Social and Technological Network Analysis

Lecture 3: Centrality Measures

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(some material from
Lada Adamic's lectures)

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In This Lecture

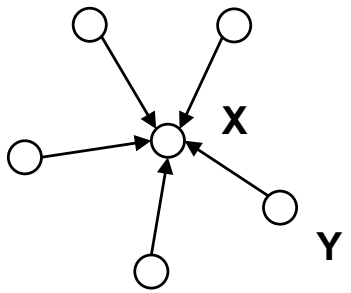
- We will introduce the concept of centrality and the various measures which have been associated to this concept.
- We will show an application.

Centrality

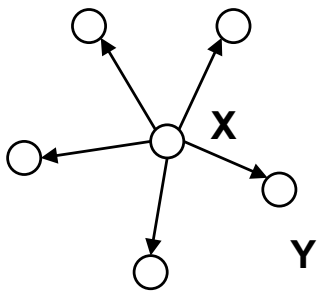
- Finding out which is the most central node is important:
  - It could help disseminating information in the network faster
  - It could help stopping epidemics
  - It could help protecting the network from breaking

Centrality: visually

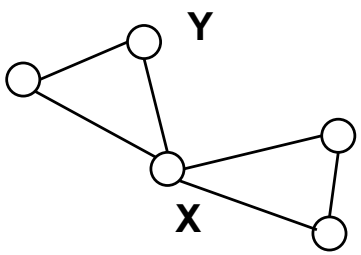
- Centrality can have various meanings:



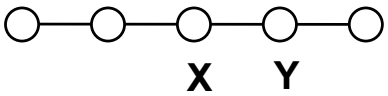
indegree



outdegree

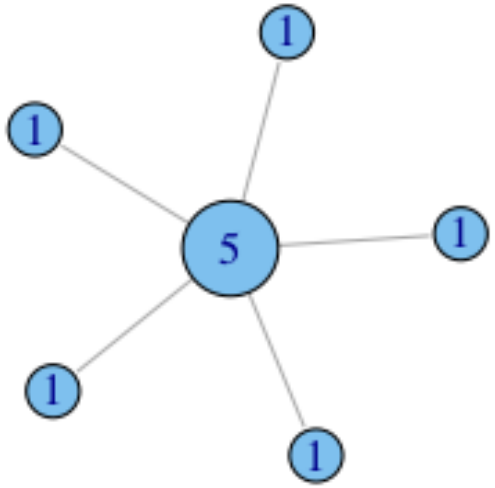


betweenness



closeness

Degree Centrality



When is the number of connections the best centrality measure?

- people who will do favors to you
- people you can talk to / have a beer with

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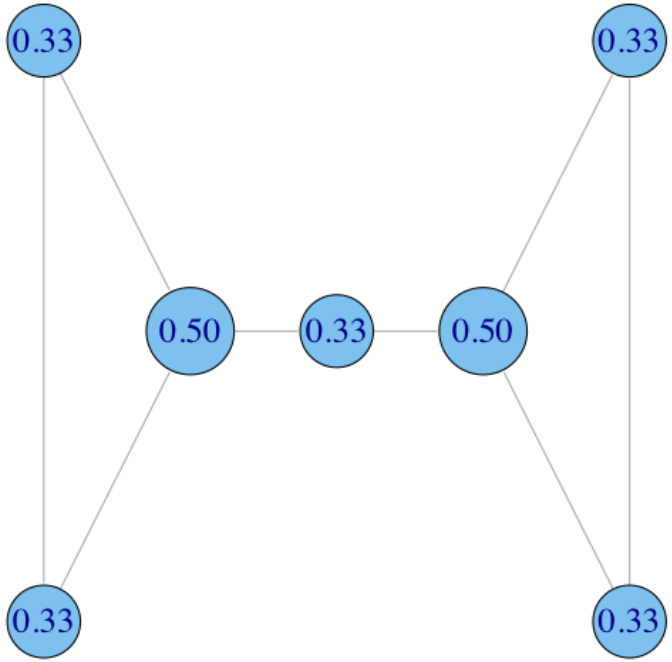
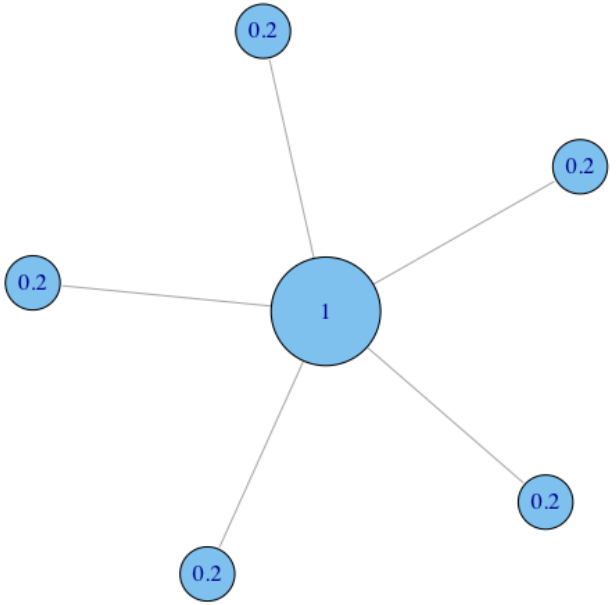
Normalization

- Divide for the max number of nodes (N-1)





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Freeman's Network Centrality

- How do we calculate the value of centrality of the network
  - To check how much variation there is among the nodes (heterogeneity?)

Max value of Degree Centrality in the Network

Max value of the above: when network is a star: 1 node has C=N-1 and all others (N-1) have 1.

$$C_D = \frac{\sum_{i=1}^{s} \left[C_D(n^*) - C_D(i) \right]}{\left[(N-1)(N-2) \right]}$$

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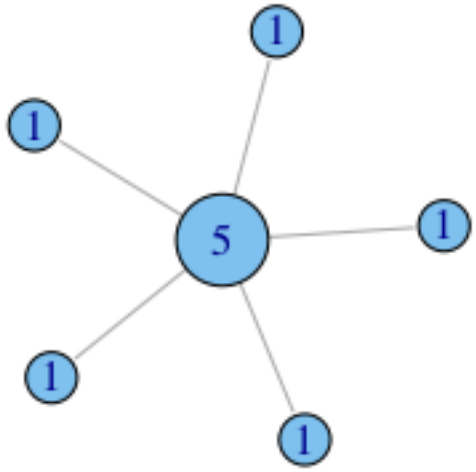
Freeman Network Centrality Explained

