MPhil in Advanced Computer Science
Advanced Category Theory in Computer Science

Leader:	Marcelo Fiore (course lecturer)
Timing:	Lent
Prerequisites:	Category Theory in Computer Science
Structure:	8 Lectures

AIMS

This module aims to train students at the forefront of research in the application of category theory to computer science.

SYLLABUS

A range of topics for the course follows.

1. Algebraic theories: universal algebra; equational logic; soundness and completeness; theory translations and constructions.

2. Algebras: initial algebras; induction principle; recursive domain equations; free algebras.

3. Presheaves: cartesian closure; essential geometric morphisms; free cocompletions; Kan extensions; coends.

4. Simply typed lambda calculus: λ-definability; Kripke logical relations; glueing; normalisation by evaluation; conservative extensions.

5. Monoidal categories: Day's convolution tensor product; substitution tensor products; operads.

6. Second-order algebraic theories: variable binding and α-equivalence; capture-avoiding substitution; metavariables; meta-substitution.

7. Joyal's species of structures: calculus of combinatorial constructions; analytic functors; the Schanuel topos.

OBJECTIVES

On completion of this module students should:

- be able to start research in theoretical computer science involving category theory.

COURSEWORK

N/A

PRACTICAL WORK

N/A

ASSESSMENT

The course will be assessed by means of an essay on one or more research papers related to the syllabus. Papers will be chosen by students on their own or from a given list of papers in accordance with the lecturer.

The essays will be marked by the lecturer and returned to the students. Subsequently, a technical discussion with each student on the material of their essay will take place. The mark for the course will be that of the essay, with an upgrade for those students that give evidence of mastering the subject during the discussion.

RECOMMENDED READING

[1] R. Crole. *Categories for types.* Cambridge University Press, 1993.

[2] B. Day. On closed categories of functors. In *Reports of the Midwest Category Seminar IV*, *Lecture Notes in Mathematics* 137, pages 1–38, 1970.

[3] M. Fiore. Rough notes on presheaves. Notes, 2001.

[4] M. Fiore. Semantic analysis of normalisation by evaluation for typed lambda calculus. In *4th International Conference on Principles and Practice of Declarative Programming* (PPDP 2002), 2002.

[5] M. Fiore. Algebraic theories and equational logics. Notes, 2008.

[6] M. Fiore and C.-K. Hur. Term equational systems and logics. In *24th Mathematical Foundations of Programming Semantics Conf.* (MFPS XXIV), *Electronic Notes in Theoretical Computer Science*, 218:171–192, 2008.

[7] M. Fiore, G. Plotkin and D. Turi. Abstract syntax and variable binding. In *14th Logic in Computer Science Conf.* (LICS'99), pages 193–202, 1999.

[8] P. Freyd. Algebraically complete categories. In *Category Theory*, *Lecture Notes in Mathematics* 1488, pages 131–156, 1991.

[9] M.J. Gabbay and A. Pitts. A new approach to abstract syntax with variable binding. *Formal Aspects of Computing*, 13:341–363, 2001.

[10] R. Harmer, M. Hyland and P.-A. Mellies. Categorical combinatorics for innocent strategies. In *22nd Logic in Computer Science Conf.* (LICS'07), pages 379–388, 2007.

[11] A. Joyal. Foncteurs analytiques et espèces de structures. In *Combinatoire Énumérative*, *Lecture Notes in Mathematics* 1234, pages 126–159, 1986.

[12] G. M. Kelly. On the operads of J. P. May. Reprints in *Theory and Applications of Categories*, 13:1–13, 2005.

[13] E. Moggi. Notions of computation and monads. *Information and Computation*, 93(1):55-92, 1991.

[14] G. Plotkin and A.J. Power. Notions of computation determine monads. In *FOSSACS 2002*, *Lecture Notes in Computer Science* 2303, pp. 342–356, 2002. (Erratum at `http://homepages.inf.ed.ac.uk/gdp/publications/`.)

[15] D. Scott. Relating theories of the λ-calculus. In *To H. B. Curry: Essays in Combinatory Logic, Lambda Calculus and Formalisms*. Academic Press, 1980.

[16] M. Smyth and G. Plotkin. The category-theoretic solution of recursive domain equations. SIAM Journal of Computing 11:761–783, 1982.

[17] D. Turi and G. Plotkin. Towards a mathematical operational semantics. In *12th Logic in Computer Science Conf.* (LICS'97), pages 280–291, 1997.

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