Prolog Lecture 3

- Symbolic evaluation of arithmetic
- Controlling backtracking: cut
- Negation

Symbolic Evaluation

Let's write some Prolog rules to evaluate symbolic arithmetic expressions such as plus(1,mult(4,5))

```
eval(plus(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 + B1.

eval(mult(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 * B1.

eval(A,A).
```

Evaluation starts with the first matching clause

Q: How does Prolog evaluate:

```
eval(plus(1,mult(4,5)),Ans)
```

A: Step 1, see if the first matching clause is true

```
eval(plus(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 + B1.
```

In this case the variable bindings are:

- A = 1, B = mult(4,5) and C = Ans

Next it looks at the body of the rule

The body of the clause with head `eval(plus(A,B),C)` and variable bindings

A = 1, B = mult(4,5) and C = Ans is:

```
eval(1,A1),
eval(mult(4,5),B1),
Ans is A1 + B1.
```

This is a conjunction: all parts must be true for the clause to be true

The body is checked term by term from left to right

First part of the body: `eval(1,A1)`

Try: eval(plus(A,B),C) :- eval(A,A1), eval(B,B1), C is A1 + B1.

Fail because 1 does not unify with plus(A,B)

Try: eval(mult(A,B),C) :- eval(A,A1), eval(B,B1), C is A1 * B1.

Fail because 1 does not unify with mult(A,B)

Try: eval(A,A).

Succeed: eval(1,A1) is true if A1 = 1

The body is checked term by term from left to right

From previous slide, `eval(1,A1)` was provable, with the side-effect of binding: `A1=1`.

So continuing through the body (note A1 is now bound):

```
eval(1,1),
eval(mult(4,5),B1),
Ans is 1 + B1.
```

The body is checked term by term from left to right

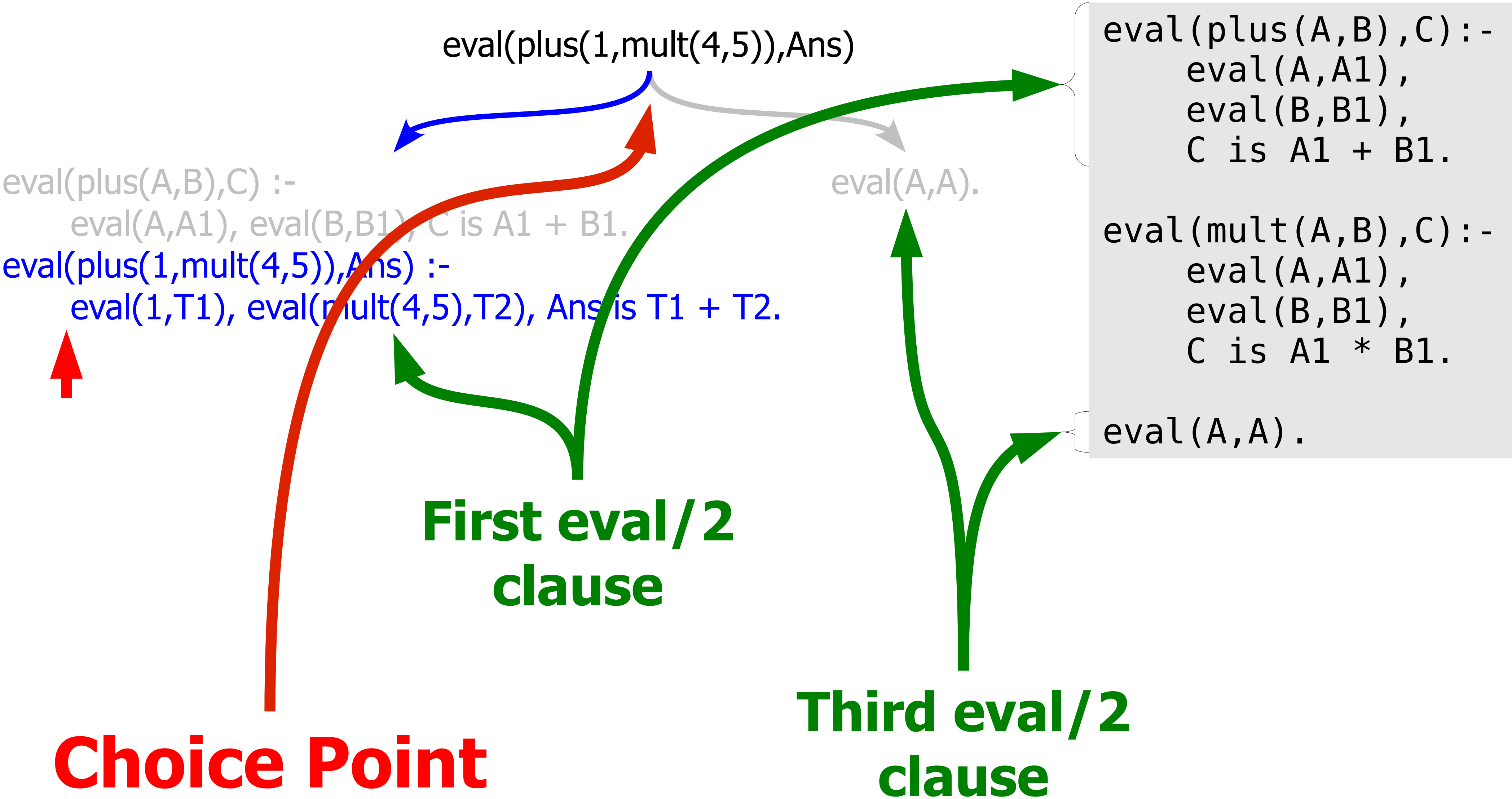
So `eval(mult(4,5),B1)` will bind B1=20:

```
eval(1,1),
eval(mult(4,5),20),
Ans is 1 + 20.
```

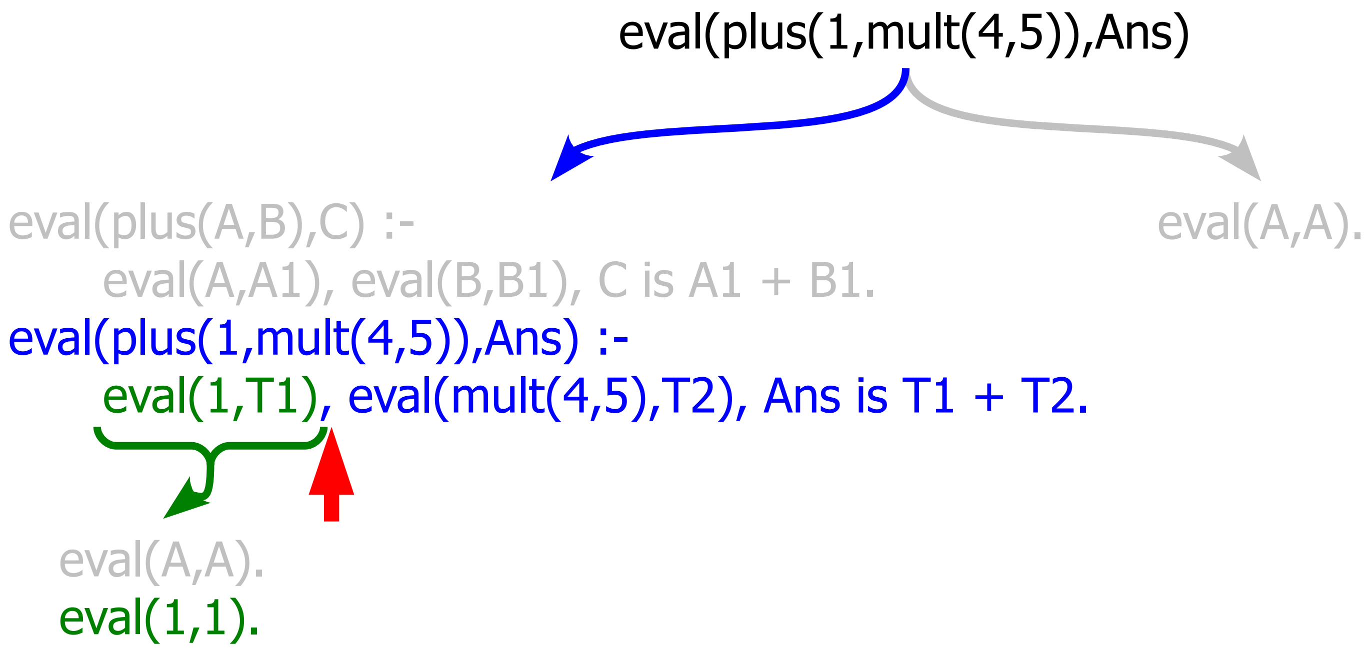
The body is checked term by term from left to right

Ans will be bound to 21, after “is” does its job.

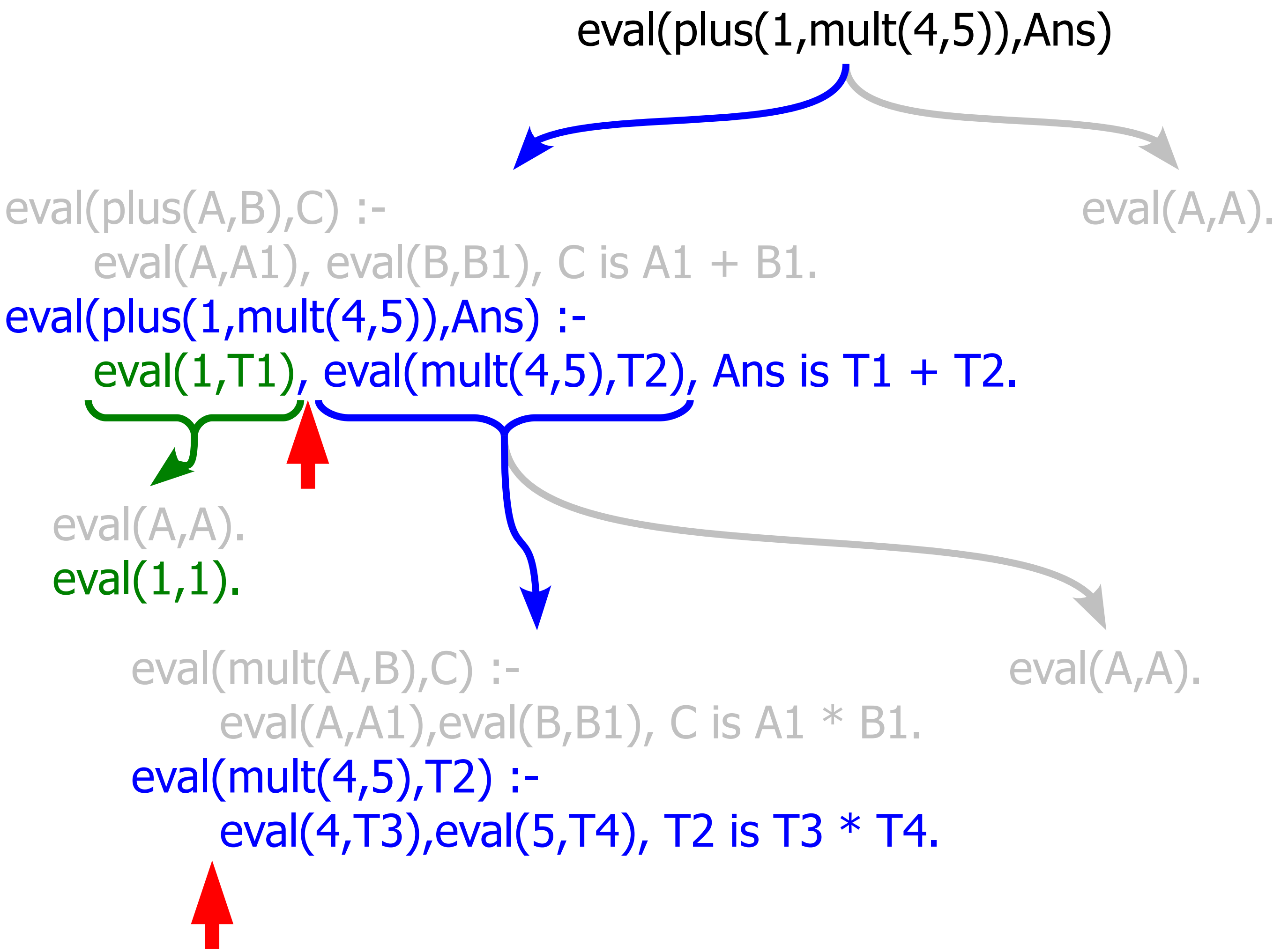
```
eval(1,1),
eval(mult(4,5),20),
21 is 1 + 20.
```



