let-polymorphism

- The standard sugaring

$$\texttt{let val } x = v \texttt{ in } e \texttt{ end} \quad \mapsto \quad (\texttt{fn } x \texttt{ => } e)(v)$$

does not respect ML type checking.

For instance

$\forall\alpha.\ \alpha \to \alpha$ — Type schemes

$$\texttt{let val } \overbrace{\texttt{f = fn x => x}} \texttt{ in f(f) end}$$

$(\beta \to \beta) \to (\beta \to \beta)$ $\quad$ $\beta \to \beta$

type checks, whilst

$$(\texttt{fn f => f(f)})(\texttt{fn x => x})$$

does not.

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For instance

$$\texttt{let val f = fn x => x in f(f) end}$$

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does not.

- Type inference for let-expressions is involved, requiring *type schemes*.

Polymorphic exceptions

Example: Depth-first search for finitely-branching trees.

```
datatype
  'a FBtree = node of 'a * 'a FBtree list ;
fun dfs P (t: 'a FBtree)
  = let
      exception Ok of 'a;
      fun auxdfs( node(n,F) )
        = if P n then raise Ok n
          else foldl (fn(t,_) => auxdfs t) NONE F ;
    in
      auxdfs t handle Ok n => SOME n
    end ;
val dfs = fn : ('a -> bool) -> 'a FBtree -> 'a option
```

When a *polymorphic exception* is declared, SML ensures that it is used with only one type. The type of a top level exception must be monomorphic and the type variables of a local exception are frozen.

Consider the following nonsense:

```
exception Poly of 'a ;    (*** ILLEGAL!!! ***)
(raise Poly true) handle Poly x => x+1 ;
```

~ Topic VII ~

Data abstraction and modularity
SML Modules[a]

References:

- **Chapter 7** of *ML for the working programmer* (2ND EDITION) by L. C. Paulson. CUP, 1996.

[a] Largely based on an *Introduction to SML Modules* by Claudio Russo `<http://research.microsoft.com/~crusso>`.

- *The Standard ML Basis Library* edited by E. R. Gansner and J. H. Reppy. CUP, 2004.

  [A useful introduction to SML standard libraries, and a good example of modular programming.]

- `<http://www.standardml.org/>`

The Core and Modules languages

SML consists of two sub-languages:

- The *Core* language is for *programming in the small*, by supporting the definition of types and expressions denoting values of those types.
- The *Modules* language is for *programming in the large*, by grouping related Core definitions of types and expressions into self-contained units, with descriptive interfaces.

The *Core* expresses details of *data structures* and *algorithms*. The *Modules* language expresses *software architecture*. Both languages are largely independent.

The Modules language

Writing a real program as an unstructured sequence of Core definitions quickly becomes unmanageable.

```
type nat = int
val zero = 0
fun succ x = x + 1
fun iter b f i =
    if i = zero then b
                else  f (iter b f (i-1))
...
(* thousands of lines later *)
fun even (n:nat) = iter true not n
```

$(\mathrm{iter}\ b\ f)\ i = f^{i}(b)$

The SML Modules language lets one split large programs into separate units with descriptive interfaces.

SML Modules

Signatures and structures

An *abstract data type* is a type equipped with a set of operations, which are the only operations applicable to that type.

Its representation can be changed without affecting the rest of the program.

- *Structures* let us *package* up declarations of related types, values, and functions.
- *Signatures* let us *specify* what components a structure must contain.

Structures

In Modules, one can encapsulate a sequence of Core type and value definitions into a unit called a *structure*.

We enclose the definitions in between the keywords

`struct ... end`.

Example: A structure representing the natural numbers, as positive integers.

```
struct
  type nat = int
  val zero = 0
  fun succ x = x + 1
  fun iter b f i = if i = zero then b
                   else f (iter b f (i-1))
end
```

The dot notation

One can name a structure by binding it to an identifier.

```
structure IntNat =
 struct
     type nat = int
     ...
     fun iter b f i = ...
 end
```

Components of a structure are accessed with the *dot notation*.