- Explanation of the denominator:
- In the star topology one node has degree N-1 and all other nodes have degree of 1:

$$0+((n-1)-1)*n-1= (n-2) *(n-1)$$

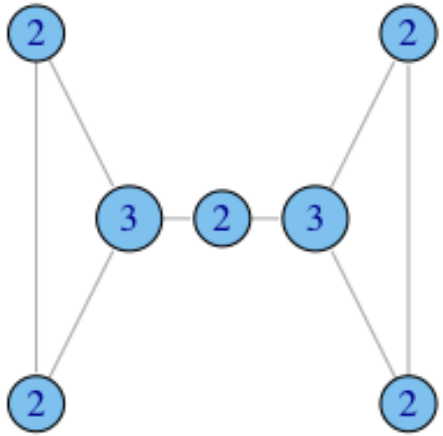
Freeman's Network Centrality



C_D = 1.0

C_D = 0.167

4+4+4+4+4/5*4
1+0+0+0+1/4*3=1/6
1+1+0+1+0+1+1/6*5=5/30



C_D = 0.167

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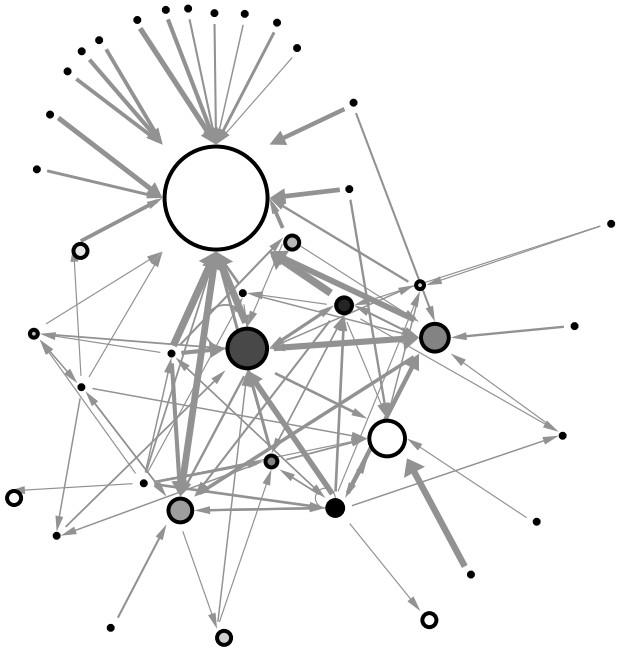
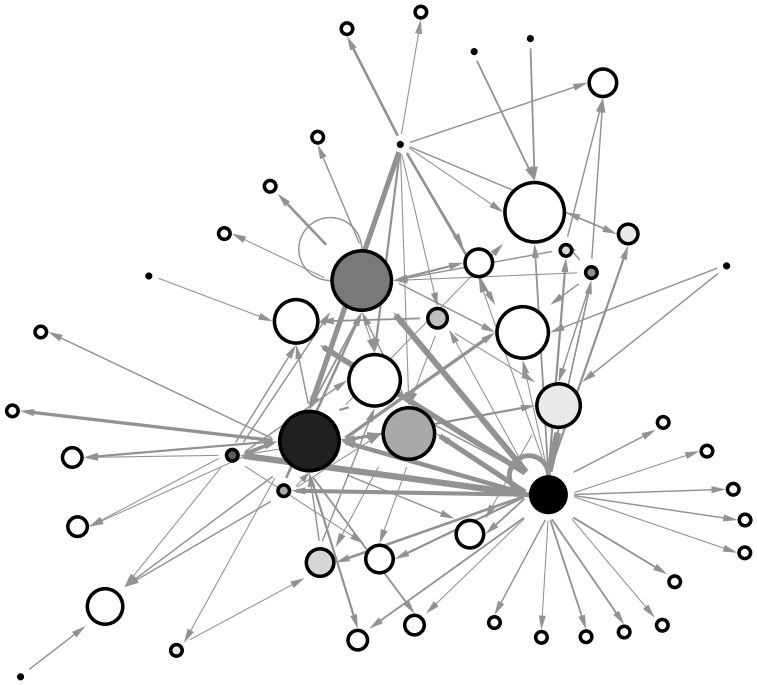
Examples: Financial Networks





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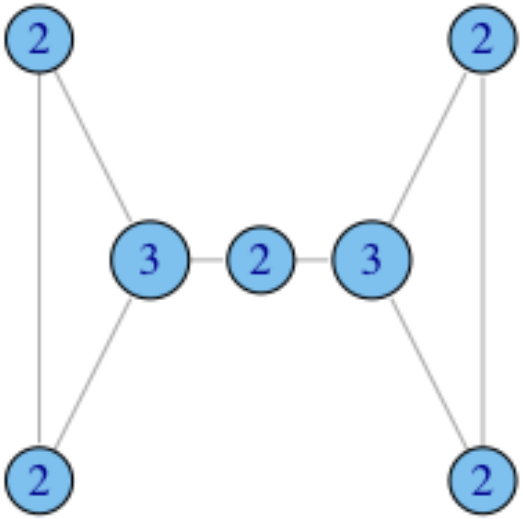
When is Degree Centrality not so good?



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When is Degree Centrality not so good (2)?

- Ability to broker between groups
- Likelihood that information originating anywhere in the network reaches you…

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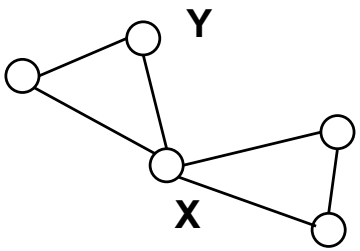
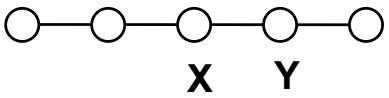
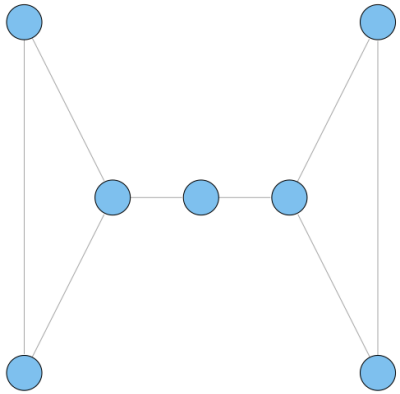
Betweenness Centrality

- Intuition: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- who has higher betweenness, X or Y?







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Betweenness (Formally)

$$C_B(i) = \sum_{j \neq k} g_{jk}(i) / g_{jk}$$

Where g_{jk} (i)= the number of shortest paths connecting *jk* passing through i

g_{jk} = total number of shortest paths

Usually normalized by:

$$C'_B(i) = C_B(i) / [(n-1)(n-2)/2]$$

number of pairs of vertices excluding the vertex itself

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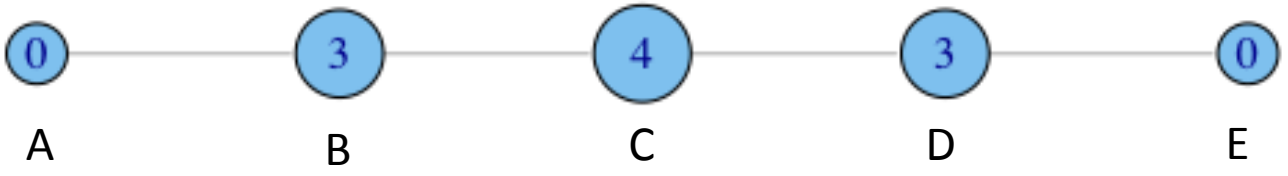
Betweenness: Example