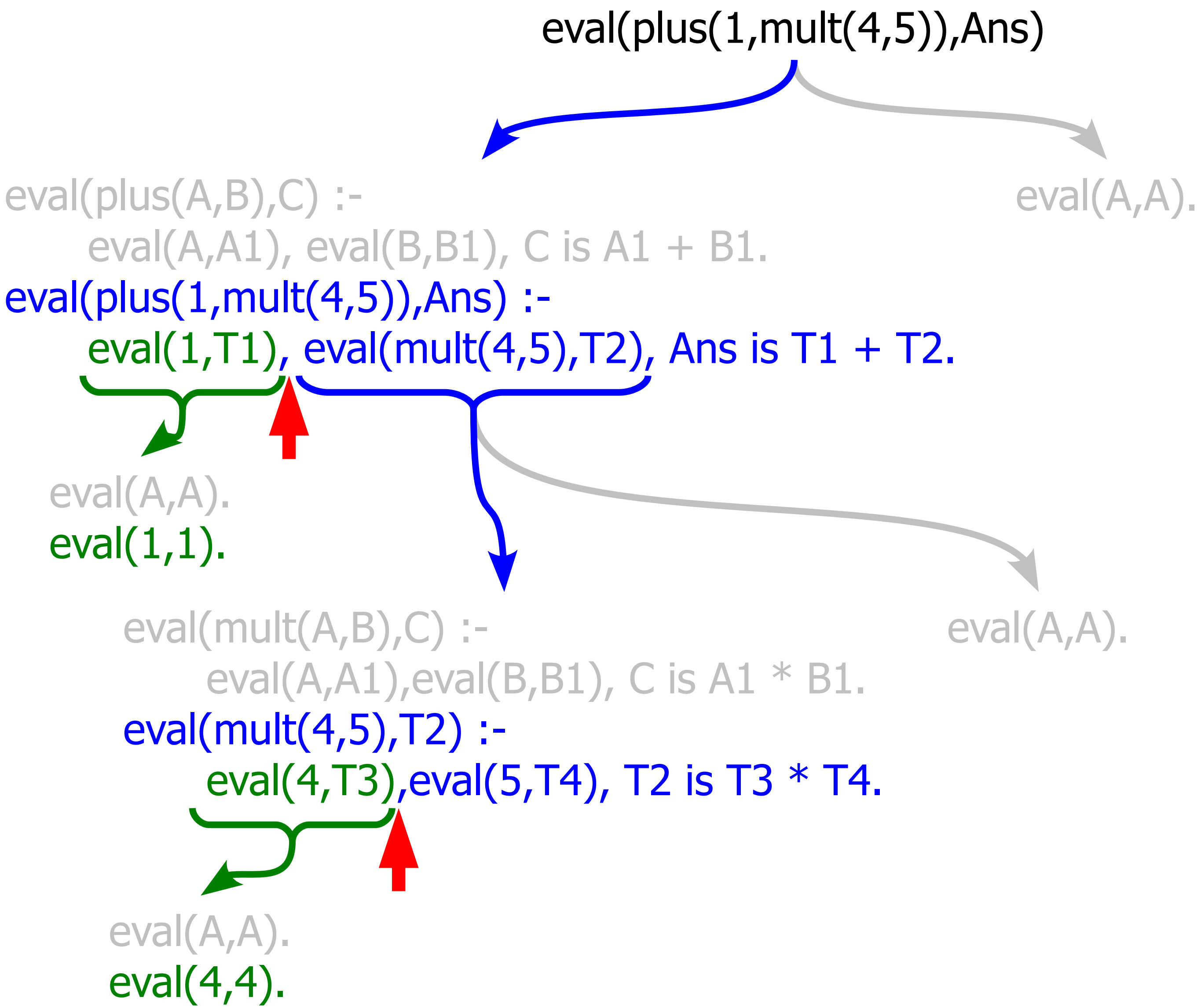
Be sure that you understand why the second eval/2 clause does not appear in this choice point



