The Model

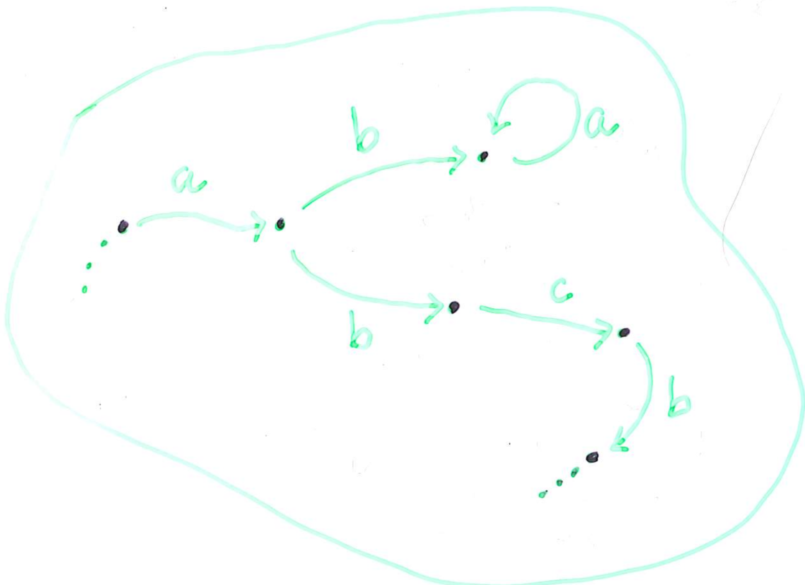
A process is modelled as a state in a labelled transition system

$$(S,\ L,\ tran)$$

S set of states / processes

L set of labels / actions

$tran \subseteq S \times L \times S$ transition relation



A process is finite state if the set of states it can reach is finite.

Finitary H-M logic

$$A ::= T \mid F \mid A_0 \wedge A_1 \mid A_0 \vee A_1 \mid \neg A \mid \langle \lambda \rangle A \mid \langle - \rangle A$$

$s \models T$ always

$s \models F$ never

$s \models A_0 \wedge A_1$ if $s \models A_0$ and $s \models A_1$

$s \models A_0 \vee A_1$ if $s \models A_0$ or $s \models A_1$

$s \models \neg A$ if not $s \models A$

$s \models \langle \lambda \rangle A$ if there exists s' s.t. $s \xrightarrow{\lambda} s'$ and $s' \models A$

$s \models \langle - \rangle A$ if there exist s', λ s.t. $s \xrightarrow{\lambda} s'$ and $s' \models A$

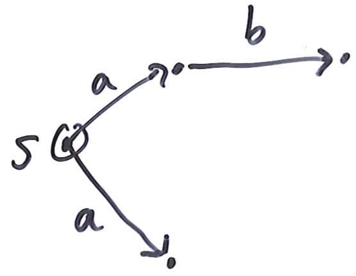
A derived assertion:

$$\neg \langle \lambda \rangle \neg A \equiv [\lambda] A$$

$s \models [\lambda] A$ iff for all s' s.t. $s \xrightarrow{\lambda} s'$, have $s' \models A$

$$\neg \langle - \rangle \neg A \equiv [-] A$$

Examples



$s \models \langle a \rangle T$?

$s \models [a] T$?

$s \models [a] \langle b \rangle T$?

$s \models \langle a \rangle \langle b \rangle T$?

Generally:

$\langle a \rangle T$

$[a] F$

$\langle - \rangle F$

$\langle - \rangle T$

$[-] T$

$[-] F$

Give a transition system with initial state satisfying:

$$\langle - \rangle [a] F$$

$$\land\ [a] \langle a \rangle T$$

Strong equivalence & Logic

Hennessy-Milner logic

$$A ::= \bigwedge_{i \in I} A_i \;\Big|\; \neg A \;\Big|\; \langle \alpha \rangle A$$

$p \models \bigwedge_{i \in I} A_i$ iff $p \models A_i$ for all $i \in I$

$p \models \neg A$ iff not $p \models A$

$p \models \langle \alpha \rangle A$ iff $p \xrightarrow{\alpha} q$ & $q \models A$ for some q.

Define

$p \asymp q$ iff for all assertions A of H-M logic

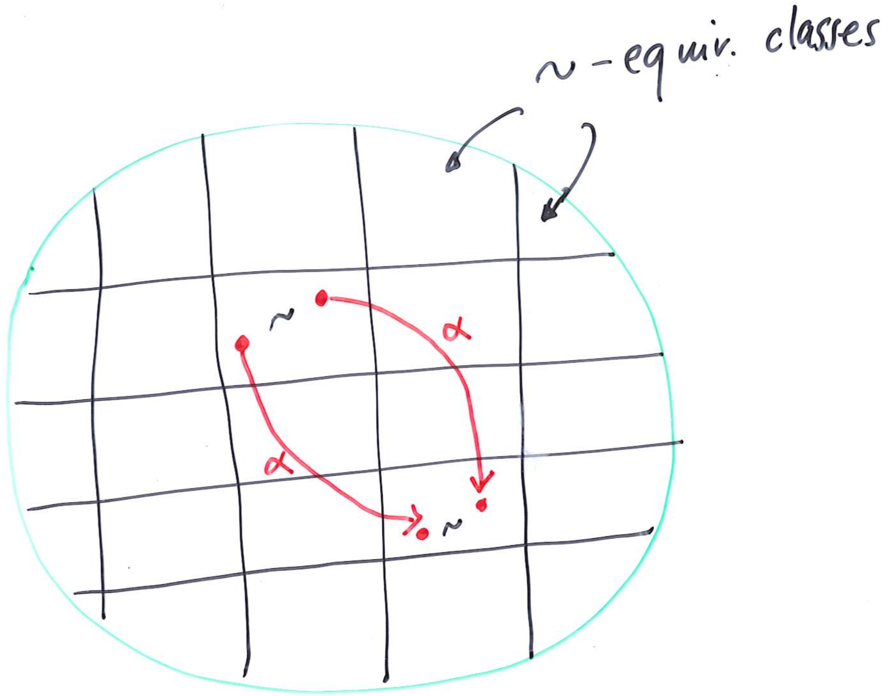
$p \models A \iff q \models A$.

Theorem: $\asymp \; = \; \sim$

in modal-μ calc or CTL

Coroll: $p \sim q$ & $p \models A \Rightarrow q \models A$

Collapsing transition systems w.r.t. bisim.



Given trans. sys. $(S, L, tran)$

form $(S, L, tran)/\sim$

states $\{s\}_\sim$

transitions $\{s\}_\sim \xrightarrow{\alpha} \{s'\}_\sim$

iff $\exists s''.\ s \xrightarrow{\alpha} s'' \sim s'$