= int

```
  fun even (n:IntNat.nat) = IntNat.iter true not n
```

NB: Type `IntNat.nat` is statically equal to `int`. Value `IntNat.iter` dynamically evaluates to a closure.

Nested structures

Structures can be nested inside other structures, in a hierarchy.

```
structure IntNatAdd =
  struct
    structure Nat = IntNat
    fun add n m = Nat.iter m Nat.succ n
  end
  ...
fun mult n m =
IntNatAdd.Nat.iter IntNatAdd.Nat.zero (IntNatAdd.add m) n
```

The dot notation (`IntNatAdd.Nat`) accesses a nested structure.

Sequencing dots provides deeper access (`IntNatAdd.Nat.zero`).

Nesting and dot notation provides *name-space* control.

Concrete signatures

Signature expressions specify the types of structures by listing the specifications of their components.

A signature expression consists of a *sequence* of component specifications, enclosed in between the keywords `sig ... end`.

```
sig  type nat = int
     val zero : nat
     val succ : nat -> nat
     val 'a iter : 'a -> ('a->'a) -> nat -> 'a
end
```

This signature fully describes the *type* of `IntNat`.

The specification of type `nat` is *concrete*: it must be `int`.

Opaque signatures

On the other hand, the following signature

```
sig type nat
    val zero : nat
    val succ : nat -> nat
    val 'a iter : 'a -> ('a->'a) -> nat -> 'a
end
```

specifies structures that are free to use *any* implementation for type `nat` (perhaps `int`, or `word`, or some recursive datatype).

This specification of type `nat` is *opaque*.

Example: Polymorphic functional stacks.

```
signature STACK =
sig
  exception E
  type 'a reptype   (* <-- INTERNAL REPRESENTATION *)
  val new: 'a reptype
  val push: 'a -> 'a reptype -> 'a reptype
  val pop: 'a reptype -> 'a reptype
  val top: 'a reptype -> 'a
end ;
```

```
structure MyStack: STACK =
struct
  exception E ;
  type 'a reptype = 'a list ;
  val new = [] ;
  fun push x s = x::s ;
  fun split( h::t ) = ( h , t )
    | split _ = raise E ;
  fun pop s = #2( split s ) ;
  fun top s = #1( split s ) ;
end ;
```

```
val MyEmptyStack = MyStack.new ;
val MyStack0 = MyStack.push 0 MyEmptyStack ;
val MyStack01 = MyStack.push 1 MyStack0 ;
val MyStack0' = MyStack.pop MyStack01 ;
MyStack.top MyStack0' ;
```

```
val MyEmptyStack = [] : 'a MyStack.reptype
val MyStack0 = [0] : int MyStack.reptype
val MyStack01 = [1,0] : int MyStack.reptype
val MyStack0' = [0] : int MyStack.reptype
val it = 0 : int
```

The implementation is exposed

Named and nested signatures

Signatures may be *named* and referenced, to avoid repetition:

```
signature NAT =
  sig type nat
      val zero : nat
      val succ : nat -> nat
      val 'a iter : 'a -> ('a->'a) -> nat -> 'a
end
```

Nested signatures specify named sub-structures:

```
signature Add =
  sig structure Nat: NAT (* references NAT *)
      val add: Nat.nat -> Nat.nat -> Nat.nat
  end
```

Signature inclusion

To avoid nesting, one can also directly `include` a signature identifier:

```
sig  include NAT
     val add: nat -> nat ->nat
end
```

NB: This is equivalent to the following signature.

```
sig type nat
    val zero: nat
    val succ: nat -> nat
    val 'a iter: 'a -> ('a->'a) -> nat -> 'a
    val add: nat -> nat -> nat
end
```

Signature matching

Q: When does a structure satisfy a signature?