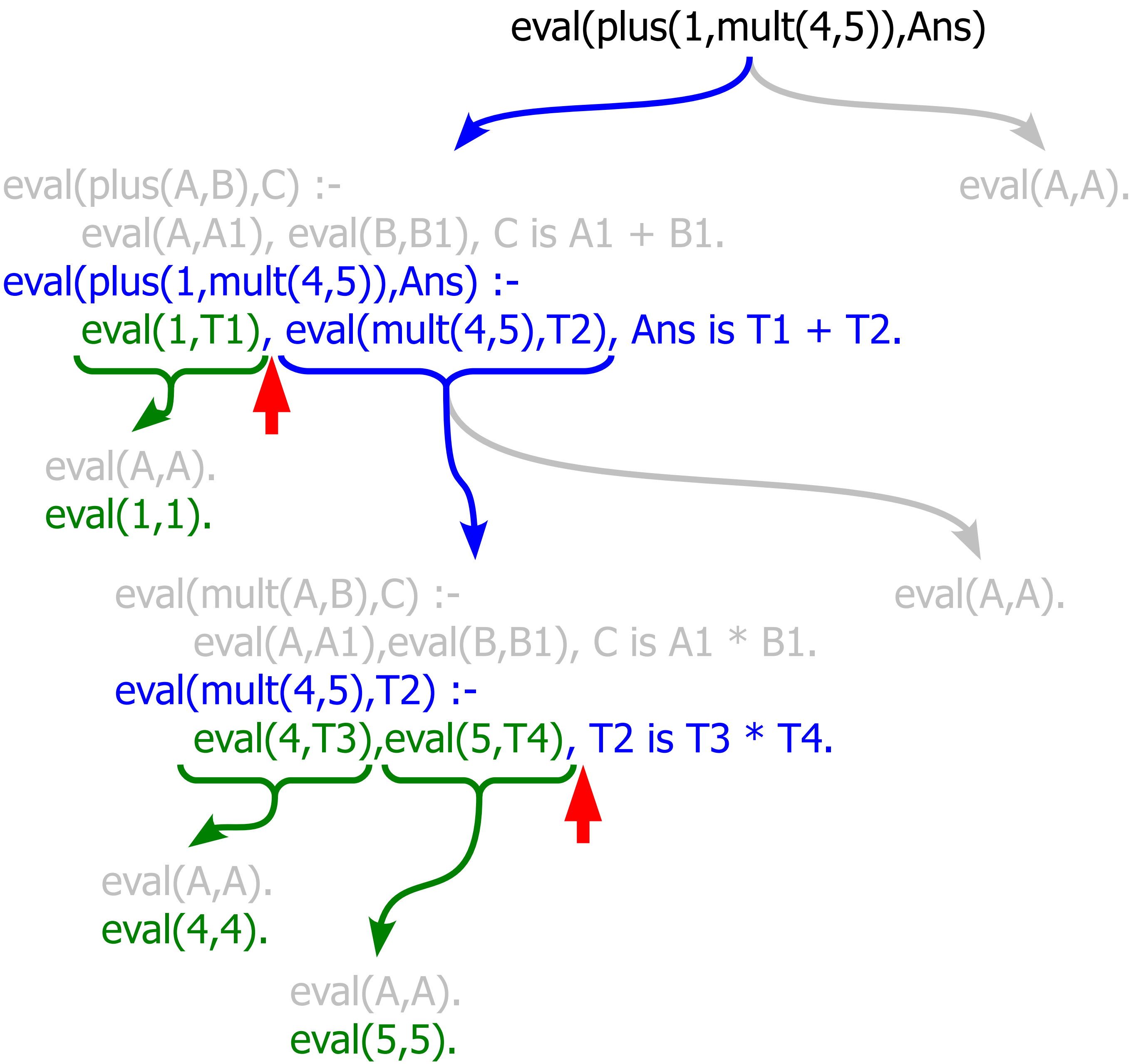

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```



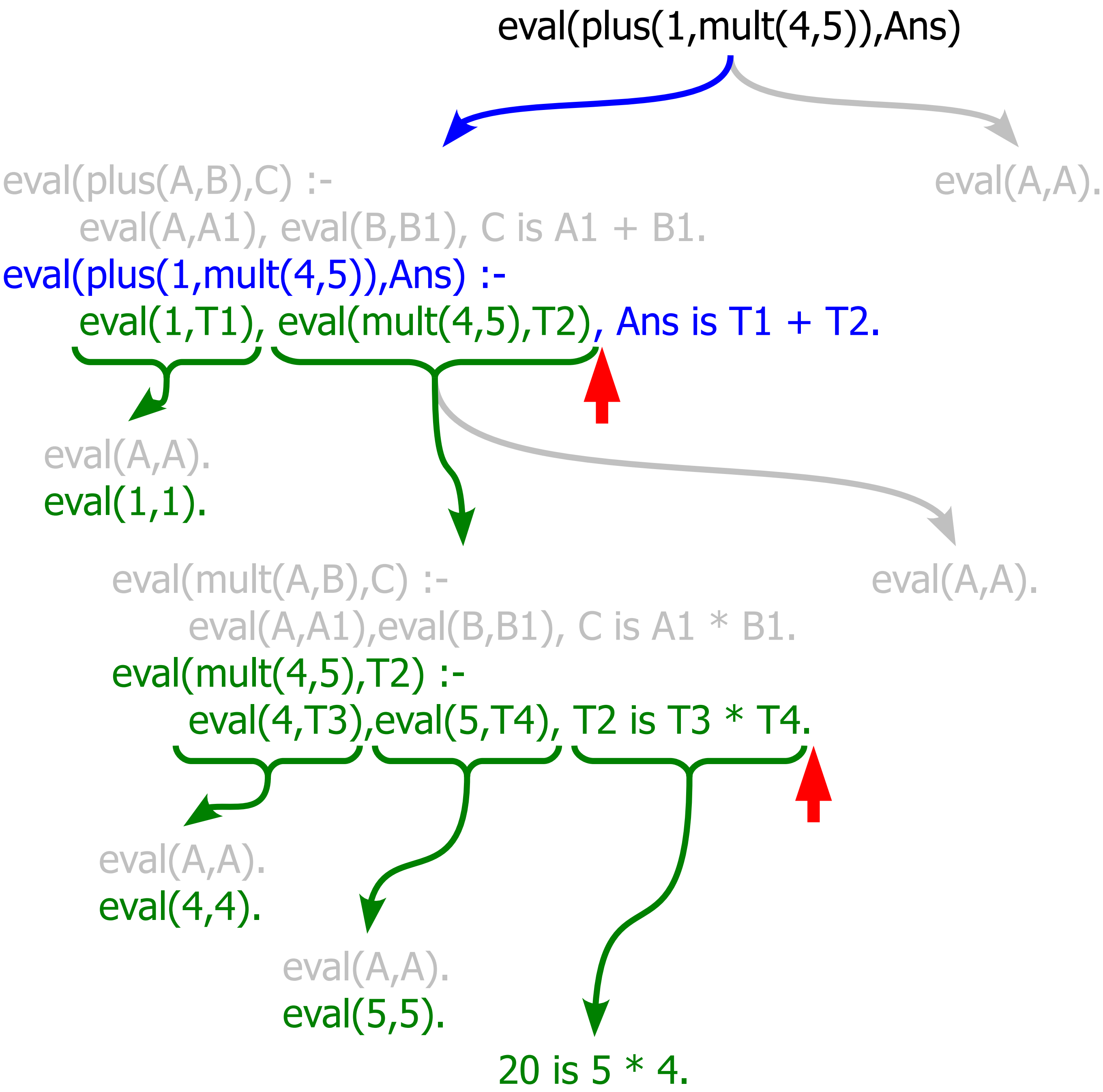

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```




```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

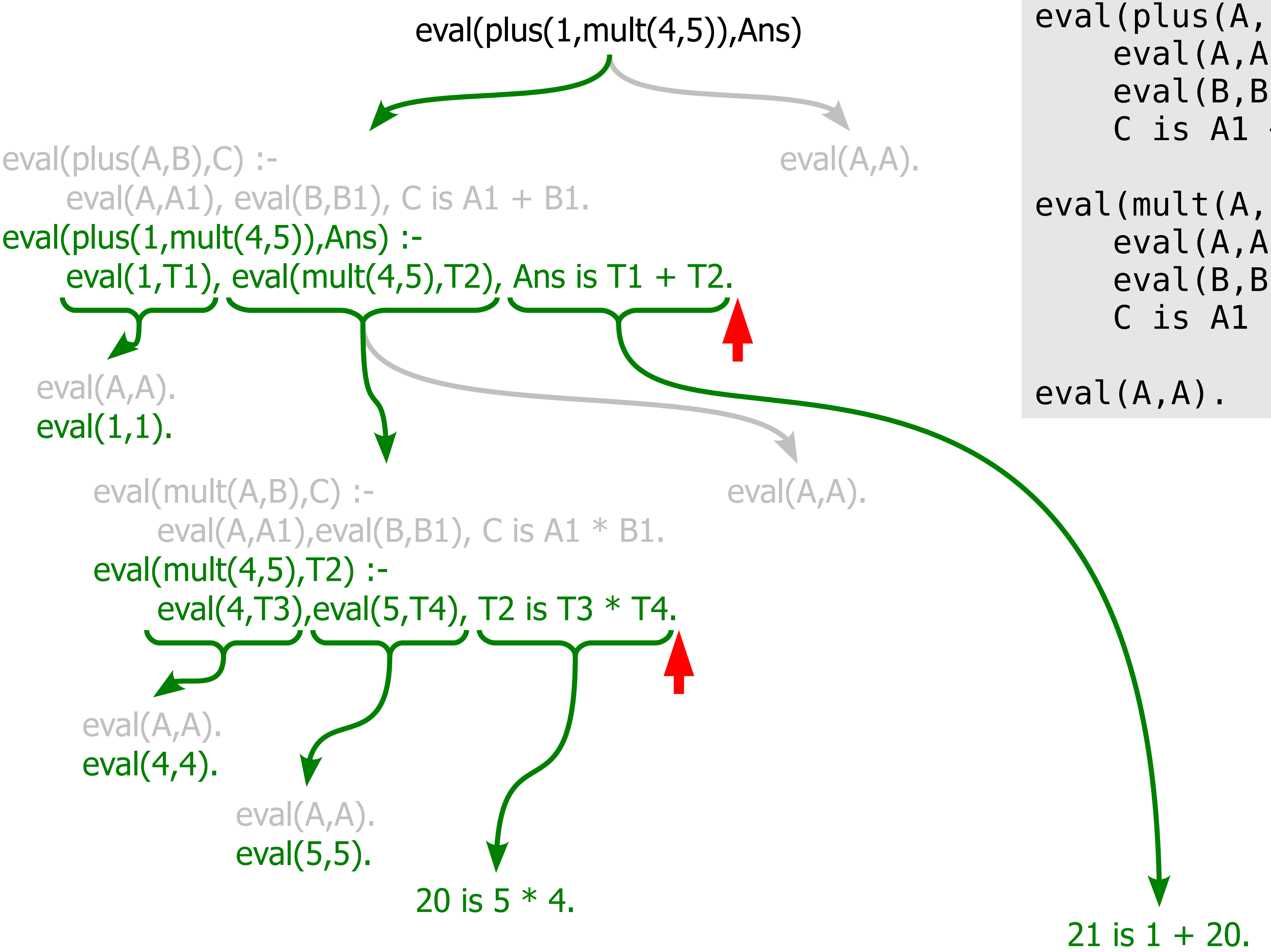



```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```



