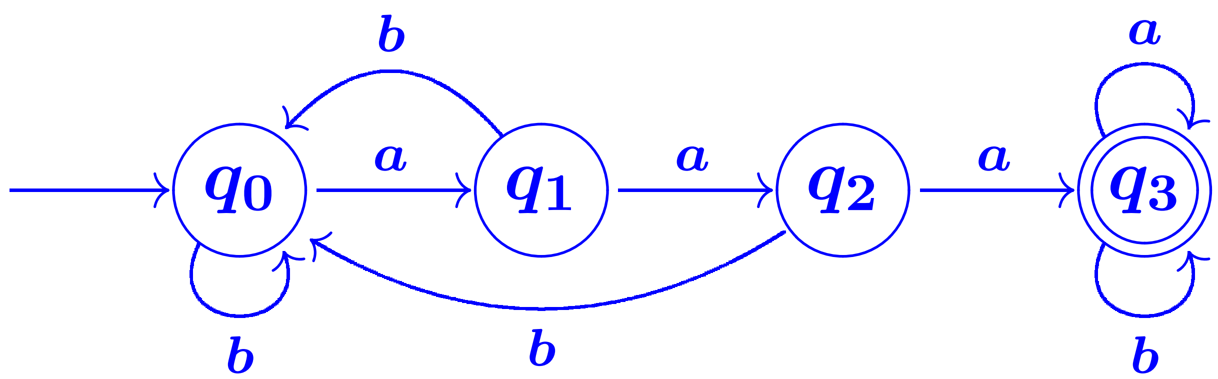
Example of a finite automaton



States: q_0, q_1, q_2, q_3.

Input symbols: a, b.

Transitions: as indicated above.

Start state: q_0.

Accepting state(s): q_3.

$L(M)$, ***language accepted*** **by a finite automaton** M

consists of all strings u over its alphabet of input symbols satisfying $q_0 \xrightarrow{u}{}^* q$ with q_0 the start state and q some accepting state. Here

$$q_0 \xrightarrow{u}{}^* q$$

means, if $u = a_1 a_2 \ldots a_n$ say, that for some states $q_1, q_2, \ldots, q_n = q$ (not necessarily all distinct) there are transitions of the form

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \cdots \xrightarrow{a_n} q_n = q.$$

N.B.

case $n = 0$: $q \xrightarrow{\varepsilon}{}^* q'$ iff $q = q'$

case $n = 1$: $q \xrightarrow{a}{}^* q'$ iff $q \xrightarrow{a} q'$.

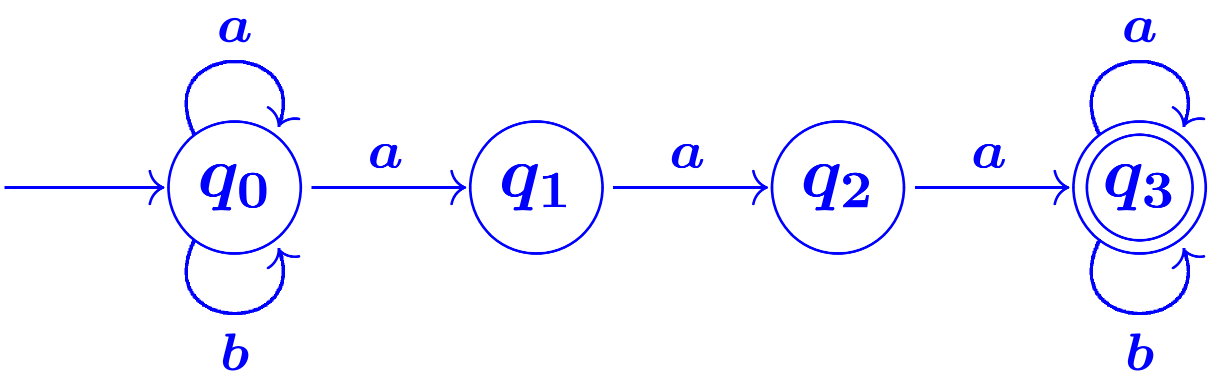
A ***non-deterministic finite automaton*** (NFA), M,
is specified by

- a finite set $States_M$ (of ***states***)
- a finite set Σ_M (the alphabet of ***input symbols***)
- for each $q \in States_M$ and each $a \in \Sigma_M$, a subset $\Delta_M(q, a) \subseteq States_M$ (the set of states that can be reached from q with a single ***transition*** labelled a)
- an element $s_M \in States_M$ (the ***start state***)
- a subset $Accept_M \subseteq States_M$ (of ***accepting states***)

Example of a non-deterministic finite automaton

Input alphabet: $\{a, b\}$.

