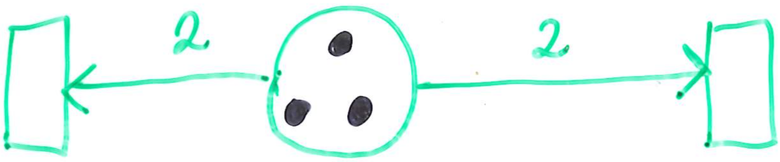
Event t has concession (is enabled) at marking $\mathcal{M}$ iff ${}^\bullet t \leq \mathcal{M}$ & $\mathcal{M} - {}^\bullet t + t^\bullet \leq Cap$

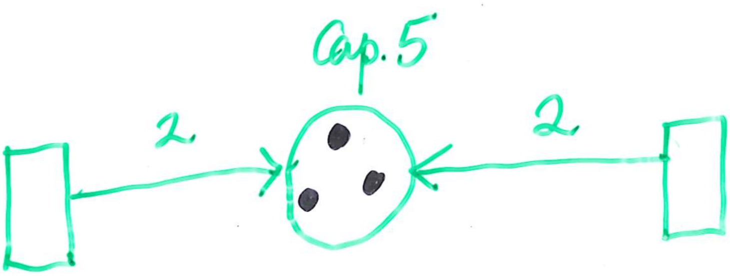

2
1
2
2
1
2


Cap : ∞
2
2
Cap : ∞
1
Cap : ∞
Cap: ∞
2
2
1
Cap: ∞
Cap: ∞


Cap. 5
1


Cap. 5
2
1


2
2


Cap. 5
2
2

A **basic net** consists of

idea: everything's a set

- a set of conditions B
- a set of events E
- a pre-condition map assigning a subset of conditions ${}^\bullet e$ to any event e
- a post-condition map assigning a subset of conditions $e^\bullet$ to any event e [AXIOM: ${}^\bullet e \cup e^\bullet \neq \emptyset$.]

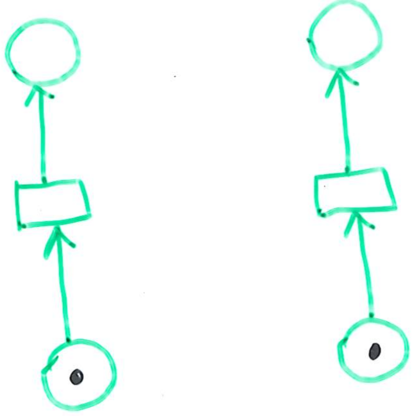
$\forall b.\ Cap(b) = 1$ ...

A marking M is a subset of conditions

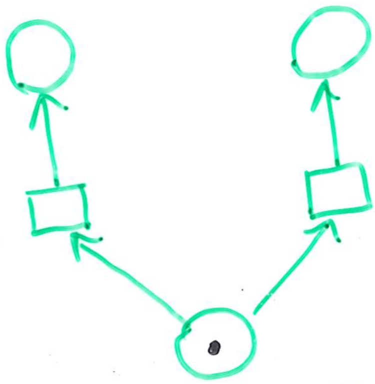
$$M \xrightarrow{e} M' \text{ iff}$$

concession ${}^\bullet e \subseteq M$ & $(M \setminus {}^\bullet e) \cap e^\bullet = \emptyset$ &

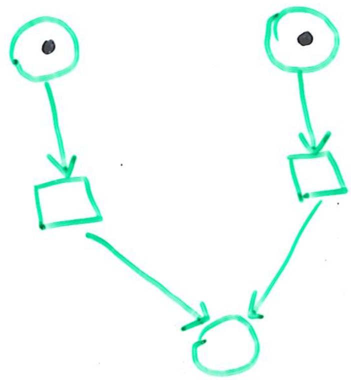
$$M' = (M \setminus {}^\bullet e) \cup e^\bullet$$