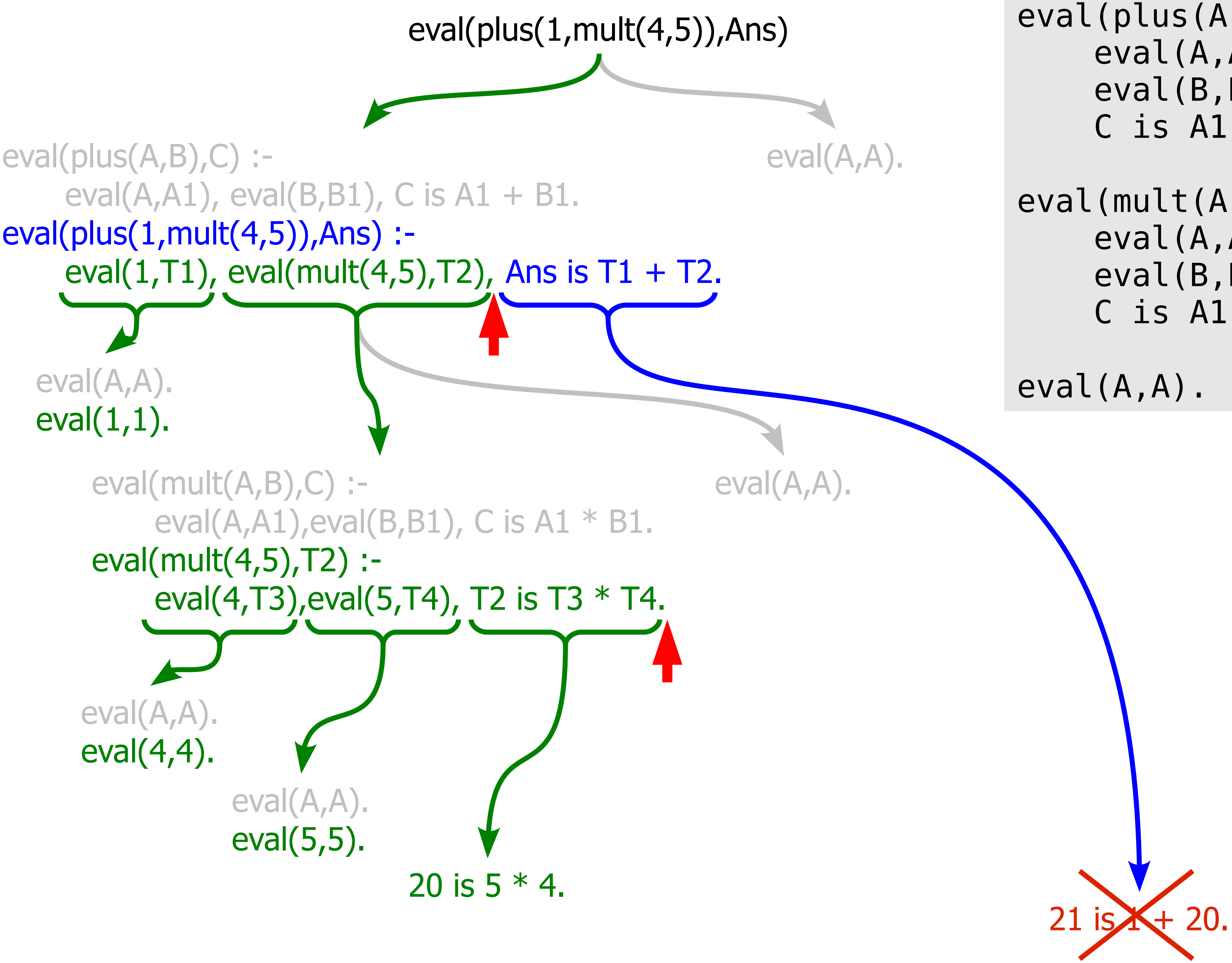

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

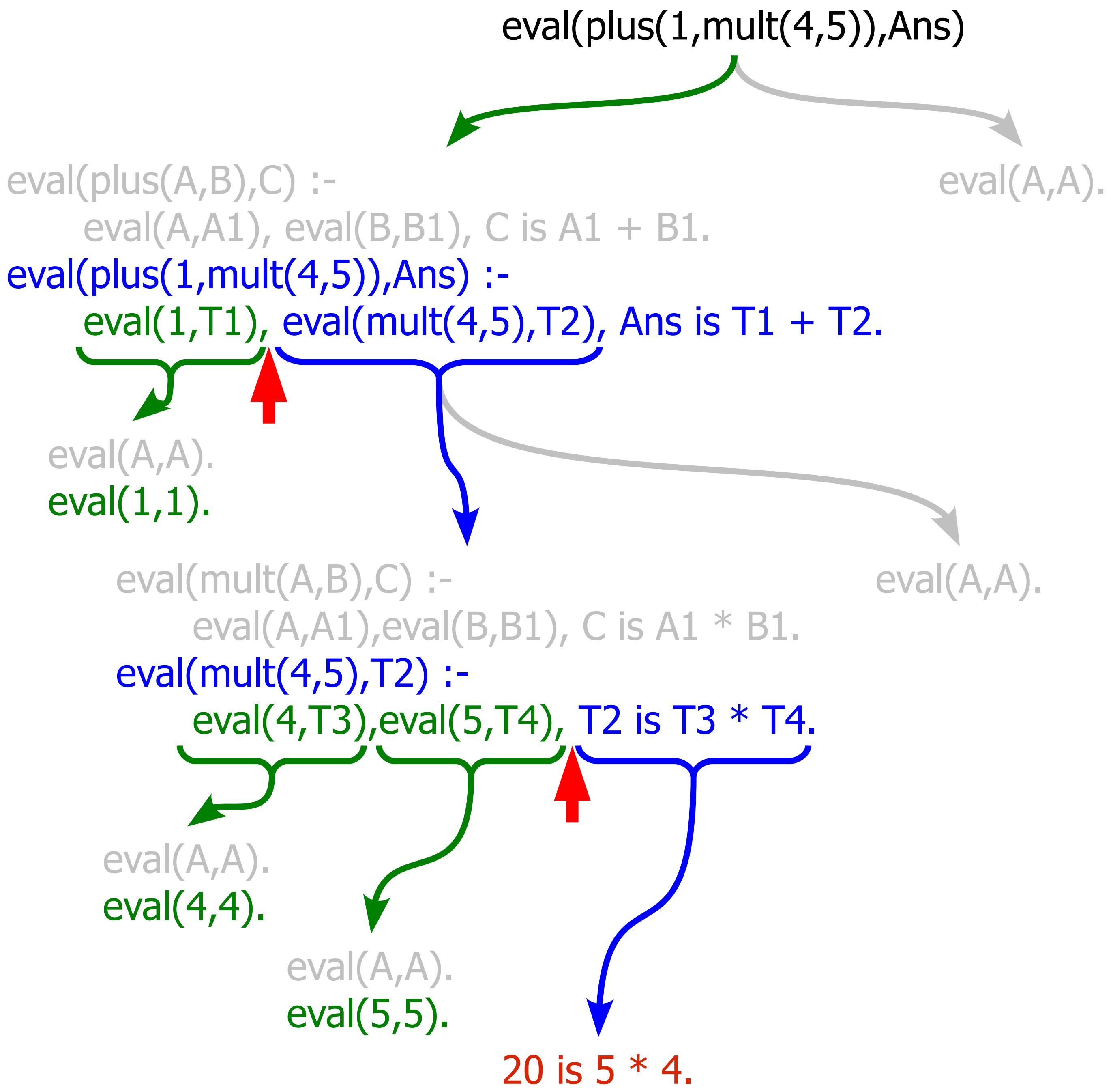
```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```



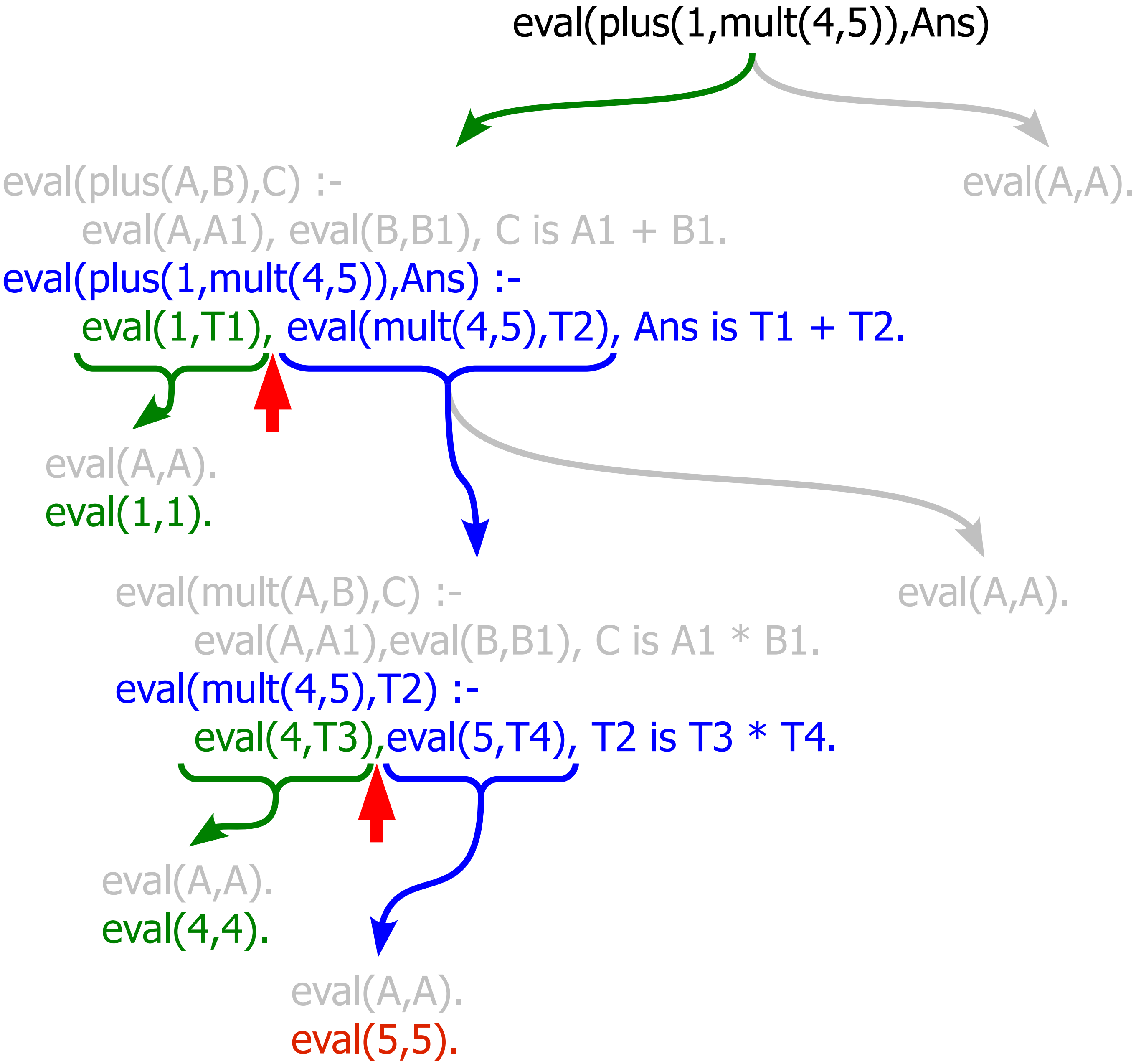
What happens if we use backtracking and ask Prolog for the next solution?

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```