States, transitions, start state, and accepting states as shown:



The language accepted by this automaton is the same as for the automaton on Slide 10, namely

$$\{u \in \{a, b\}^* \mid u \text{ contains three consecutive } a\text{'s}\}.$$

A ***deterministic finite automaton*** (DFA) is an NFA M with the property that for each $q \in States_M$ and $a \in \Sigma_M$, the finite set $\Delta_M(q, a)$ contains exactly one element—call it $\delta_M(q, a)$.

Thus in this case transitions in M are essentially specified by a ***next-state function***, δ_M, mapping each (state, input symbol)-pair (q, a) to the unique state $\delta_M(q, a)$ which can be reached from q by a transition labelled a:

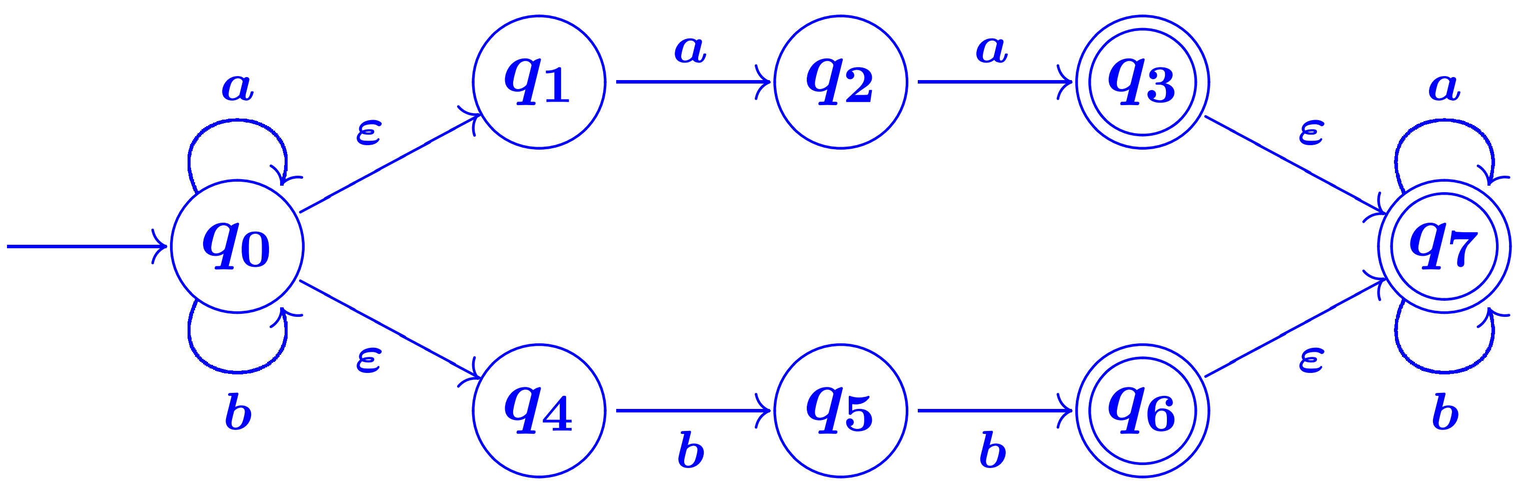
$$q \xrightarrow{a} q' \quad \text{iff} \quad q' = \delta_M(q, a)$$

An ***NFA with ε-transitions*** (NFA$^{\varepsilon}$)
is specified by an NFA M together with a binary relation, called the ***ε-transition relation***, on the set $States_M$. We write

$$q \xrightarrow{\varepsilon} q'$$

to indicate that the pair of states (q, q') is in this relation.