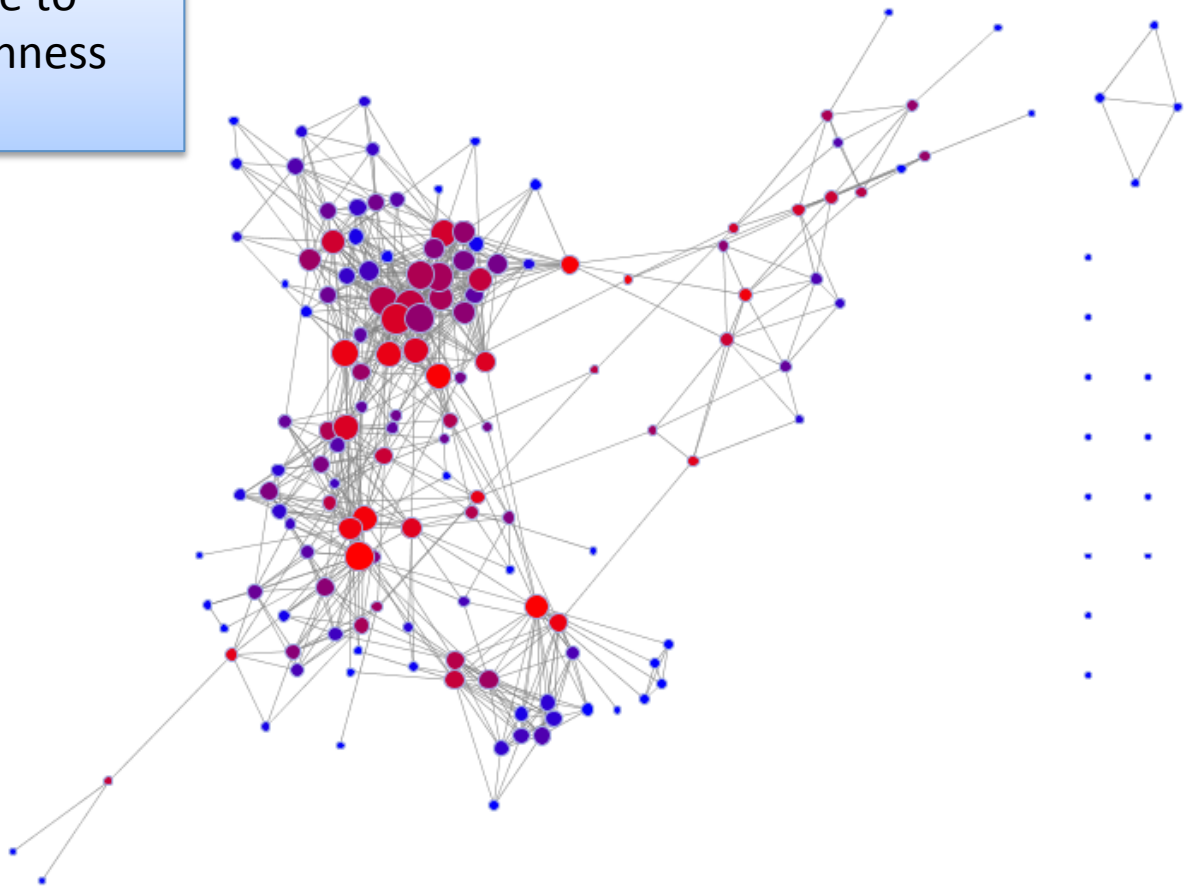
- A lies between no two other vertices
- B lies between A and 3 other vertices: C, D, and E
- C lies between 4 pairs of vertices (A,D),(A,E), (B,D),(B,E)

- note that there are no alternative paths for these pairs to take, so C gets full credit



Facebook Example (Adamic)

Color(from blue to red) is betweenness
Size is degree.



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Closeness Centrality

- What if it is not so important to have many direct friends?
- Or be "between" others

- But one still wants to be in the "middle" of things, not too far from the center

Closeness Centrality (Formally)

- Closeness is based on the length of the average shortest path between a vertex and all vertices in the graph

$$C_c(i) = \left[\sum_{j=1}^{N} d(i,j)\right]^{-1}$$

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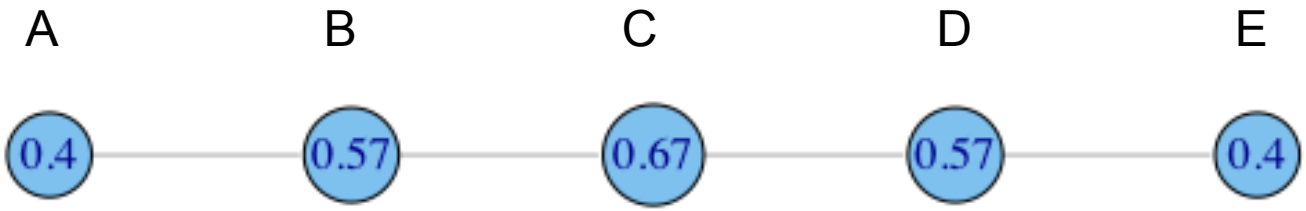
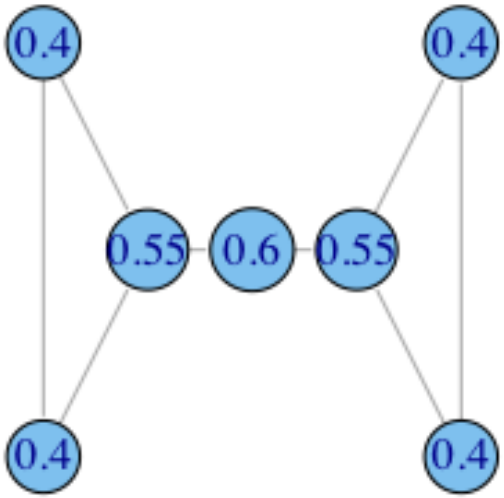
$$C_C'(i) = (C_C(i))/(N-1)$$

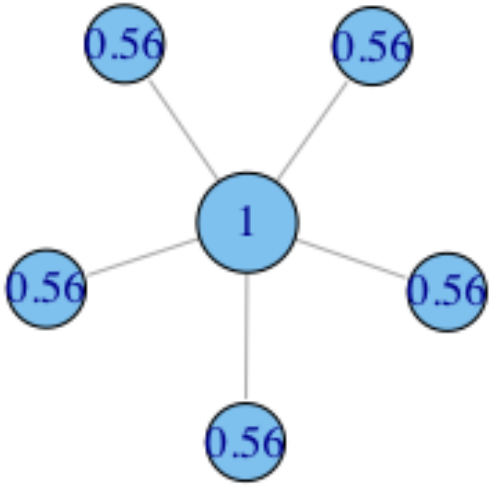
Closeness: Example



$$C_c'(A) = \left[\frac{\sum_{j=1}^{N} d(A,j)}{N-1} \right]^{-1} = \left[\frac{1+2+3+4}{4} \right]^{-1} = \left[\frac{10}{4} \right]^{-1} = 0.4$$