```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```




```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

eval(A,A).
eval(1,1).

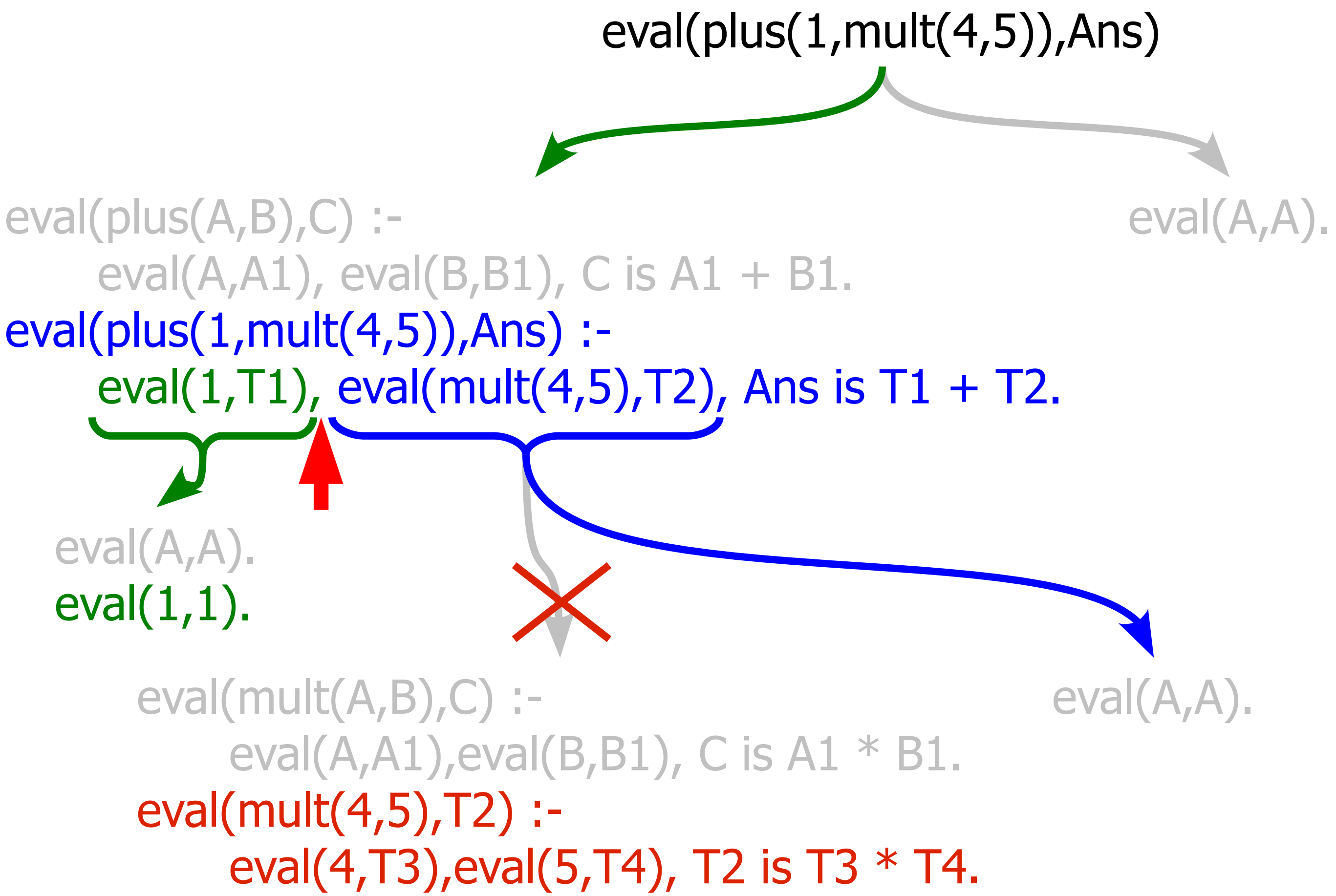
eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.

eval(A,A).

eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(4,4).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```




```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

eval(plus(1,mult(4,5)),Ans)

eval(plus(A,B),C) :-
    eval(A,A1), eval(B,B1), C is A1 + B1.

eval(A,A).

eval(plus(1,mult(4,5)),Ans) :-
    eval(1,T1), eval(mult(4,5),T2), Ans is T1 + T2.

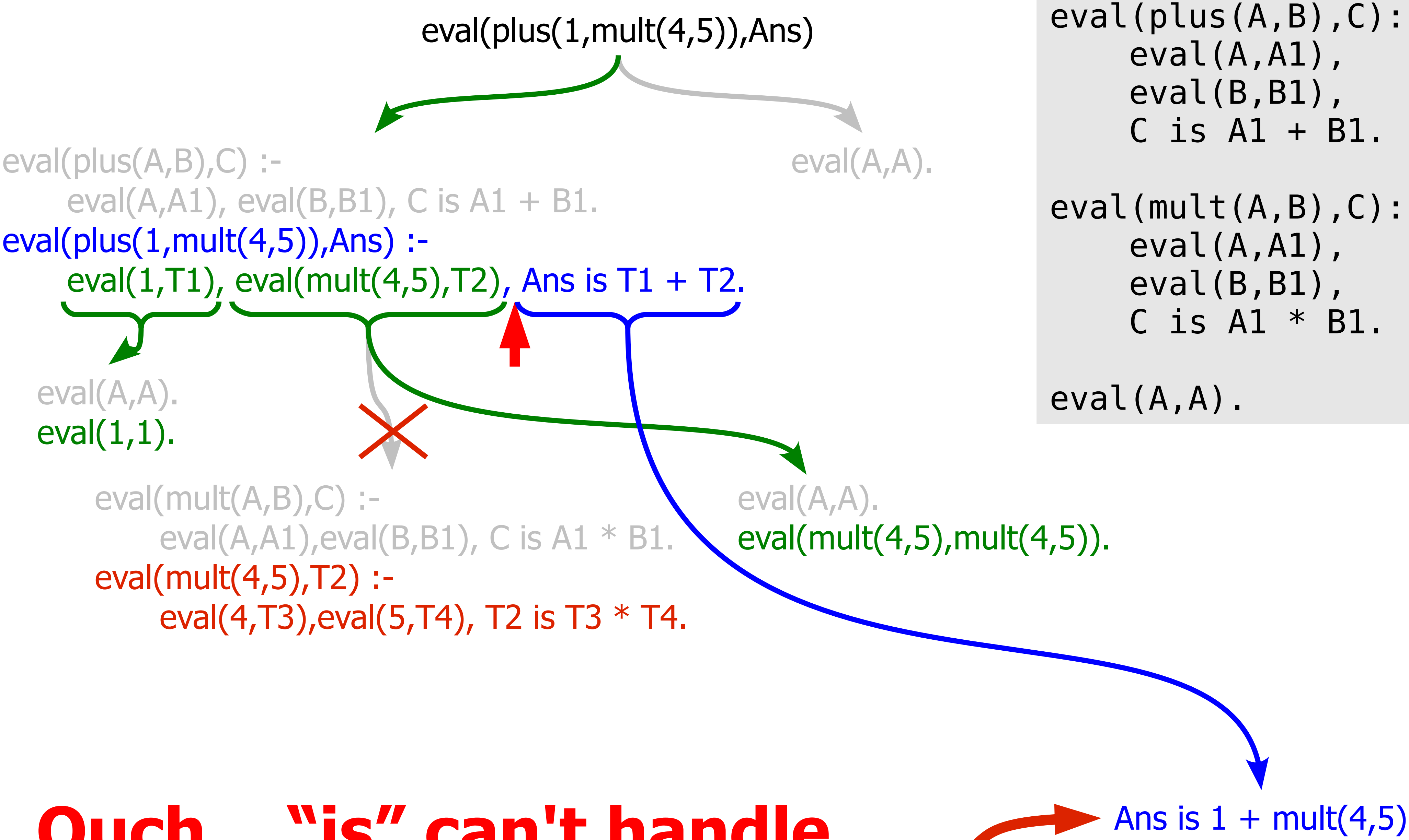
eval(A,A).
eval(1,1).

eval(mult(A,B),C) :-
    eval(A,A1),eval(B,B1), C is A1 * B1.
eval(mult(4,5),T2) :-
    eval(4,T3),eval(5,T4), T2 is T3 * T4.

eval(A,A).
eval(mult(4,5),mult(4,5)).

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```

```
eval(plus(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 + B1.

eval(mult(A,B),C):-
    eval(A,A1),
    eval(B,B1),
    C is A1 * B1.

eval(A,A).
```



Ouch... "is" can't handle the mult(4,5) term!

(a) Eliminate spurious solutions by making your clauses orthogonal

Need to eliminate the (unwanted) choice point

A way to do this: make sure only one clause matches: **eval(A,A)** becomes **eval(gnd(A),A)**.

```
eval(plus(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 + B1.
eval(mult(A,B),C) :- eval(A,A1),
                     eval(B,B1),
                     C is A1 * B1.
eval(gnd(A),A).
```