A: The type of a structure *matches* a signature whenever it implements at least the components of the signature.

- The structure must *realise* (i.e. define) all of the opaque type components in the signature.
- The structure must *enrich* this realised signature, component-wise:
  - ⋆ every concrete type must be implemented equivalently;
  - ⋆ every specified value must have a more general type scheme;
  - ⋆ every specified structure must be enriched by a substructure.

Properties of signature matching

- The components of a structure can be defined in a different order than in the signature; names matter but ordering does not.
- A structure may contain more components, or components of more general types, than are specified in a matching signature.
- Signature matching is *structural*. A structure can match many signatures and there is no need to pre-declare its matching signatures (unlike “interfaces” in Java and C#).
- Although similar to record types, signatures actually play a number of different roles.

Subtyping

Signature matching supports a form of *subtyping* not found in the Core language:

- A structure with more type, value, and structure components may be used where fewer components are expected.
- A value component may have a more general type scheme than expected.

Using signatures to restrict access

The following structure uses a *signature constraint* to provide a restricted view of `IntNat`:

```
structure ResIntNat =
  IntNat : sig type nat
                val succ : nat->nat
                val iter : nat->(nat->nat)->nat->nat
           end
```

NB: Not of type

$\alpha \to (\alpha \to \alpha) \to nat \to \alpha$

NB: The constraint `str:sig` prunes the structure `str` according to the signature `sig`:

- `ResIntNat.zero` is *undefined*;
- `ResIntNat.iter` is *less* polymorphic that `IntNat.iter`.

Transparency of `_:_`

Although the `_:_` operator can hide names, it does not conceal the definitions of opaque types.

Thus, the fact that `ResIntNat.nat` = `IntNat.nat` = `int` remains *transparent*.

For instance the application `ResIntNat.succ(~3)` is still well-typed, because `~3` has type `int` ...but `~3` is negative, so not a valid representation of a natural number!

SML Modules
Information hiding

In SML, we can limit outside access to the components of a structure by *constraining* its signature in *transparent* or *opaque* manners.

Further, we can *hide* the representation of a type by means of an `abstype` declaration.

The combination of these methods yields abstract structures.

(The first language mechanism
- it is obsolete

Using signatures to hide the identity of types

With different syntax, signature matching can also be used to enforce *data abstraction*:

```
structure AbsNat =
  IntNat :> sig type nat
                val zero: nat
                val succ: nat->nat
                val 'a iter: 'a->('a->'a)->nat->'a
            end
```

Has the effect of creating new abstract types for the opaque types of the signature

The constraint `str :> sig` prunes `str` but also generates a new, *abstract* type for each opaque type in `sig`.

- The actual implementation of `AbsNat.nat` by `int` is *hidden*, so that `AbsNat.nat` $\neq$ `int`.

  `AbsNat` is just `IntNat`, but with a hidden type representation.

- `AbsNat` defines an *abstract datatype* of natural numbers: the only way to construct and use values of the abstract type `AbsNat.nat` is through the operations, `zero`, `succ`, and `iter`.

  E.g., the application `AbsNat.succ(~3)` is ill-typed: `~3` has type `int`, not `AbsNat.nat`. This is what we want, since `~3` is not a natural number in our representation.

  In general, abstractions can also prune and specialise components.

1. Opaque signature constraints

```
structure MyOpaqueStack :> STACK = MyStack ;

val MyEmptyOpaqueStack = MyOpaqueStack.new ;
val MyOpaqueStack0 = MyOpaqueStack.push 0 MyEmptyOpaqueStack ;
val MyOpaqueStack01 = MyOpaqueStack.push 1 MyOpaqueStack0 ;
val MyOpaqueStack0' = MyOpaqueStack.pop MyOpaqueStack01 ;
MyOpaqueStack.top MyOpaqueStack0' ;

val MyEmptyOpaqueStack = - : 'a MyOpaqueStack.reptype
val MyOpaqueStack0 = - : int MyOpaqueStack.reptype
val MyOpaqueStack01 = - : int MyOpaqueStack.reptype
val MyOpaqueStack0' = - : int MyOpaqueStack.reptype
val it = 0 : int
```