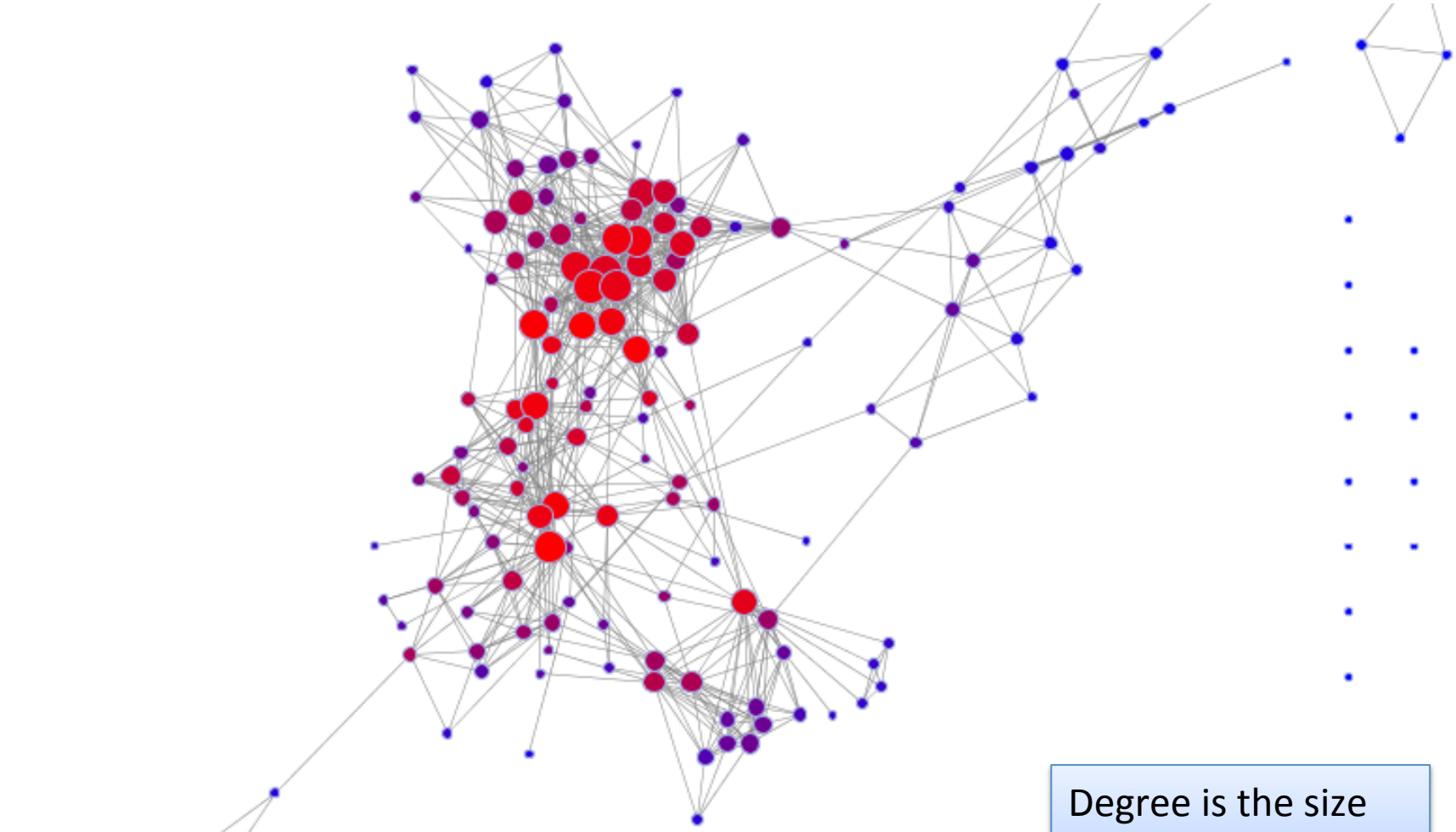
Examples





(1+1+2+3+4+4/6)^-1=6/15=0.4

Example: Facebook (Adamic)



Degree is the size
Color is closeness

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Eigenvector Centrality

- Degree Centrality depends on having many connections: but what if these connections are pretty isolated?
- A central node should be one connected to powerful nodes

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in G} a_{v,t} x_t$$

$$\mathbf{Ax} = \lambda \mathbf{x}$$

Neighbourhood of Xv

Adjacency Matrix of the graph

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Eigenvector Centrality Algorithm

- 1. Start by assigning centrality score of 1 to all nodes (v_i = 1 for all i in the network)
- 2. Recompute scores of each node as weighted sum of centralities of all nodes in a node's neighborhood: v_i = sum_{j \in N} a_{ij}*v_j
- 3. Normalize v by dividing each value by the largest value
- 4. Repeat steps 2 and 3 until values of v stop changing.

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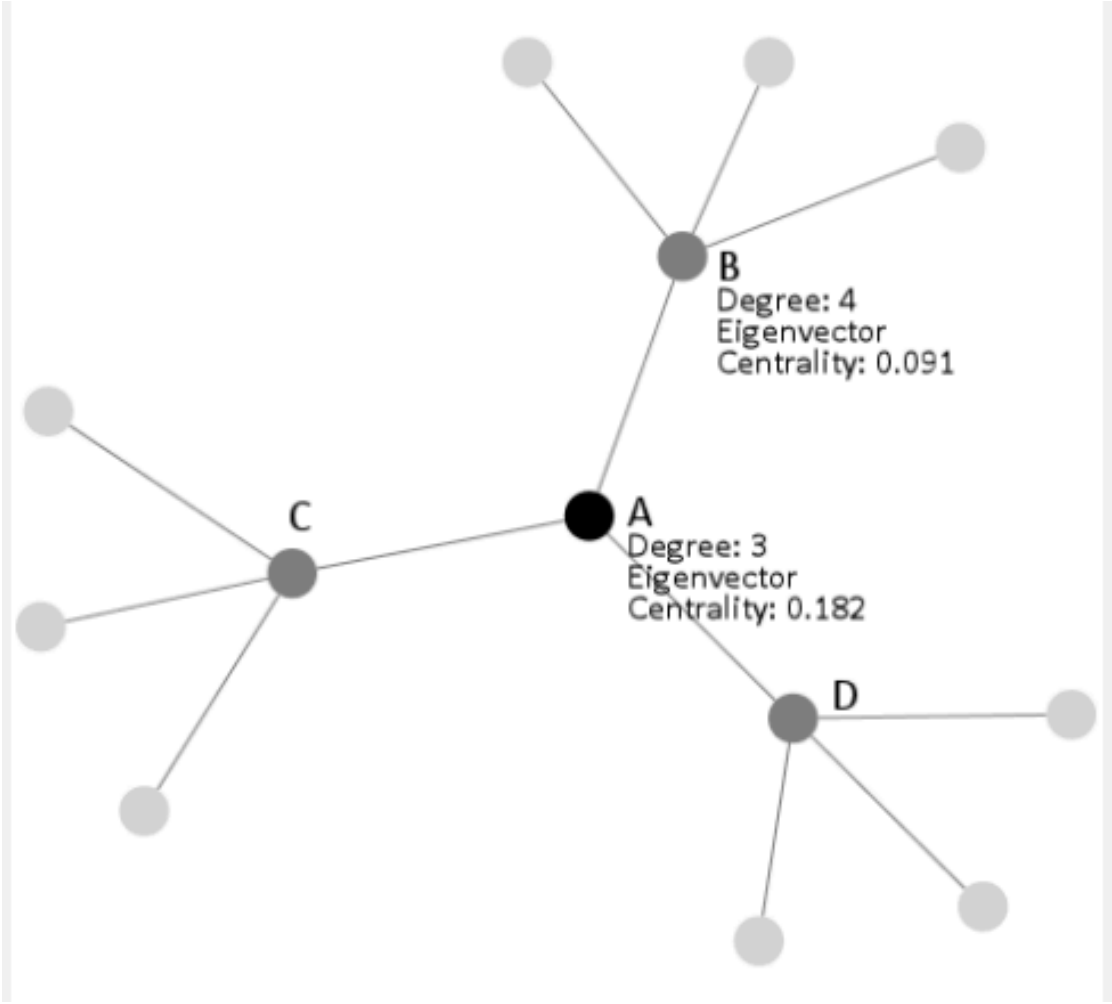
Example