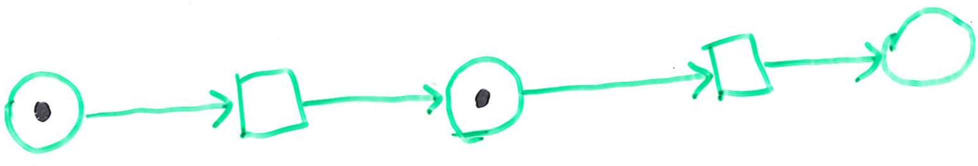
Concurrency



Forwards conflict



Backwards conflict

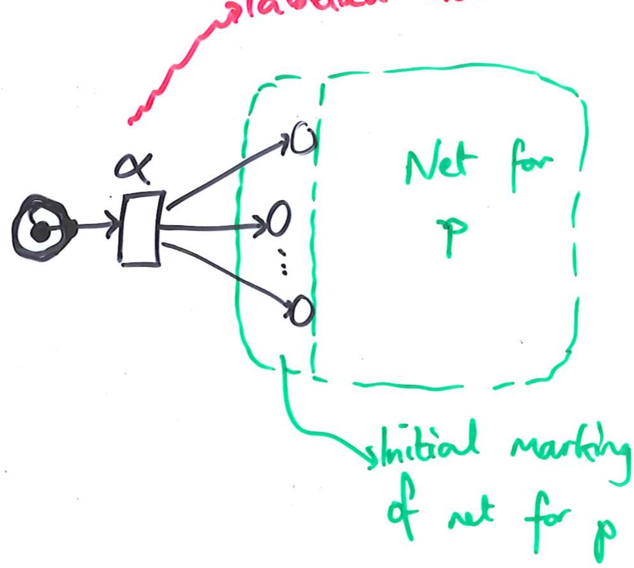


Contact

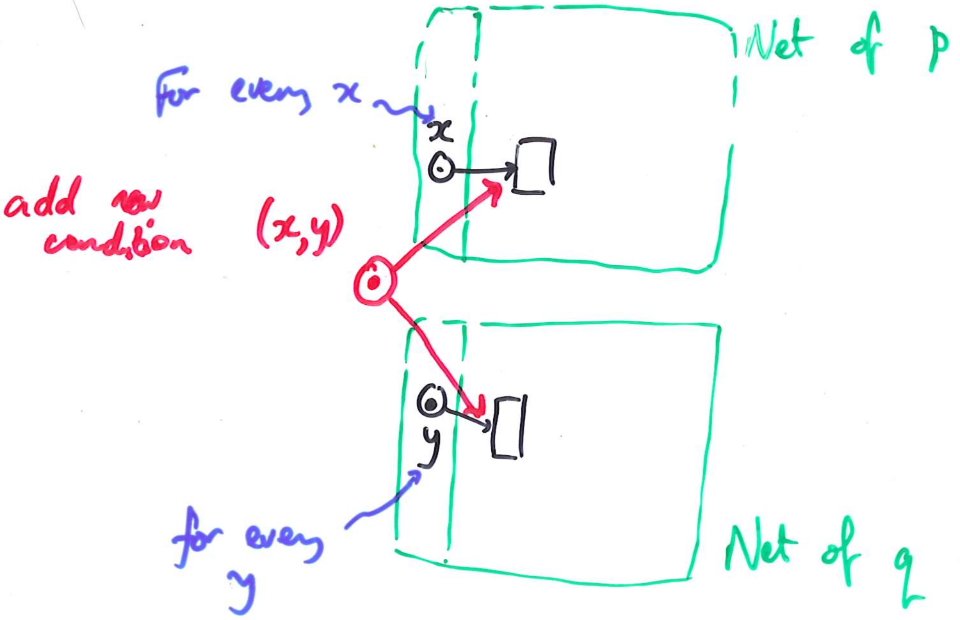
CCS operations on basic Petri nets.