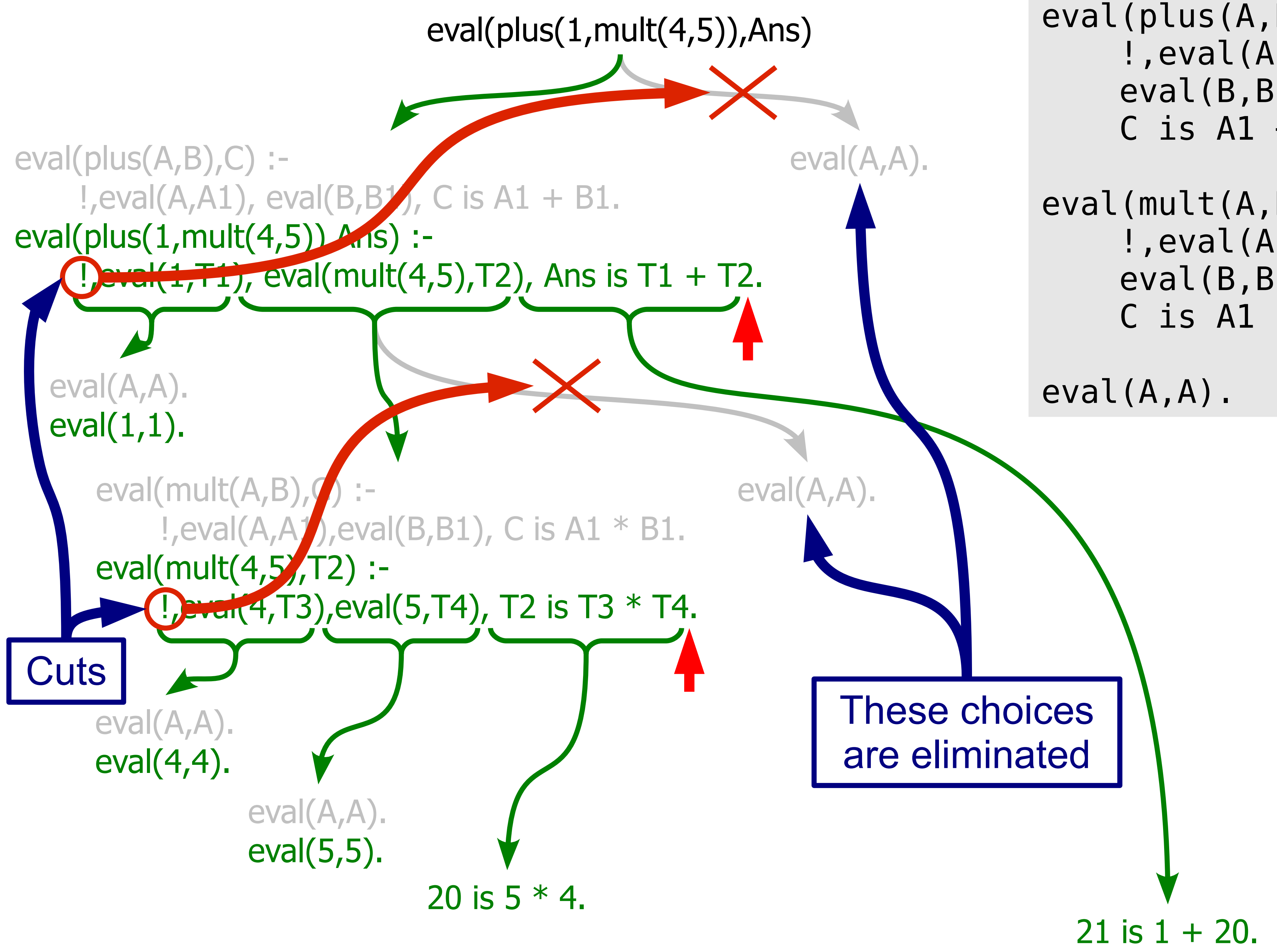
(b) Eliminate spurious solutions by explicitly discarding choice points

Alternatively we can tell Prolog to commit to its first choice and discard the choice point (p114/126)

We do this with the cut operator. Written: !

```
eval(plus(A,B),C) :- !,eval(A,A1),
                       eval(B,B1),
                       C is A1 + B1.
eval(mult(A,B),C) :- !,eval(A,A1),
                       eval(B,B1),
                       C is A1 * B1.
eval(A,A).
```

```
eval(plus(A,B),C):-
     !,eval(A,A1),
     eval(B,B1),
     C is A1 + B1.

eval(mult(A,B),C):-
     !,eval(A,A1),
     eval(B,B1),
     C is A1 * B1.

eval(A,A).
```

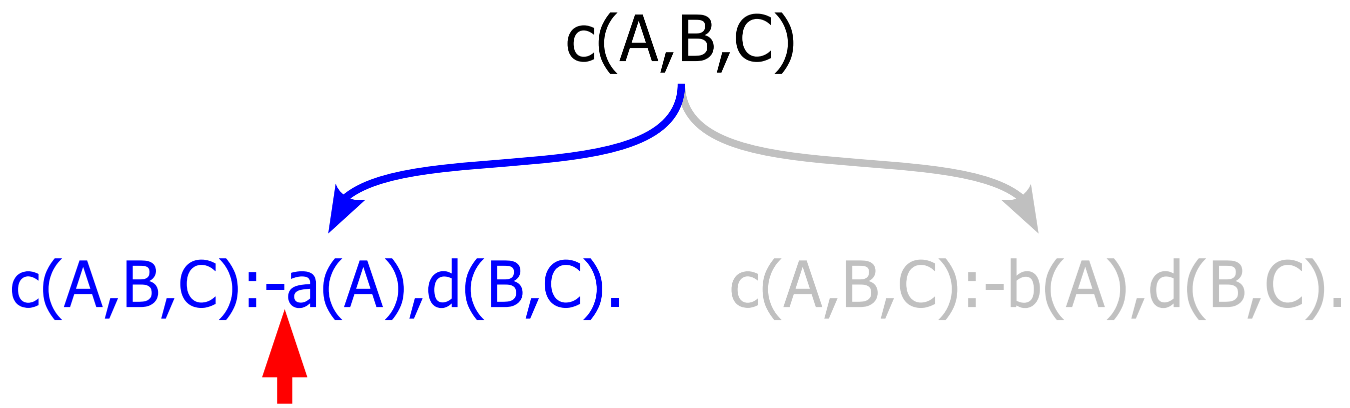


Cutting out choice

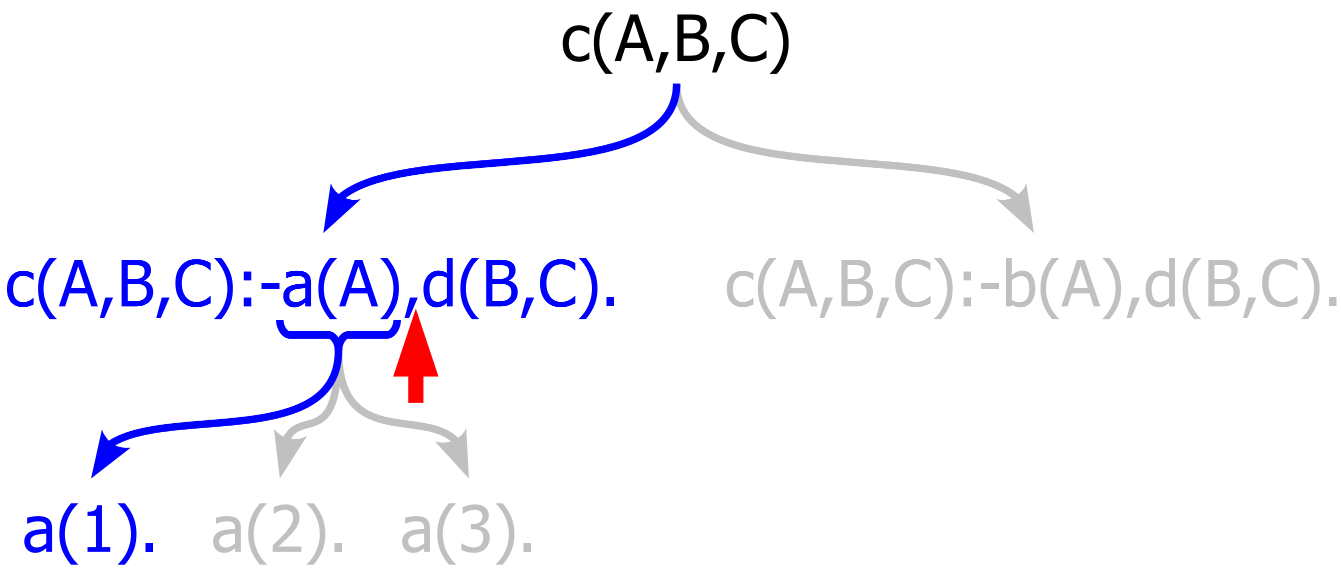
Whenever Prolog evaluates a cut it discards all choice points back to the parent clause

An example:

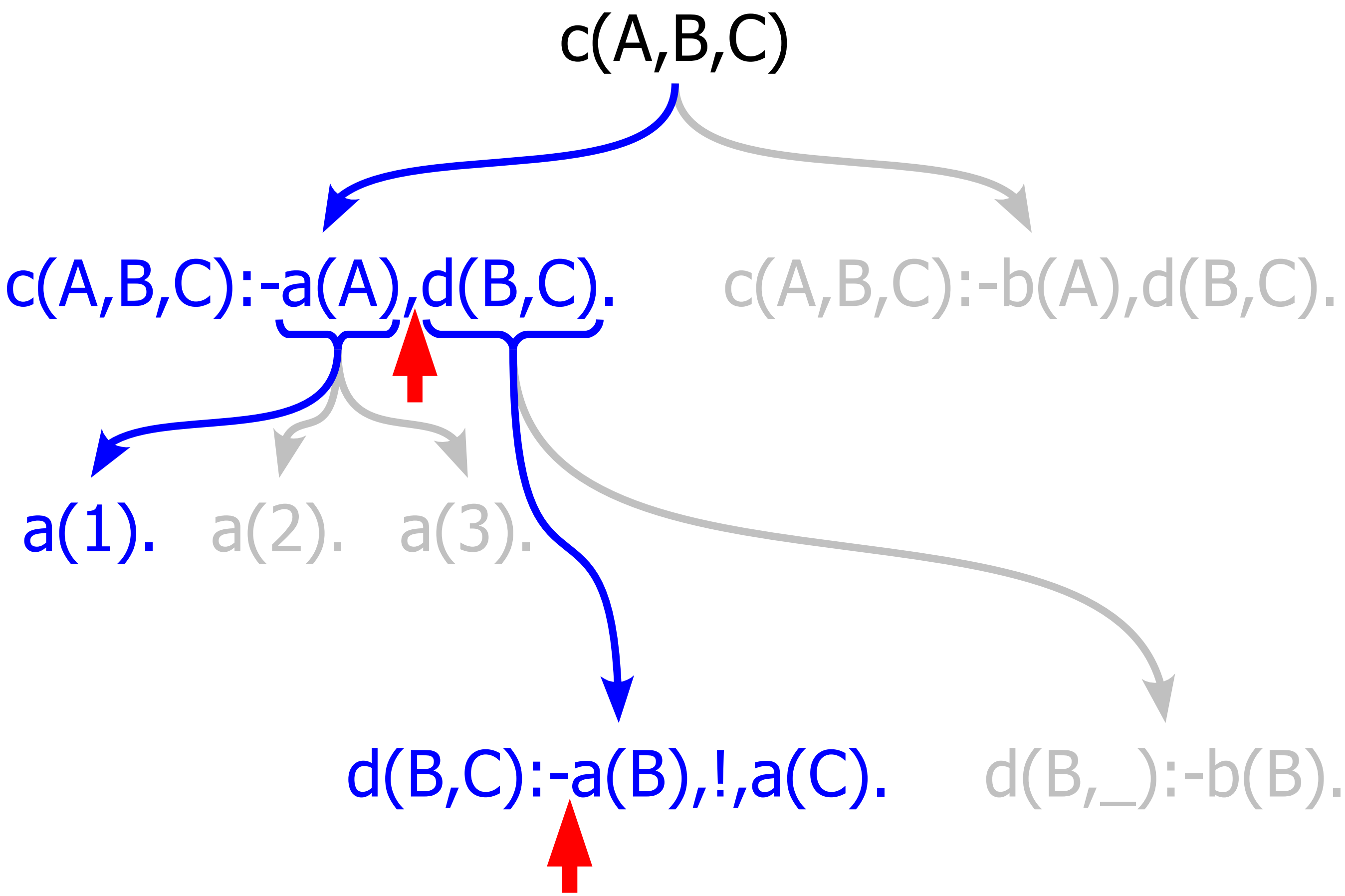
```
a(1).          c(A,B,C) :- a(A),d(B,C).
a(2).          c(A,B,C) :- b(A),d(B,C).
a(3).          d(B,C) :- a(B),!,a(C).
b(apple).      d(B,_) :- b(B).
b(orange).
```