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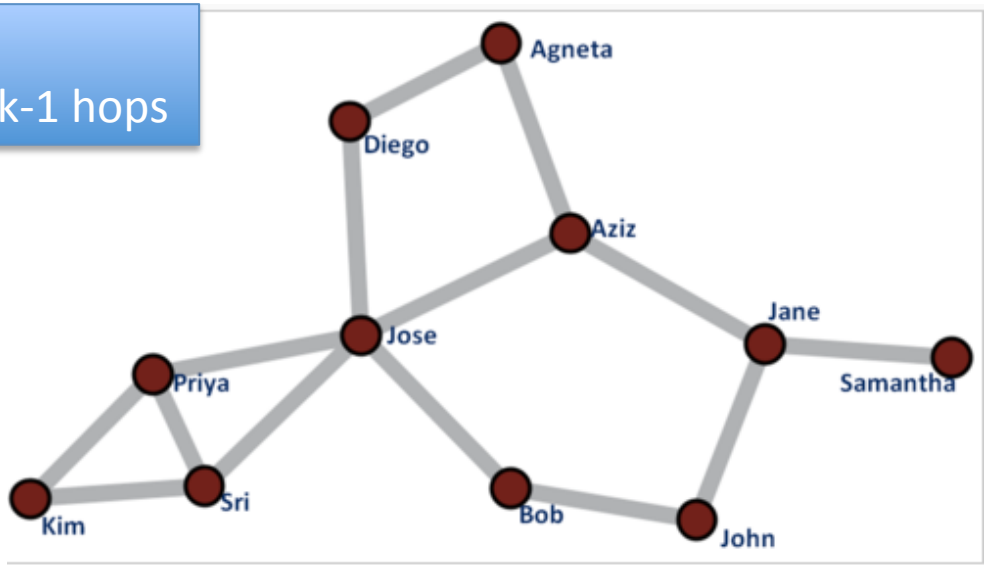
Katz Centrality

- Closeness counts the number of shortest paths, but one could count the **number of paths**.

$$C_{\text{Katz}}(i) = \sum_{k=1}^{\infty} \sum_{j=1}^{n} \alpha^k (A^k)_{ji}$$

Alpha is an attenuation factor
A^k (ij) indicates if ij are connected by k-1 hops



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Application of Centrality Measures to Big Data

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People Tweet While They Watch

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Repeated (retweet) content starts with RT

Address other users with an @

Rich Media embeds via links

Tags start with #

RT @jowyang If you are watching the debate you're invited to participate in #tweetdebate Here is the 411 http://tinyurl.com/3jdy67

ANATOMY OF A TWEET

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Tweet Crawl

- Three hashtags: ***#current #debate08 #tweetdebate***
- 97 mins debate + 53 mins following = ***2.5 hours*** total.
- ***3,238 tweets*** from ***1,160 people***.
  - 1,824 tweets from 647 people during the debate.
  - 1,414 tweets from 738 people post debate.
- 577 @ mentions *(reciprocity!)*
  - **266** mentions **during** the debate
  - **311 afterwards**.
- Low RT: **24 retweets** in total
  - 6 during
  - 18 afterwards.

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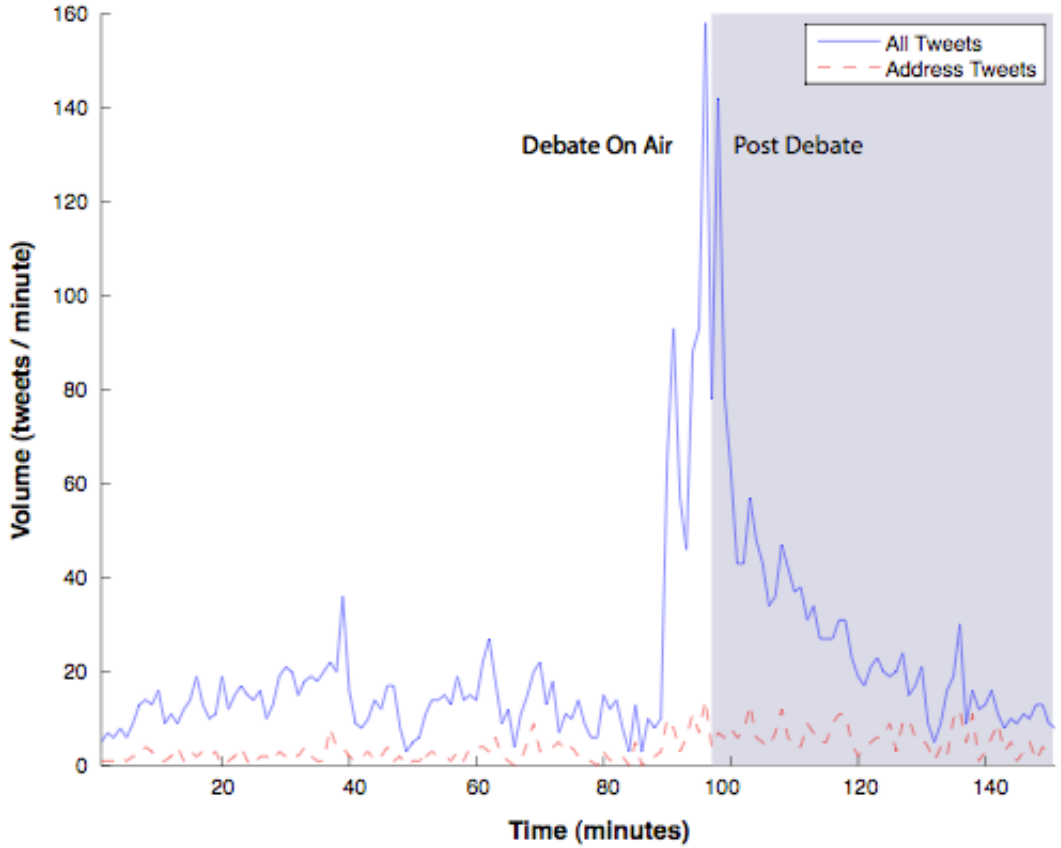
Volume of Tweets



