

Category Theory Exercises: Week 1

October 2009

These exercises are not compulsory, and they will not contribute to your final grade. Please send your solutions or questions by e-mail to bk291@cam.ac.uk, or leave them in Bartek Klin's pigeonhole next to Reception.

Exercise 1. For any object A in a category \mathbf{C} , define the *coslice category* A/\mathbf{C} by analogy to the slice category \mathbf{C}/A , as suggested in the lecture. Then check that

$$(A/\mathbf{C})^{op} = \mathbf{C}^{op}/A.$$

Exercise 2. Prove that for any two subcategories \mathbf{D}, \mathbf{E} of a fixed category \mathbf{C} , their *intersection* $\mathbf{D} \cap \mathbf{E}$ defined by:

$$\begin{aligned} |\mathbf{D} \cap \mathbf{E}| &= |\mathbf{D}| \cap |\mathbf{E}| \\ \text{Ar}(\mathbf{D} \cap \mathbf{E}) &= \text{Ar}(\mathbf{D}) \cap \text{Ar}(\mathbf{E}) \end{aligned}$$

is a well-defined category and a subcategory of \mathbf{C} . How about the *union* of \mathbf{D} and \mathbf{E} ?