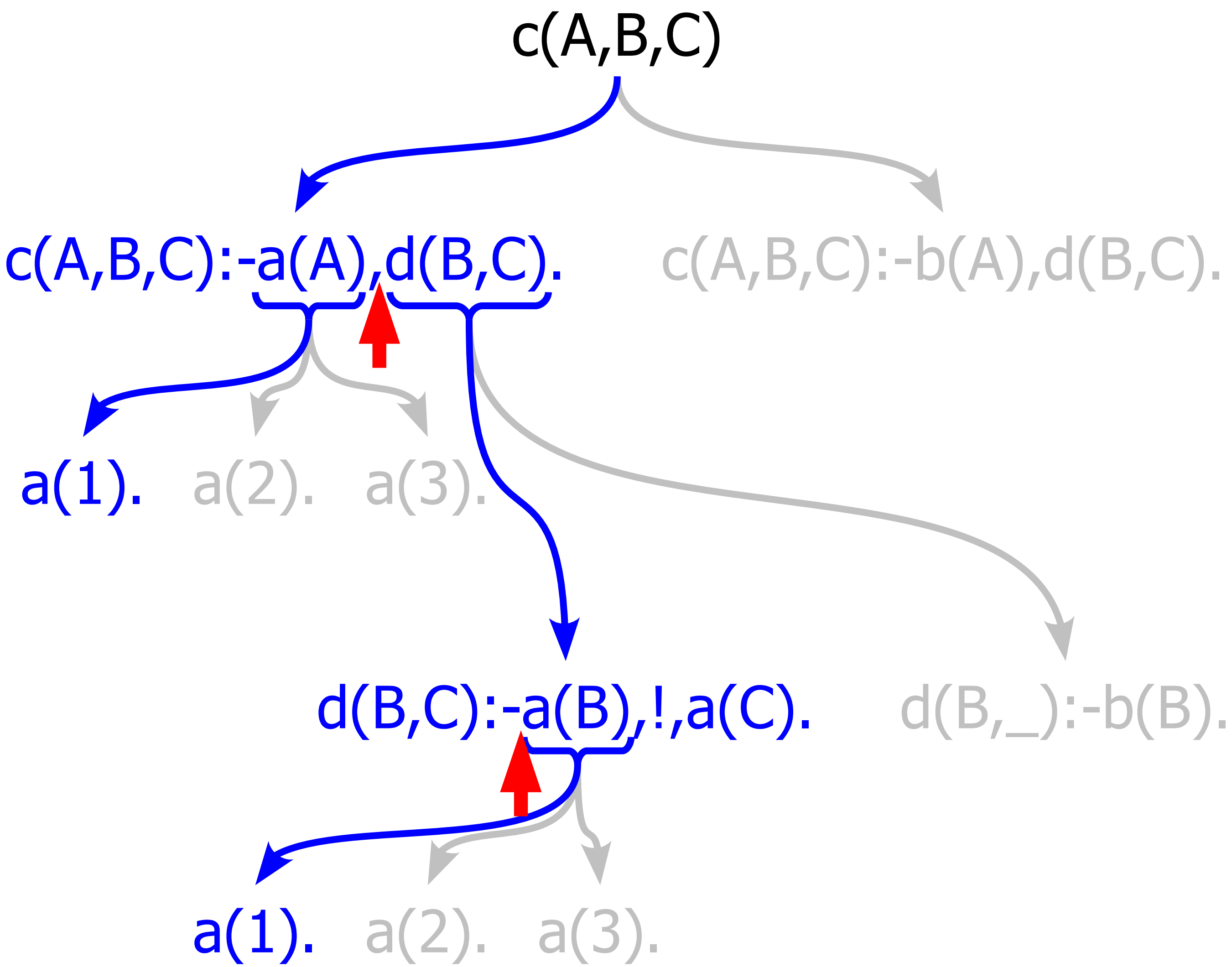

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



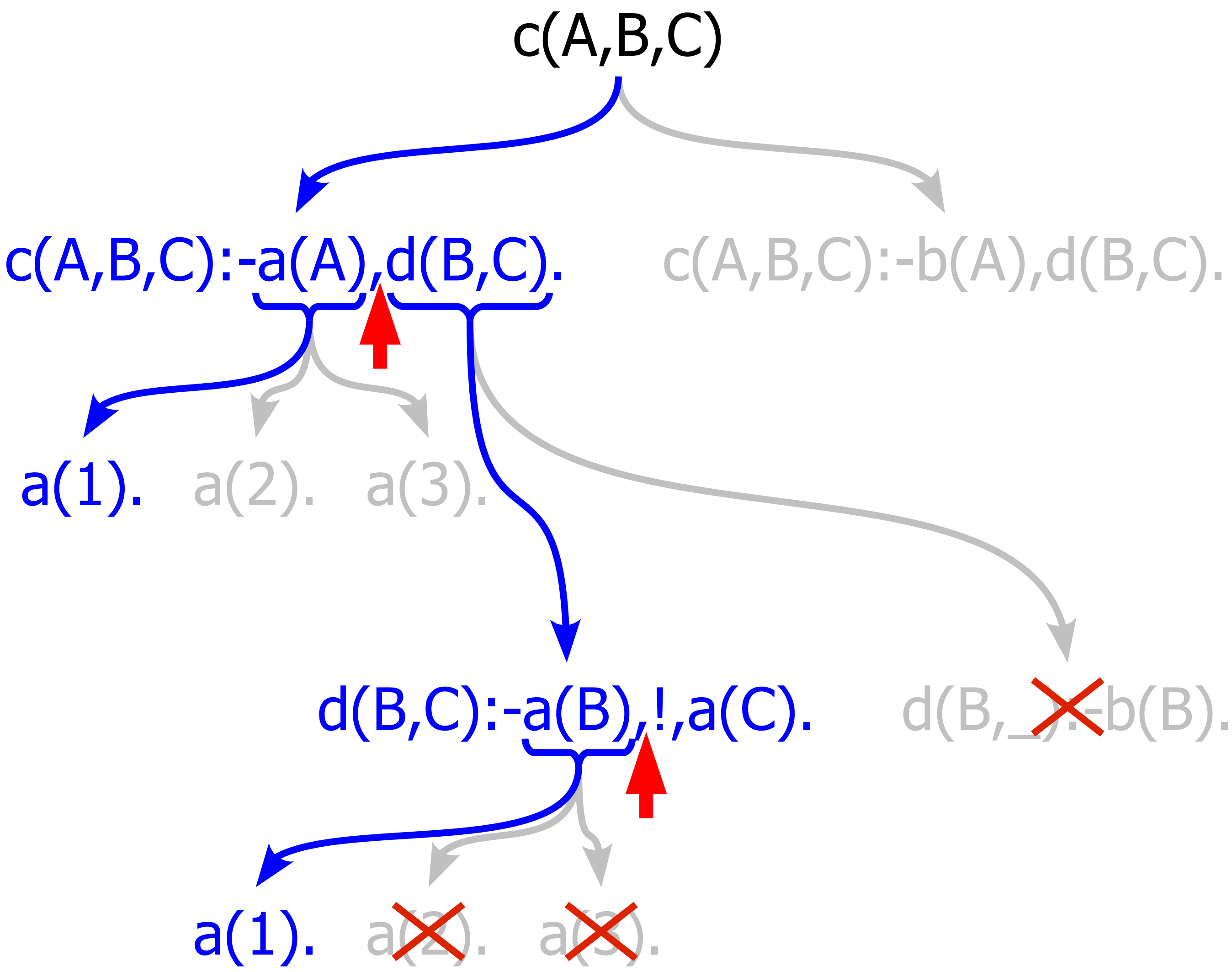

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



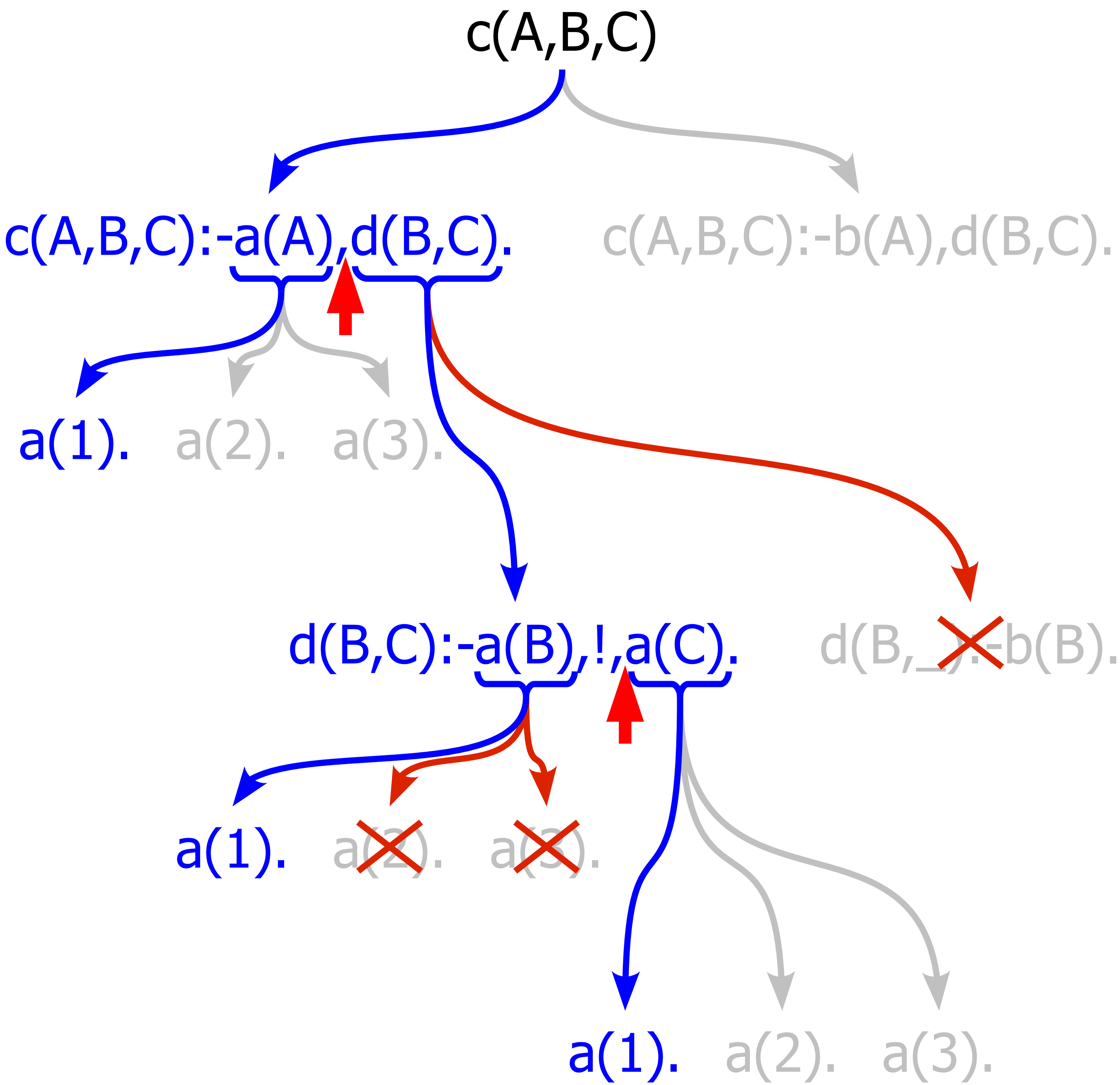

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



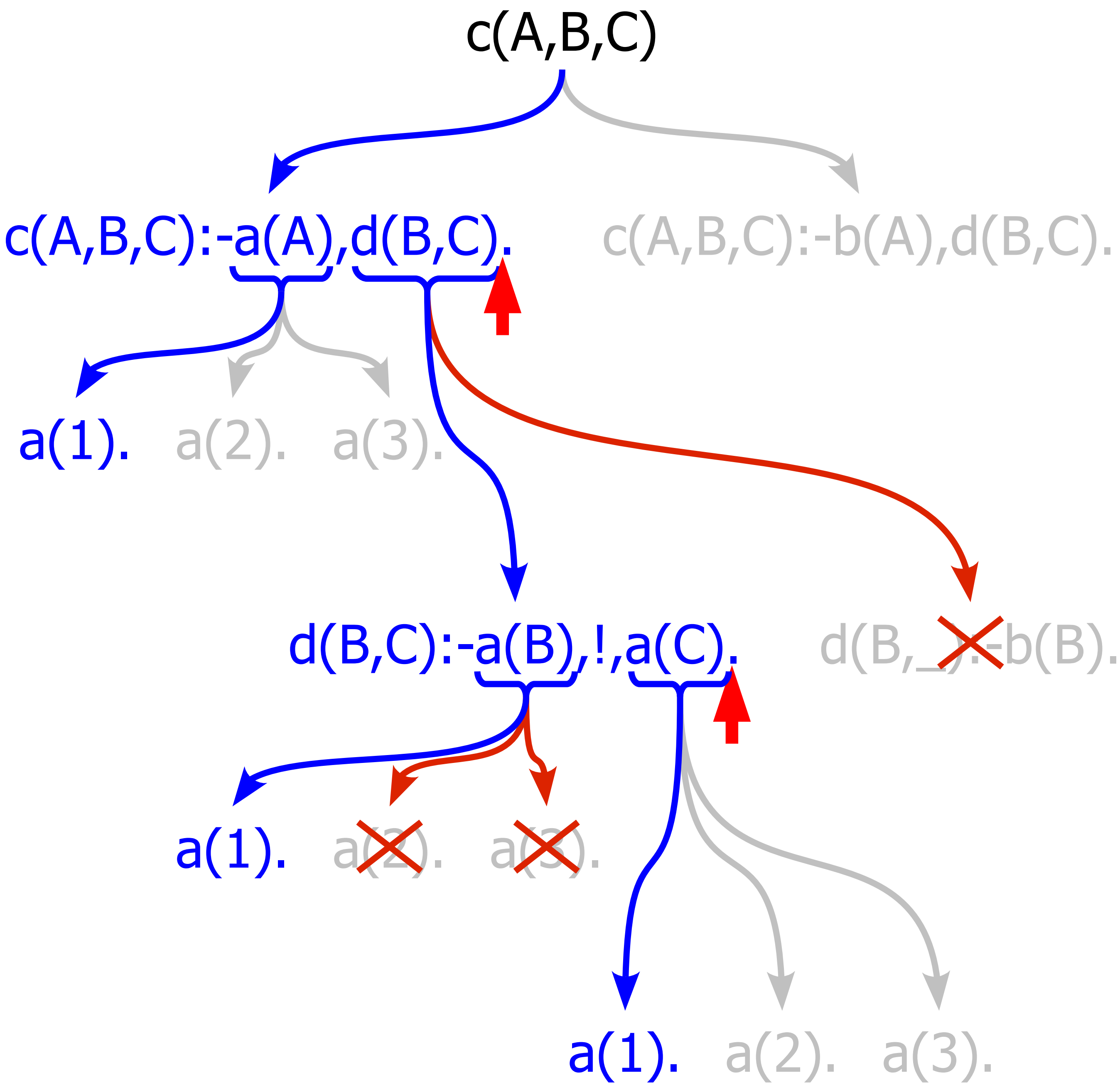

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



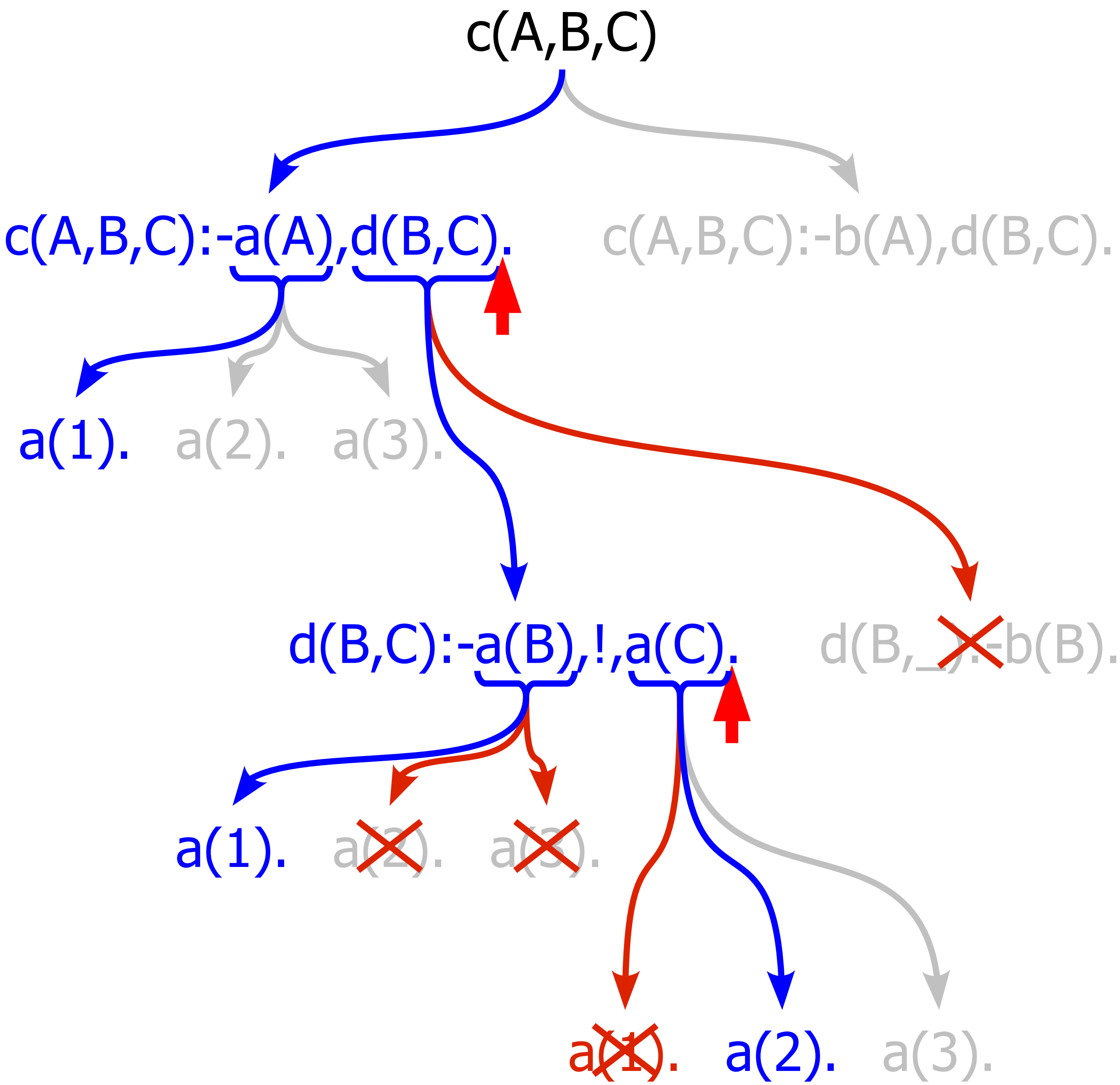

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```




```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

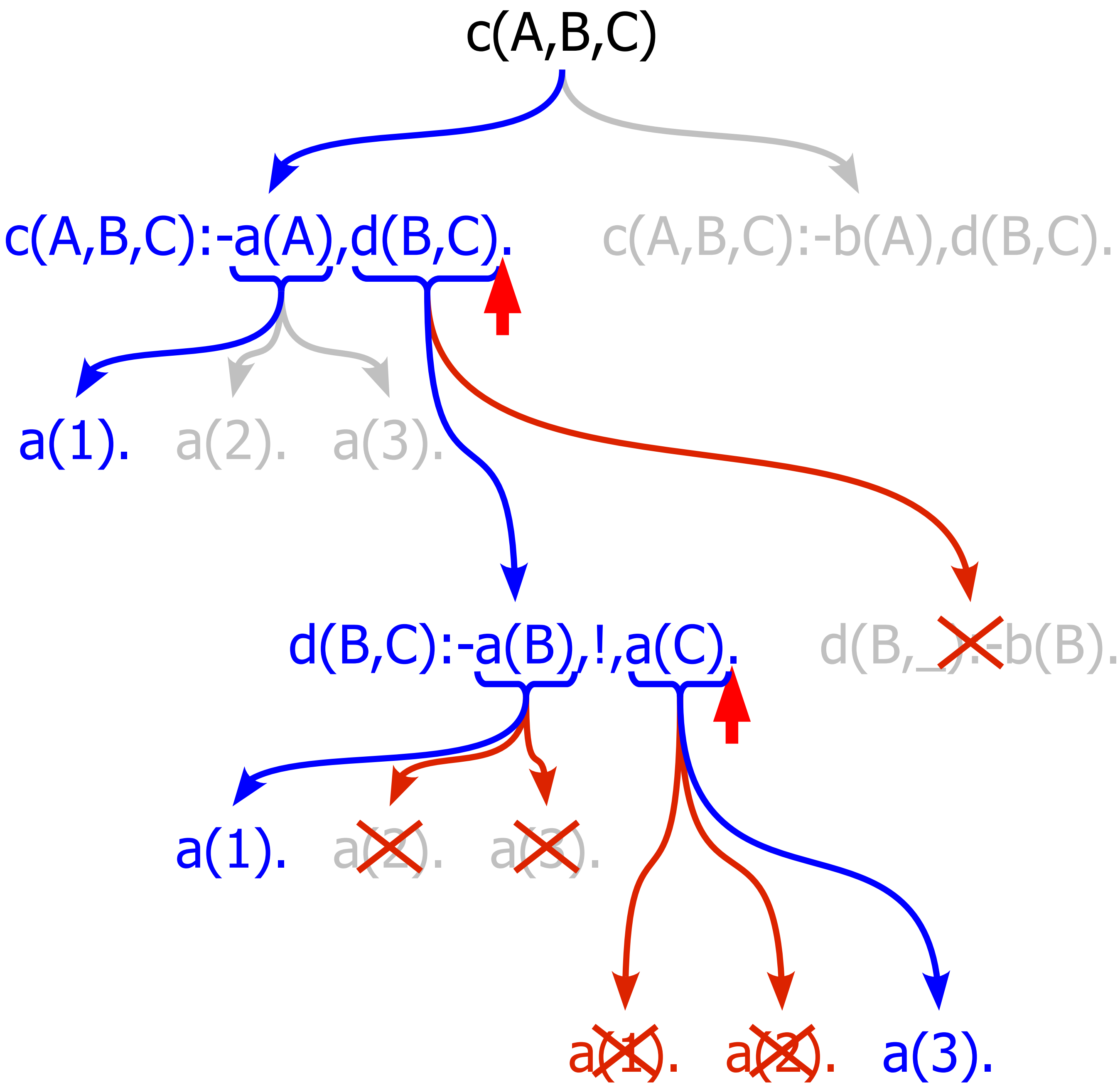