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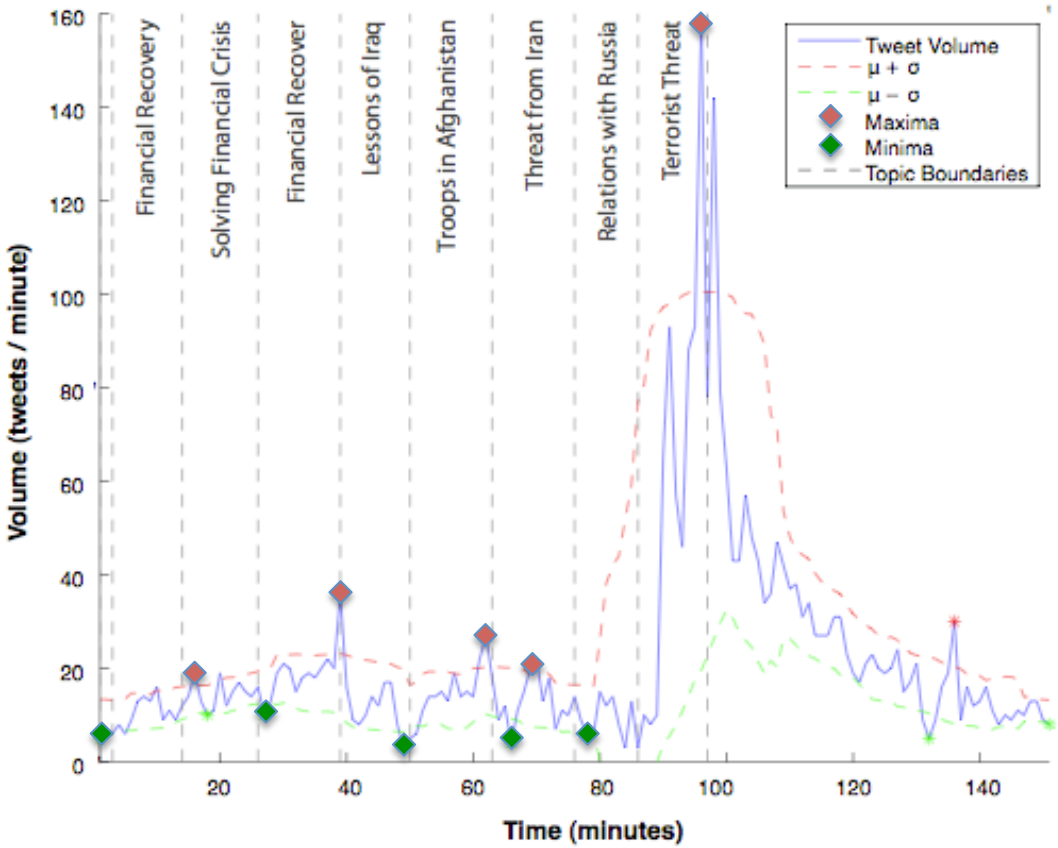
Discussion Segments



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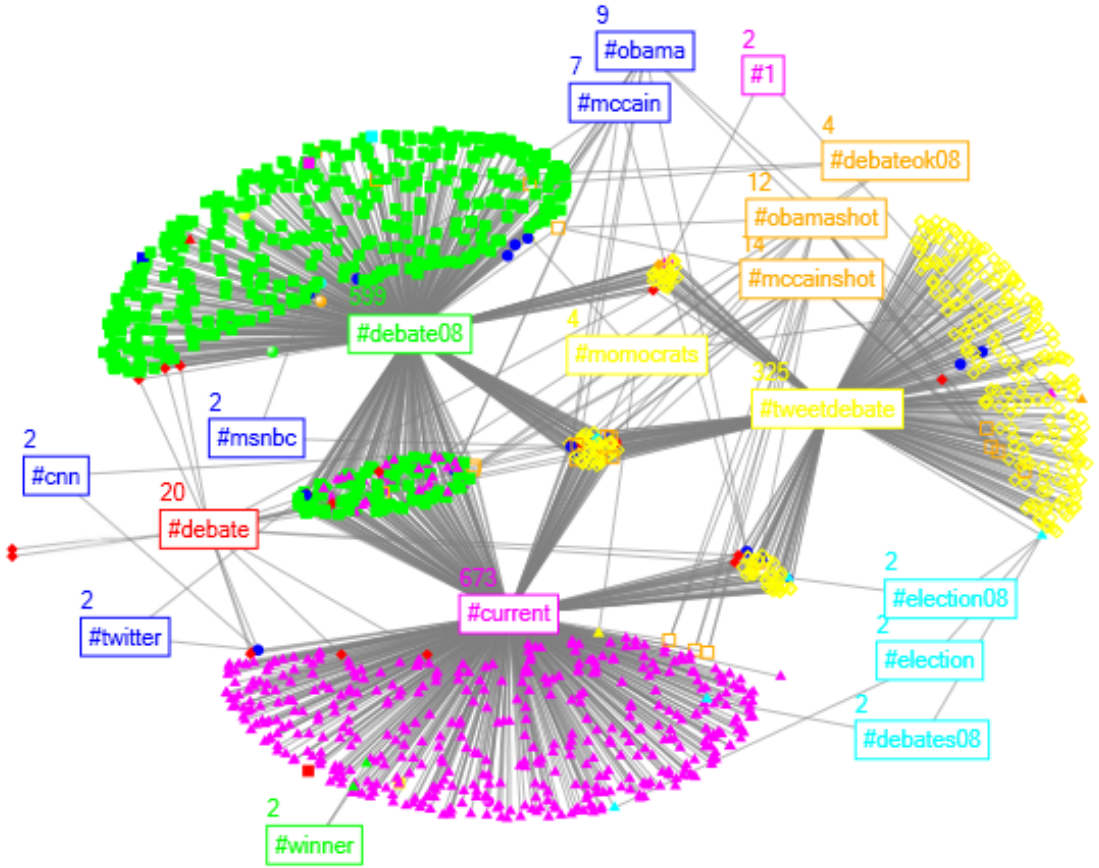
Cluster of tags



