Artificial Intelligence II

Additional exercises, part II: making decisions, HMMs and Bayes learning

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1 Making decisions

1. Exam question: 2007, paper 8, question 9.
2. Exam question: 2011, paper 8, question 8.

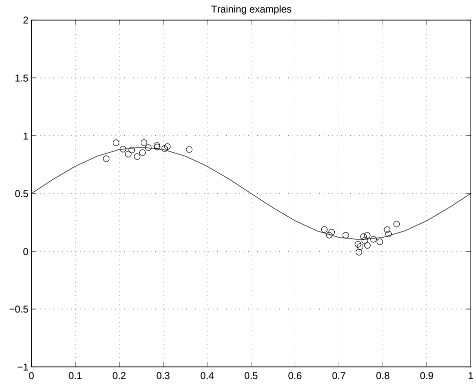
2 HMMs

1. Derive the equation
$$b_{T+1:t} = \mathbf{SE}_{T+1} b_{T+2:t}$$
for the backward message in a hidden Markov model (lecture slide 207).
2. Establish how the prior $\Pr(S_0)$ should be included in the derivation of the Viterbi algorithm. (This is mentioned on slide 191, but no detail is given.)
3. Exam question: 2005, paper 9, question 8.
4. Exam question: 2008, paper 9, question 5.
5. Exam question: 2010, paper 7, question 4.

3 Bayesian learning

1. Derive the *weight decay* training algorithm
$$\mathbf{w}_{\text{MAP}} = \underset{\mathbf{w}}{\operatorname{argmin}} \frac{\alpha}{2}||\mathbf{w}||^2 + \frac{\beta}{2}\sum_{i=1}^{m}(y_i - f(\mathbf{w};\mathbf{x}_i))^2$$
given on slide 267.
2. Use the standard Gaussian integral to derive the final equation for Bayesian regression
$$p(Y|\mathbf{y},\mathbf{x}) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(y - f(\mathbf{w}_{\text{MAP}};\mathbf{x}))^2}{2\sigma_y^2}\right)$$
where
$$\sigma_y^2 = \frac{1}{\beta} + \mathbf{g}^T\mathbf{A}^{-1}\mathbf{g}$$
given on slide 283.
3. This question asks you to produce a version of the graph on slide 285, but using the Metropolis algorithm instead of the solution obtained by approximating the integral. Any programming language is fine, although Matlab is probably the most straightforward.

   The data is simple artificial data for a one-input regression problem. Use the target function
$$f(x) = \frac{1}{2} + 0.4\sin 2\pi x$$
and generate 30 examples clustered around $x = 0.25$ and $x = 0.75$. Then label these examples
$$y(x) = f(x) + n$$
where n is Gaussian noise of standard deviation 0.05. Plot the data as follows:



Let $\mathbf{w} \in \mathbb{R}^W$ be the vector of all the weights in a network. Your supervised learner should be based on a prior density

$$p(\mathbf{w}) = \left(\frac{2\pi}{\alpha}\right)^{-W/2} \exp\left(-\frac{\alpha}{2}||\mathbf{w}||^2\right)$$

on the weights. A value of $\alpha = 1$ is reasonable. The likelihood used should be

$$p(\mathbf{y}|\mathbf{w}) = \left(\frac{2\pi}{\beta}\right)^{-m/2} \exp\left(-\frac{\beta}{2}\sum_{i=1}^{m}(y(x_i) - h(\mathbf{w}; x_i))^2\right)$$

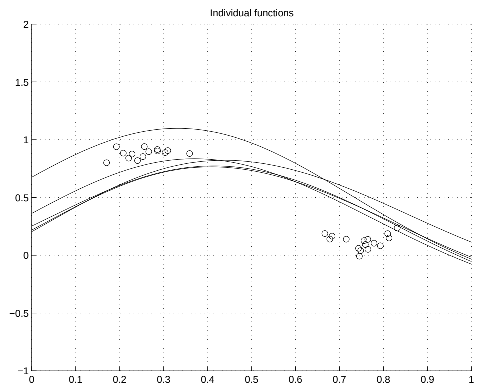
where m is the number of examples and $h(\mathbf{w}; x)$ is the function computed by the neural network with weights $\mathbf{w}$. A value of $\beta = 1/(0.05)^2$ is appropriate. Note that we are assuming that hyperparameters α and β are known, and the prior and likelihood used are the same as those used in the lectures.

Complete the following steps:

(a) Write a function `simpleNetwork` function implementing a multilayer perceptron with a single hidden layer, a basic feedforward structure as illustrated in the AI I lectures, and a single output node. The network should use sigmoid activation functions for the hidden units and a linear activation function for its output. You should use a network having 4 hidden units.

(b) Starting with a weight vector chosen at random, use the Metropolis algorithm to sample the posterior distribution $p(\mathbf{w}|\mathbf{y})$. You should generate a sequence of 100 weight vectors $\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_{100}$.

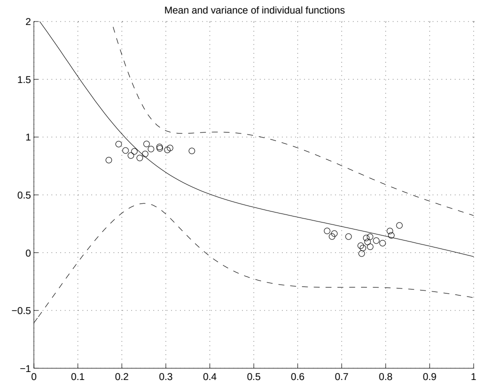
(c) Plot the function $h(\mathbf{w}_i; x)$ computed by the neural network for a few of the weight vectors obtained.



(d) Discard the first 50 weight vectors generated. Using the remainder, calculate the mean and variance of the corresponding functions using

$$\text{mean}(x) = \frac{1}{50}\sum_{i=51}^{100} h(\mathbf{w}_i; x)$$

and a similar expression for the variance. Plot the mean function along with error bars provided by the variance.



4. Explain how the Gibbs algorithm might be applied to the Bayesian network developed earlier for the *roof-climber alarm* problem.

5. Exam question: 2010, paper 8, question 2.