Nil nil

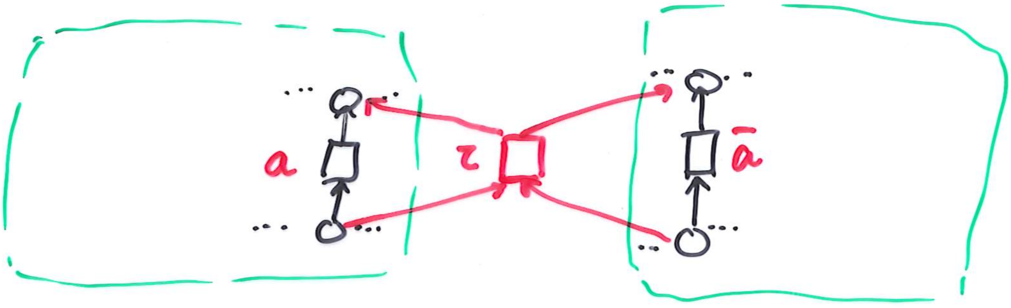
Prefix $\alpha . p$



Sum $p + q$



Parallel composition. $p \| q$

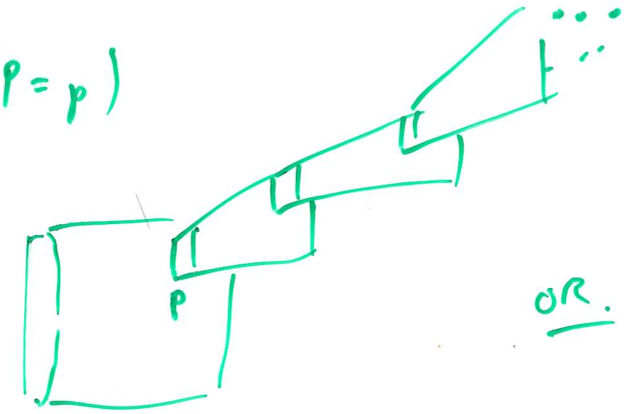


Net of p

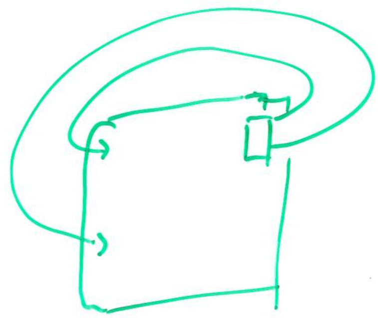
Net of q

Add a τ-event for every pair of complementary actions.

Recursion $rec(P = p)$



OR.



Exercise:

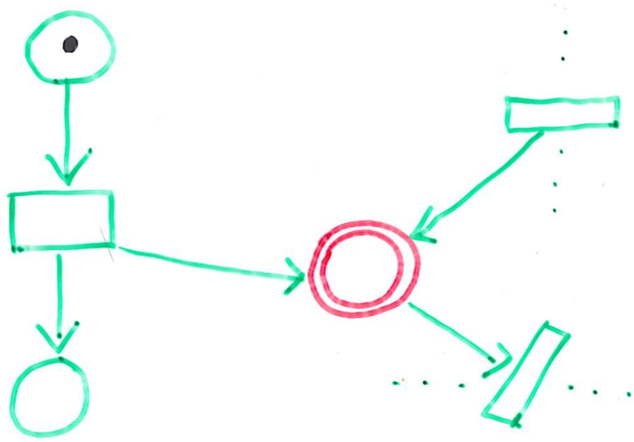
Draw the net for

$$a \mid (b \mid d) + \bar{a} \mid c$$

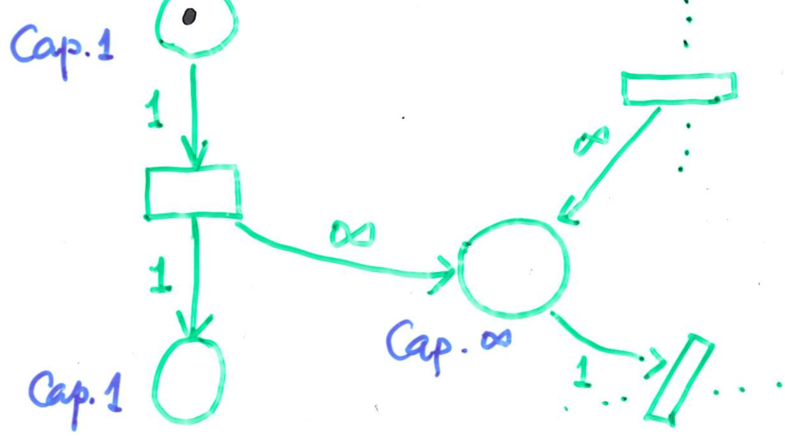
Persistent conditions, conditions which when they hold persist in holding, and can be used repeatedly as preconditions.