```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```



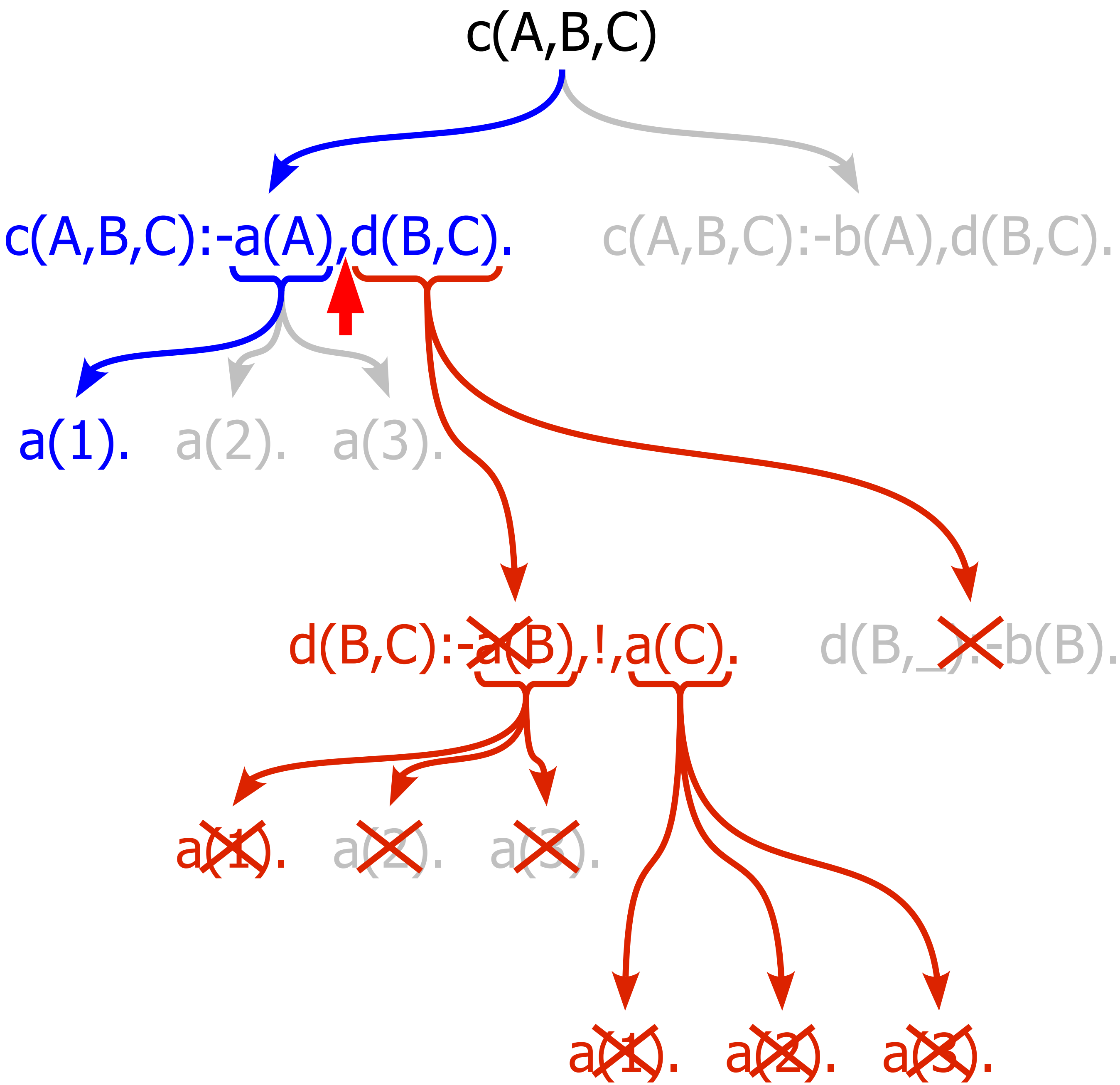

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Backtrack once



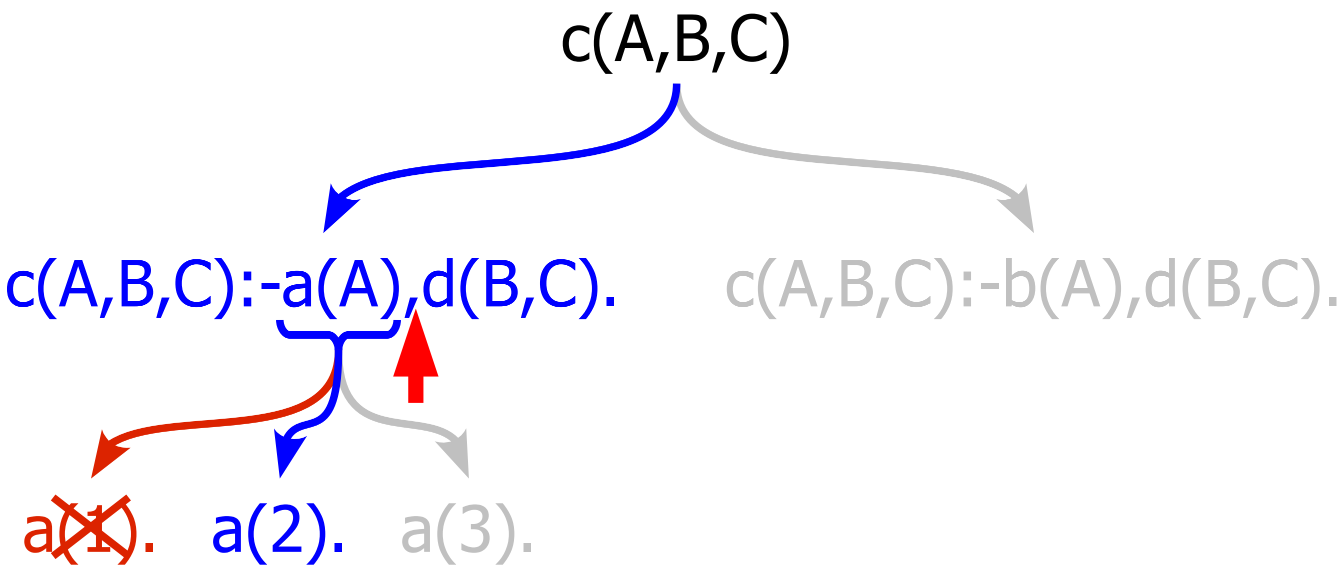

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Backtrack twice



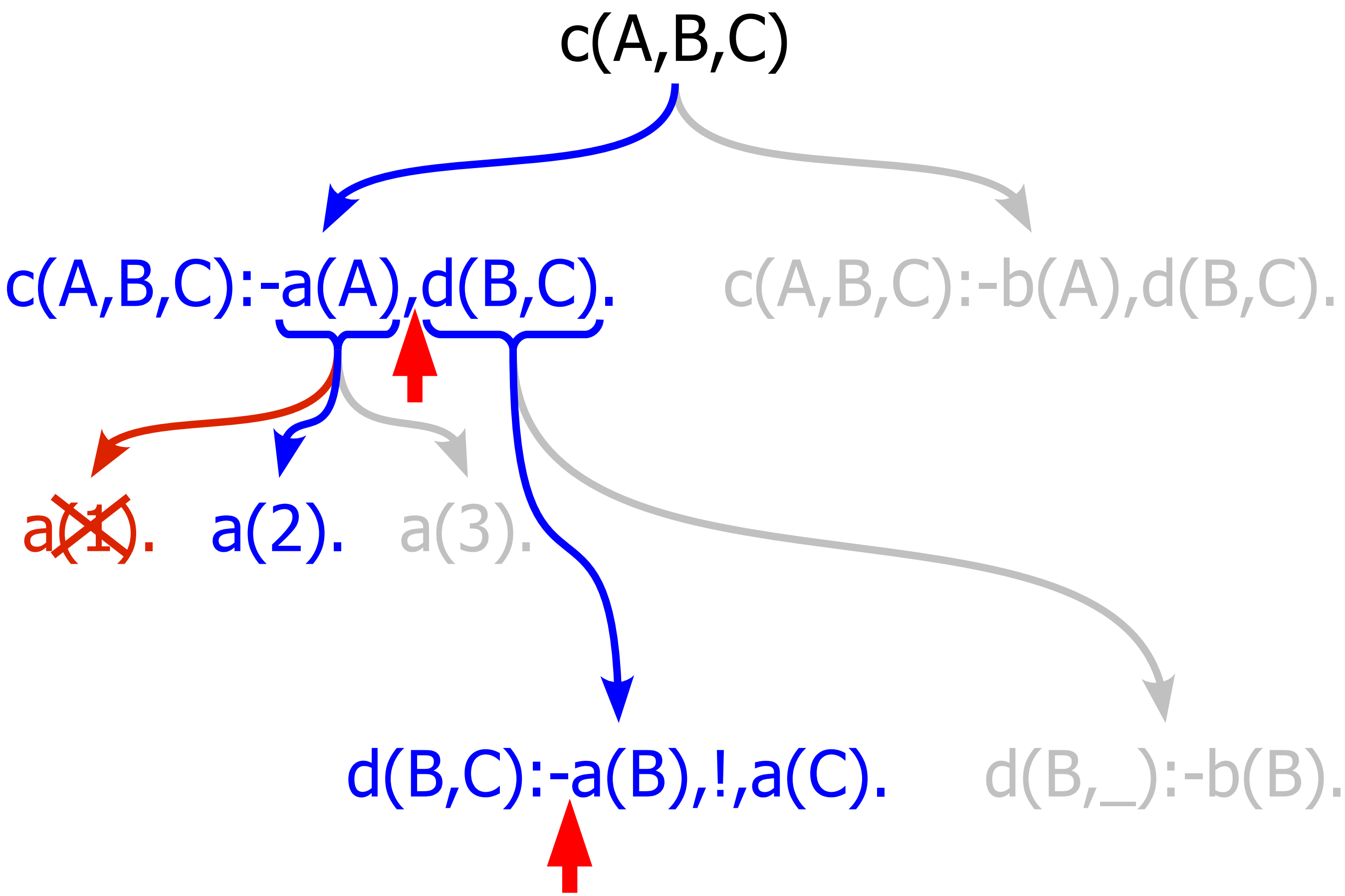

```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Backtrack three times




```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

First a/1 has other solutions




```
a(1).
a(2).
a(3).
b(apple).
b(orange).
c(A,B,C):-a(A),d(B,C).
c(A,B,C):-b(A),d(B,C).
d(B,C):-a(B),!,a(C).
d(B,_):-b(B).
```

Can try to derive d/2 afresh...

Cut can change the logical meaning of your program

```
p :- a,b.
p :- c.
```

$p \Leftrightarrow (a \wedge b) \vee c$

```
p :- a,!,b.
p :- c.
```

$p \Leftrightarrow (a \wedge b) \vee (\neg a \wedge c)$

This is a red cut – **DANGER!** (p127/138)

Cut can be used for efficiency reasons

```
split([],[],[]).
split([H|T],[H|L],R) :- H < 5, split(T,L,R).
split([H|T],L,[H|R]) :- H >= 5, split(T,L,R).
```

If the second clause succeeds the third cannot

- we don't need to keep a choice point
- yet the interpreter cannot infer this on its own

Cut can be used for efficiency reasons

```
split([],[],[]).
split([H|T],[H|L],R) :- H < 5,!, split(T,L,R).
split([H|T],L,[H|R]) :- H >= 5, split(T,L,R).
```

Add a cut to make the orthogonality explicit

- This is a green cut – it just helps program execution go faster

We could go one step further at the expense of readability

```
split([],[],[]).
split([H|T],[H|L],R) :- H < 5,!, split(T,L,R).
split([H|T],L,[H|R]) :- split(T,L,R).
```

The comparison in the third clause is no longer necessary

- but now each clause does not stand on its own
- stylistic preference – I avoid doing this

Programmers new to Prolog often cause determinism errors

Check that your predicates return the correct number of answers!

- e.g. providing a correct answer multiple times is likely to cause bugs that are difficult to find

Below is a predicate you can use in debugging

- (Note that findall is not discussed in lectures)

```
numsol(Predicate,NumberOfSolutions):-
     findall(dummy,Predicate,AnsList),
     length(AnsList,NumberOfSolutions).
```

Testing using the numsol/2 predicate

A warning will be generated about mergesplat/3.

- What is wrong with the mergesplat/3 predicate?

```
mergesplat([],[],[]).
mergesplat(A,[],A).
mergesplat([],B,B).
mergesplat([A|As],[B|Bs],[A,B|Rest]):-
        mergesplat(As,Bs,Rest).

:-numsol(merge([1,2],[a,b],_),1).
:-numsol(mergesplat([1,2],[a,b],_),1).
```

Cut gives us more expressive power

```
isDifferent(A,A) :- !,fail.
isDifferent(_,_).
```

isDifferent(A,B) is true iff A and B do not unify

Questions that you should be able to answer:

- Is this a red or a green cut?
- How can you define the fail/0 predicate?

Using cut, we can implement “not” (Negation by failure)

```
not(A) :- A,!,fail.
not(_).
```

not(A) is true if A cannot be shown to be true

- This is negation by failure (p124/135)

Negation by failure is based on the closed world assumption: (p129/138)

Everything that is true in the “world” is stated (or can be derived from) the clauses in the program

Negation Example

```
good_food(theWrestlers).
good_food(theCambridgeLodge).
expensive(theCambridgeLodge).

bargain(R) :- good_food(R),
              not(expensive(R)).
```

we can ask:

- bargain(R)

and Prolog replies:

- R = theWrestlers

Negation Gotcha!

```
good_food(theWrestlers).
good_food(theCambridgeLodge).
expensive(theCambridgeLodge).

bargain(R) :- not(expensive(R)),
              good_food(R).
```

we can ask the same query:

- bargain(R)

and Prolog replies:

- false.

Clause body terms have been swapped around!

Why?

```
good_food(theWrestlers).
good_food(theCambridgeLodge).
expensive(theCambridgeLodge).

bargain(R) :- not(expensive(R)),
              good_food(R).
```

Prolog first tries to find an R such that expensive(R) is true.

- therefore not(expensive(R)) will fail if there are any expensive restaurants

We sometimes identify the way to use parameters of a rule

Prolog's non-logical properties can make it important whether or not an argument to a predicate is bound

% indicates a comment to the end of that line

```
% this comment in some hypothetical code is
% describing how to query myrule(+A,+B,-C,-D)
```

The convention for comments about rule parameters:

+X is a ground term (must be instantiated)

-X is a variable term (must be unbound)

?X means it does not matter (roughly)

Query “myrule” with two ground (input) terms A and B and two variable (output) terms C and D

Prolog variables and quantifiers

When R is not bound, quantifiers need attention

`expensive(R)`

- "There exists an R that is expensive".

`not(expensive(R))`

- "There does not exist an R that is expensive".
- In other words, "for all R, not expensive(R)".

Databases

Information can be stored as tuples in Prolog's internal database

```
tName(dme26,'David Eyers').
tName(awm22,'Andrew Moore').

tGrade(dme26,'IA',2.1).
tGrade(dme26,'IB',1).
tGrade(dme26,'II',1).
tGrade(awm22,'IA',2.1).
tGrade(awm22,'IB',1).
tGrade(awm22,'II',1).
```

Databases

We can now write a program to find all names:

```
qName(N) :- tName(_,N).
```

Or a program to find the full name and all grades for dme26.

```
qGrades(F,C,G) :- tName(I,F), tGrade(I,C,G).
```

Further exercises are in the problem sheet...