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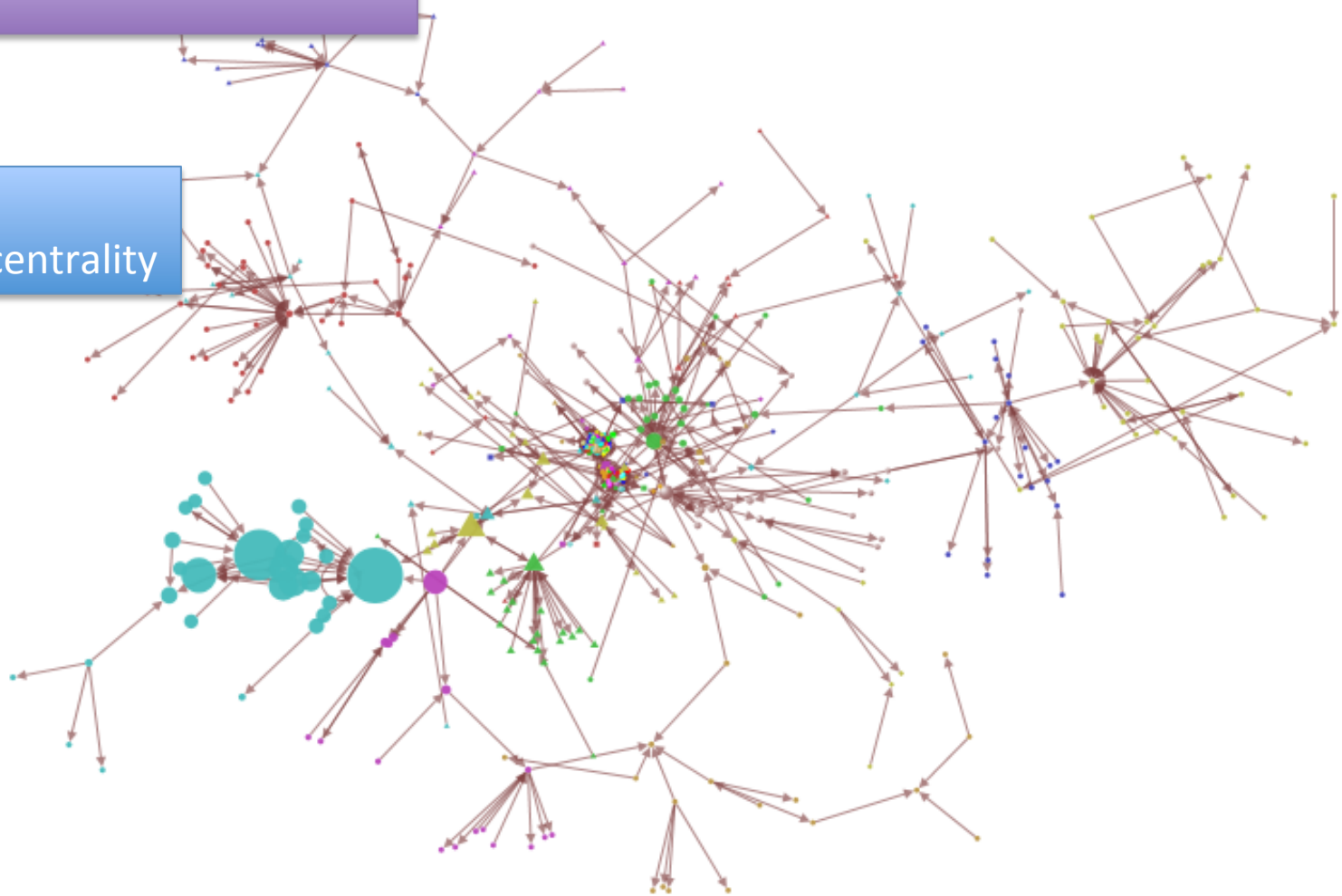
Twitter Mentions

John Tweets: “Hey @mary, my person is winning!”

Large node size=
High eigenvector centrality



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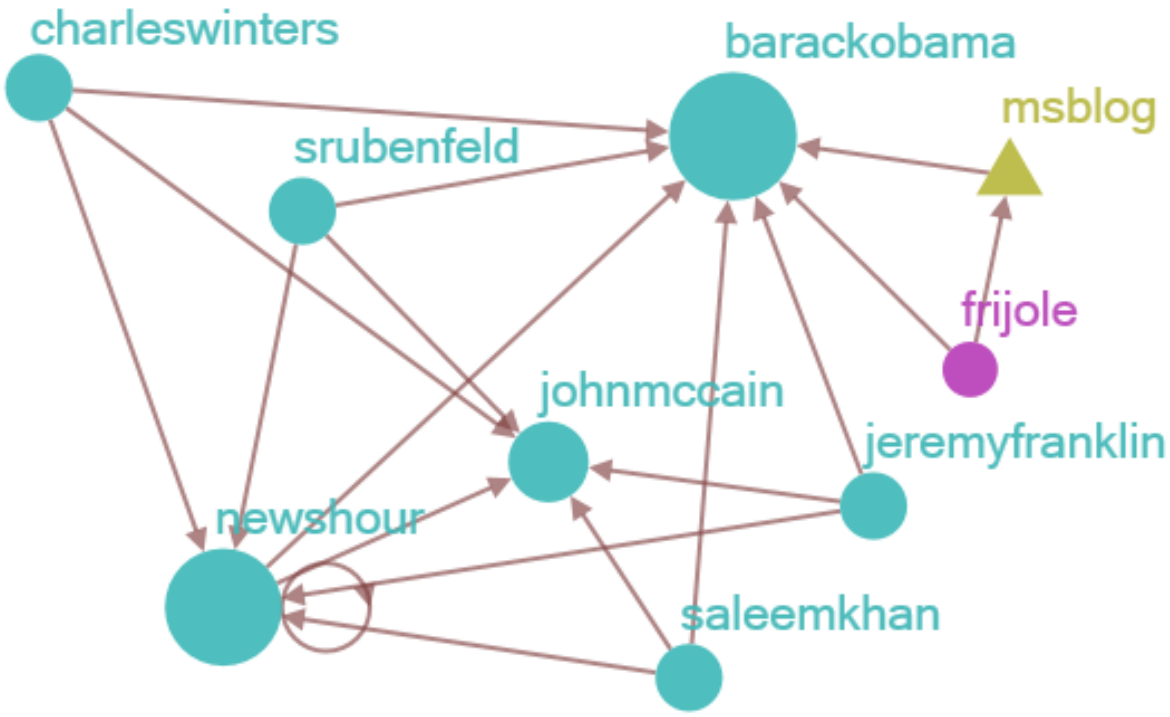
Automatic Discovery through Centrality

High Eigenvector Centrality Figures on Twitter from the First US Presidential Debate of 2008.



