University of Cambridge
Computer Laboratory
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Easter Term 2010/11
Exercises 1
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Interactive Formal Verification (L21)

1 Replace, Reverse and Delete

▷ Define a function *replace*, such that *replace x y zs* yields *zs* with every occurrence of *x* replaced by *y*.

```
replace :: "'a ⇒ 'a ⇒ 'a list ⇒ 'a list"
```

▷ Prove or disprove (by counterexample) the following theorems. You may have to prove some lemmas first.

theorem *"rev (replace x y zs) = replace x y (rev zs)"*
theorem *"replace x y (replace u v zs) = replace u v (replace x y zs)"*
theorem *"replace y z (replace x y zs) = replace x z zs"*

▷ Define two functions for removing elements from a list: *del1 x xs* deletes the first occurrence (from the left) of *x* in *xs*, *delall x xs* all of them.

```
del1   :: "'a ⇒ 'a list ⇒ 'a list"
delall :: "'a ⇒ 'a list ⇒ 'a list"
```

▷ Prove or disprove (by counterexample) the following theorems.

theorem *"del1 x (delall x xs) = delall x xs"*
theorem *"delall x (delall x xs) = delall x xs"*
theorem *"delall x (del1 x xs) = delall x xs"*
theorem *"del1 x (del1 y zs) = del1 y (del1 x zs)"*
theorem *"delall x (del1 y zs) = del1 y (delall x zs)"*
theorem *"delall x (delall y zs) = delall y (delall x zs)"*
theorem *"del1 y (replace x y xs) = del1 x xs"*
theorem *"delall y (replace x y xs) = delall x xs"*
theorem *"replace x y (delall x zs) = delall x zs"*
theorem *"replace x y (delall z zs) = delall z (replace x y zs)"*
theorem *"rev (del1 x xs) = del1 x (rev xs)"*
theorem *"rev (delall x xs) = delall x (rev xs)"*