Eg. Assertions in mathematics

Broadcast messages



As general net:

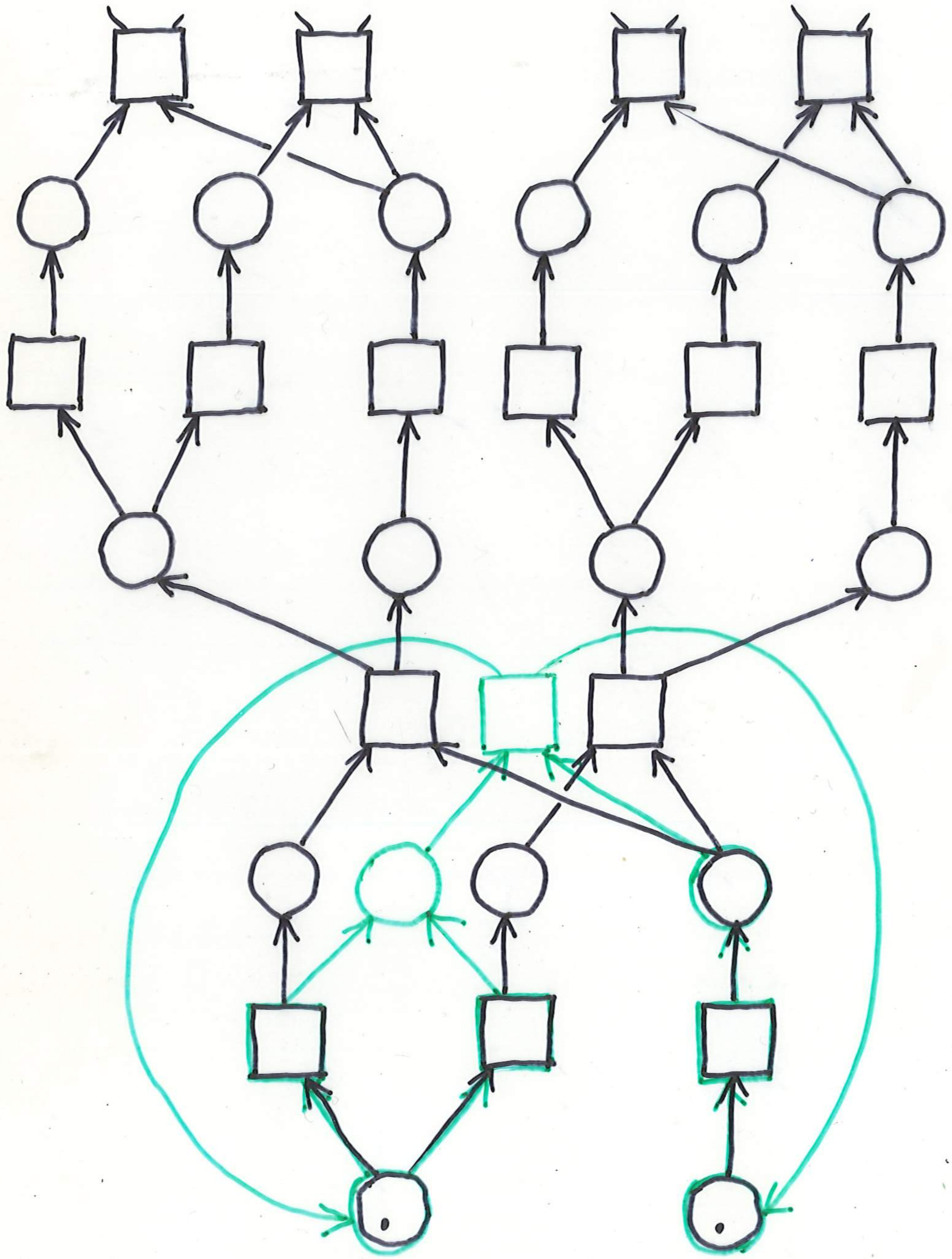


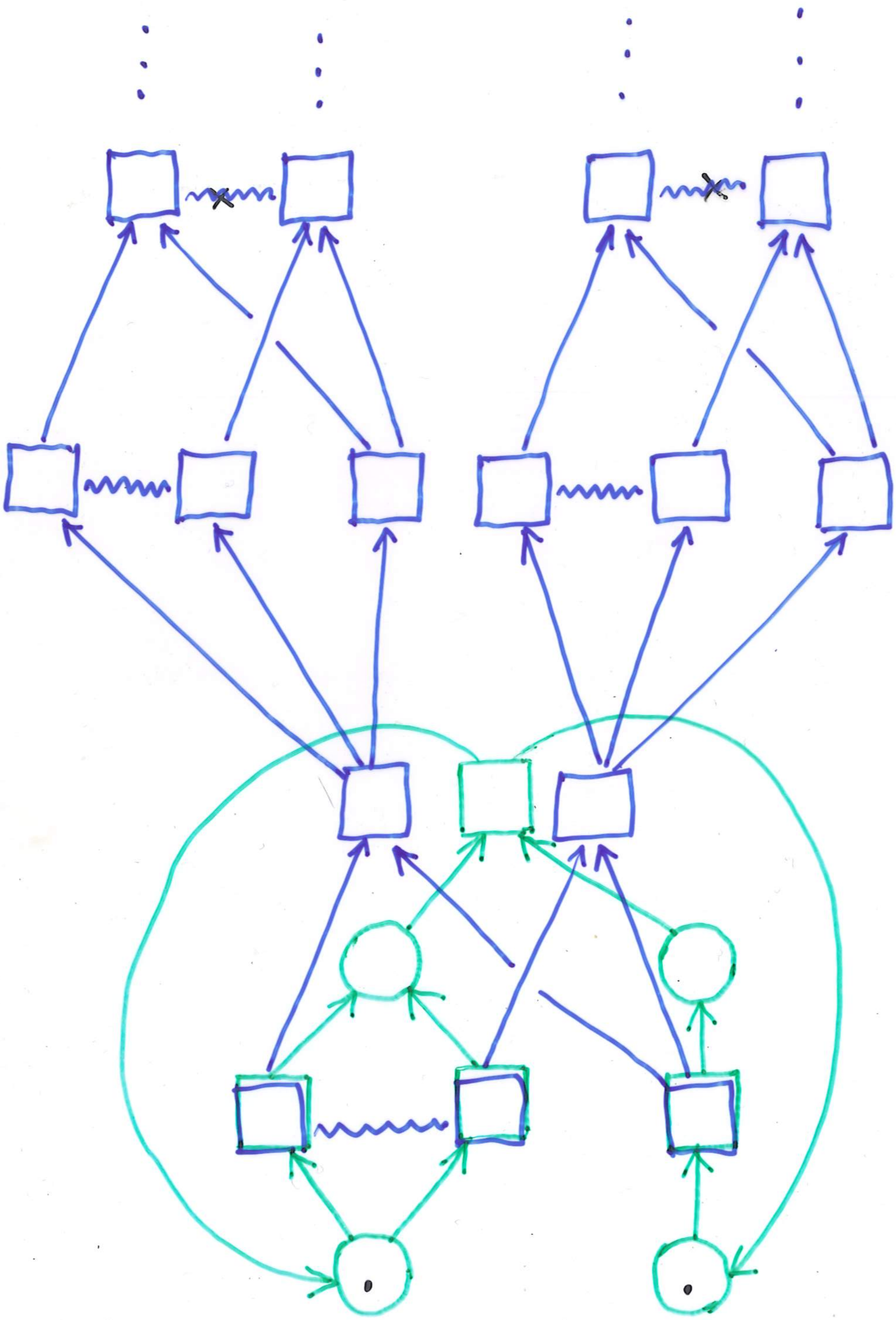
$M \xrightarrow{e} M'$ iff

${}^\bullet e \subseteq M$ & $(e^\bullet \cap (M \setminus (\mathrm{Persistent} \cup {}^\bullet e))) = \emptyset$ &

$M' = (M \setminus {}^\bullet e) \cup e^\bullet \cup (M \cap \mathrm{Persistent})$.

Unfolding nets, partial order models





Event structure : causal dependency, conflict