

HW1

- 1 Give an example of an interesting binary operation that is NOT associative.
- 2 Prove that matrix multiplication (over a semiring) is associative. Which semiring axioms are required?
- 3 When is matrix multiplication commutative?
- 4 Prove Fact 1 (last slide of Lecture 3).
- 5 Prove Fact 2 (last slide of Lecture 3).
- 6 Let

$$A = \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix}.$$

Prove that A^* is

$$\begin{pmatrix} (A_{1,1} \oplus A_{1,2}A_{2,2}^*A_{2,1})^* & A_{1,1}^*A_{1,2}(A_{2,2} \oplus A_{2,1}A_{1,1}^*A_{1,2})^* \\ A_{2,2}^*A_{2,1}(A_{1,1} \oplus A_{1,2}A_{2,2}^*A_{2,1})^* & (A_{2,2} \oplus A_{2,1}A_{1,1}^*A_{1,2})^* \end{pmatrix}.$$