Example (with input alphabet = $\{a, b\}$):



$L(M)$, ***language accepted*** **by an NFA$^{\varepsilon}$ M**

consists of all strings u over the alphabet Σ_M of input symbols satisfying $q_0 \overset{u}{\Rightarrow} q$ with q_0 the initial state and q some accepting state.

Here $\cdot \overset{-}{\Rightarrow} \cdot$ is defined by:

$q \overset{\varepsilon}{\Rightarrow} q'$ iff $q = q'$ or there is a sequence $q \overset{\varepsilon}{\rightarrow} \cdots q'$ of one or more ε-transitions in M from q to q'

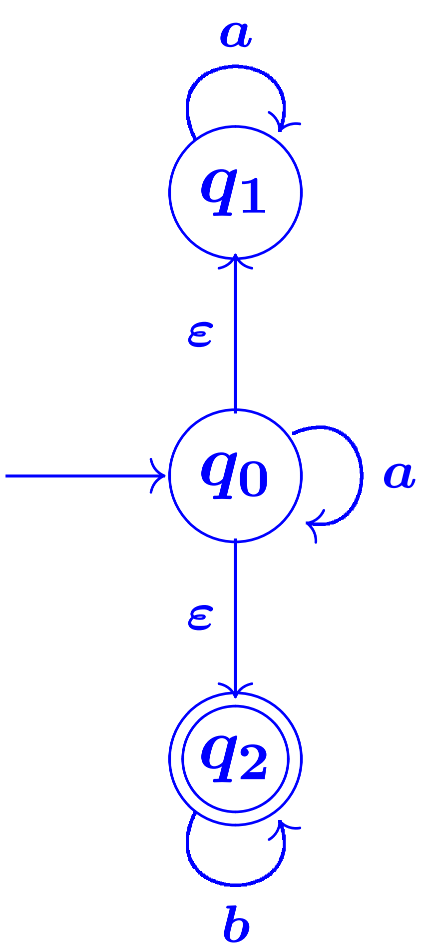
$q \overset{a}{\Rightarrow} q'$ (for $a \in \Sigma_M$) iff $q \overset{\varepsilon}{\Rightarrow} \cdot \overset{a}{\rightarrow} \cdot \overset{\varepsilon}{\Rightarrow} q'$

$q \overset{ab}{\Rightarrow} q'$ (for $a, b \in \Sigma_M$) iff $q \overset{\varepsilon}{\Rightarrow} \cdot \overset{a}{\rightarrow} \cdot \overset{\varepsilon}{\Rightarrow} \cdot \overset{b}{\rightarrow} \cdot \overset{\varepsilon}{\Rightarrow} q'$

and similarly for longer strings

Example of the subset construction

M :



δ_{PM} :	a	b
$\emptyset$	$\emptyset$	$\emptyset$
$\{q_0\}$	$\{q_0, q_1, q_2\}$	$\{q_2\}$
$\{q_1\}$	$\{q_1\}$	$\emptyset$
$\{q_2\}$	$\emptyset$	$\{q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_2\}$
$\{q_1, q_2\}$	$\{q_1\}$	$\{q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_2\}$

Theorem. For each NFA$^{\varepsilon}$ M there is a DFA PM with the same alphabet of input symbols and accepting exactly the same strings as M, i.e. with $L(PM) = L(M)$

Definition of PM (refer to Slides 12 and 14):

- $States_{PM} \stackrel{\text{def}}{=} \{S \mid S \subseteq States_M\}$
- $\Sigma_{PM} \stackrel{\text{def}}{=} \Sigma_M$
- $S \stackrel{a}{\to} S'$ in PM iff $S' = \delta_{PM}(S, a)$, where $\delta_{PM}(S, a) \stackrel{\text{def}}{=} \{q' \mid \exists q \in S\ (q \stackrel{a}{\Rightarrow} q' \text{ in } M)\}$
- $s_{PM} \stackrel{\text{def}}{=} \{q \mid s_M \stackrel{\varepsilon}{\Rightarrow} q\}$
- $Accept_{PM} \stackrel{\text{def}}{=}$ $\{S \in States_{PM} \mid \exists q \in S\ (q \in Accept_M)\}$