

## 1991 Paper 4 Question 6

### Formal Languages and Automata

- (a) Prove or disprove each of the following statements. You may use any well-known results provided you state them clearly.
- (i) Given regular languages  $L_1$  and  $L_2$ , the set of strings which are in both  $L_1$  and  $L_2$  forms a regular language.
  - (ii) The set  $\{ww^R \mid w \in \Sigma^*\}$  is a regular language over the alphabet  $\Sigma = \{a, b\}$  (where  $w^R$  denotes the reverse of the string  $w$ ).
  - (iii) The set of strings  $w \in \{0, 1\}^*$  for which the number of occurrences of 0 and the number of occurrences of 1 in  $w$  are both even forms a regular language.
- (b)  $M$  is a finite deterministic automaton with  $n$  states. Show that the language accepted by  $M$  is non-empty if and only if  $M$  accepts a string of length  $(n - 1)$  or less.

[Hint: consider the shortest word accepted by  $M$ , if any.]