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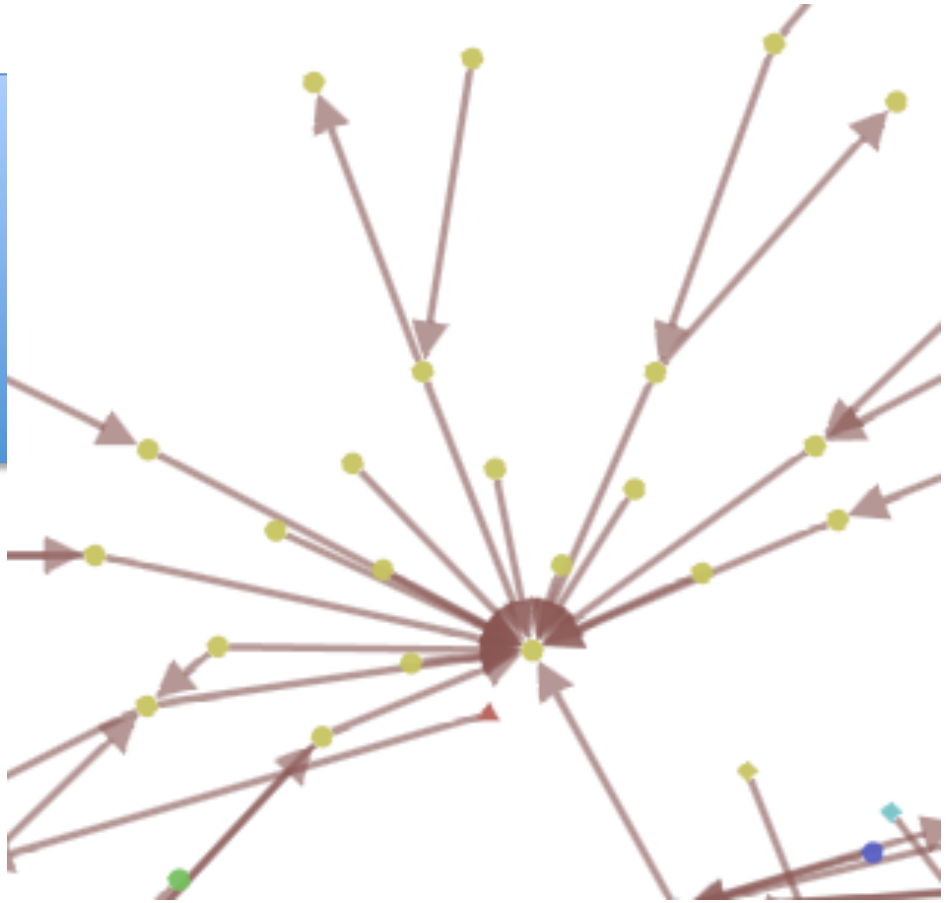
Centrality

Twitter User	Eigenvector Centrality	In Degree	Out Degree
@barackobama	0.472	15	0
@newshour	0.427	11	5
@johnmccain	0.277	6	0
@charleswinters	0.223	0	3
@jeremyfranklin	0.223	0	3
@saleemkhan	0.223	0	3
@srubenfeld	0.223	0	3
@msblog	0.221	5	6
@frijole	0.175	0	7

Sinks

High in-degree but poor centrality:

@current, who ran the Hack the Debate program, and
@jowyang, an employee of Forrester Research who uses Twitter as a personal, not corporately related microblog.



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Tweets to Terms

Twitter	drinking candidates wins ***minute*** lehrer letting tv mccains tweet moderator	kennedy plan hope hand comment moderator tie home audience republicans	cut ***earmarks*** compared ***tax*** joke bear dollars problems personal american	screen bailout energy economic power condoulo nuclear giving freeze looked	senate difference ***winning*** strategy iraq festooned fact war giving hand	pulling ***pakistan*** understand drinking republican strategy sounds coming telling story	lousy democracies lol government story wars league times ***iran*** -5	***georgia*** video blog league ***russia*** ha lot condescending issue oil	security experience 9/11 ***safe*** country tactics attack video pakistan bringing
Topic	**Opening**	**Financial Recovery**	**Solving Financial Crisis**	**Financial Recovery**	**Lessons of Iraq**	**Troops in Afghanistan**	**Threat from Iran**	**Relations with Russia**	**Terrorist Threat**
Debate	presidential debates ***minutes*** eisenhower financial direct policy news mississippi university	street greed main how's house package accountable wall rewarded crisis	$18 requests ***earmarks*** gateway loopholes $5,000 pork-barrel employer business ***tax***	programs medicare cost eliminate $700 hard decisions agency rescue tom	funding ***winning*** leave timetable violence lessons baghdad surge succeed started	border taliban prepared supported muddle u.s bombing ***pakistan*** qaeda army	henry kissinger contacts preparation legitimize table ***iranians*** sanctions ahmadinejad precondition	***georgia*** international putin ukraine ***russia*** world's offshore aggression resurgent nato	restore knowledge missile ***safer*** veterans terms focused earth billions challenges

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Twitter as Reaction

Twitter	drinking	kennedy	cut	screen	senate	pulling	lousy	***georgia***	security
	[illegible]	plan	***earmarks***	bailout	difference	***pakistan***	democracies	video	experience
	wins	hope	compared	energy	***winning***	understand	lol	blog	9/11
	minute	hand	***tax***	economic	strategy	drinking	government	league	***safe***
	lehrer	comment	joke	power	iraq	republican	story	***russia***	country
	letting	moderator	bear	condoulo	festooned	strategy	wars	ha	tactics
	tv	tie	dollars	nuclear	fact	sounds	league	lot	attack
	mccains	home	problems	giving	war	coming	times	condescending	video
	tweet	audience	personal	freeze	giving	telling	iran	issue	pakistan
	moderator	republicans	american	looked	hand	story	-5	oil	bringing
Topic	**Opening**	**Financial Recovery**	**Solving Financial Crisis**	**Financial Recovery**	**Lessons of Iraq**	**Troops in Afghanistan**	**Threat from Iran**	**Relations with Russia**	**Terrorist Threat**
Debate	presidential	street	$18	programs	funding	border	henry	***georgia***	restore
	debates	greed	requests	medicare	***winning***	taliban	kissinger	international	knowledge
	minutes	main	***earmarks***	cost	leave	prepared	contacts	putin	missile
	eisenhower	how's	gateway	eliminate	timetable	supported	preparation	ukraine	***safer***
	financial	house	loopholes	$700	violence	muddle	legitimize	***russia***	veterans
	direct	package	$5,000	hard	lessons	u.s	table	world's	terms
	policy	accountable	pork-barrel	decisions	baghdad	bombing	***iranians***	offshore	focused
	news	wall	employer	agency	surge	***pakistan***	sanctions	aggression	earth
	mississippi	rewarded	business	rescue	succeed	qaeda	ahmadinejad	resurgent	billions
	university	crisis	***tax***	tom	started	army	precondition	nato	challenges

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Summary

- We have introduced various measures of centrality and explained the pros and cons
- We have illustrated one example of use of centrality in a Twitter related example

References

- Kleinberg's book: Chapter 3.
- **Ack: L. Adamic's slides**

- M. Newmann. **Networks**. Oxford University Press. April 2010.

- David A. Shamma, Lyndon Kennedy, and Elizabeth F. Churchill. 2009. *Tweet the debates: understanding community annotation of uncollected sources.* In Proceedings of the first SIGMM workshop on Social media (WSM '09). ACM, New York, NY, USA

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