2. abstypes

```
structure MyHiddenStack: STACK =
struct
  exception E ;
  abstype 'a reptype = S of 'a list (* <-- HIDDEN *)
  with                              (* REPRESENTATION *)
    val new = S [] ;
    fun push x (S s) = S( x::s ) ;
    fun pop( S [] ) = raise E
      | pop( S(_::t) ) = S( t ) ;
    fun top( S [] ) = raise E
      | top( S(h::_) ) = h ;
  end ;
end ;
```

```
val MyHiddenEmptyStack = MyHiddenStack.new ;
val MyHiddenStack0 = MyHiddenStack.push 0 MyHiddenEmptyStack ;
val MyHiddenStack01 = MyHiddenStack.push 1 MyHiddenStack0  ;
val MyHiddenStack0' = MyHiddenStack.pop MyHiddenStack01  ;
MyHiddenStack.top MyHiddenStack0' ;
```

```
val MyHiddenEmptyStack = - : 'a MyHiddenStack.reptype
val MyHiddenStack0 = - : int MyHiddenStack.reptype
val MyHiddenStack01 = - : int MyHiddenStack.reptype
val MyHiddenStack0' = - : int MyHiddenStack.reptype
val it = 0 : int
```

SML Modules

Functors

- An SML *functor* is a structure that takes other structures as parameters.
- Functors let us write program units that can be combined in different ways. Functors can also express generic algorithms.

Functors

Modules also supports *parameterised structures*, called *functors*.

Example: The functor `AddFun` below takes any implementation, `N`, of naturals and re-exports it with an addition operation.

```
functor AddFun(N:NAT) =
        struct
          structure Nat = N
          fun add n m = Nat.iter n (Nat.succ) m
        end
```

- A functor is a *function* mapping a formal argument structure to a concrete result structure.

- The body of a functor may assume no more information about its formal argument than is specified in its signature.

  In particular, opaque types are treated as distinct type parameters.

  Each actual argument can supply its own, independent implementation of opaque types.

Functor application

A functor may be used to create a structure by *applying* it to an actual argument:

```
structure IntNatAdd = AddFun(IntNat)
structure AbsNatAdd = AddFun(AbsNat)
```

The actual argument must match the signature of the formal parameter—so it can provide more components, of more general types.

Above, `AddFun` is applied twice, but to arguments that differ in their implementation of type `nat` (`AbsNat.nat` $\neq$ `IntNat.nat`).

Example: Generic imperative stacks.

```
signature STACK =
 sig
  type itemtype
  val push: itemtype -> unit
  val pop: unit -> unit
  val top: unit -> itemtype
end ;
```

```
exception E ;
functor Stack( T: sig type atype end ) : STACK =
struct
  type itemtype = T.atype
  val stack = ref( []: itemtype list )
  fun push x
    = ( stack := x :: !stack )
  fun pop()
    = case !stack of [] => raise E
      | _::s => ( stack := s )
  fun top()
    = case !stack of [] => raise E
      | t::_ => t
end ;
```

```
structure intStack
  = Stack(struct type atype = int end) ;
```

```
structure intStack : STACK
```

```
intStack.push(0) ;
intStack.top() ;
intStack.pop() ;
intStack.push(4) ;
```

```
val it = () : unit
val it = 0 : intStack.itemtype
val it = () : unit
val it = () : unit
```

Why functors?

Functors support:

Code reuse.

`AddFun` may be applied many times to different structures, reusing its body.

Code abstraction.

`AddFun` can be compiled before any argument is implemented.

Type abstraction.

`AddFun` can be applied to different types `N.nat`.