Databases

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Outline

Re-ordered Syllabus

Lecture 01 **Basic Concepts.** Relations, attributes, tuples, and relational schema. Tables in SQL.

Lecture 02 **Query languages.** Relational algebra, relational calculi (tuple and domain). Examples of SQL constructs that mix and match these models.

Lecture 03 **More on SQL.** Null values (and three-valued logic). Inner and Outer Joins. Views and integrity constraints.

Lecture 04 **Redundancy is a Bad Thing.** Update anomalies. Capturing redundancy with functional and multivalued dependencies.

Re-ordered Syllabus

Lecture 05 **Analysis of Redundancy.** Implied functional dependencies, logical closure. Reasoning about functional dependencies.

Lecture 06 **Eliminating Redundancy.** Schema decomposition. Lossless join decomposition. Dependency preservation. 3rd normal form. Boyce-Codd normal form.

Lecture 07 **Schema Decomposition.** Decomposition examples. Multivalued dependencies and Fourth normal form.

Re-ordered Syllabus

Lectures 08, 09, 10 **Redundancy can be a Good Thing!** Database updates. The main issue: query response vs. update throughput. Locking vs. update throughput. Indices are derived data! Selective de-normalization. Materialized views. The extreme case: “read only” database, data warehousing, data-cubes, and OLAP vs OLTP.

Lecture 11 **Entity-Relationship Modeling.** High-level modeling. Entities and relationships. Representation in relational model. Reverse engineering as a common application.

Lecture 12 **What is a DBMS?** Different levels of abstraction, data independence. Other data models (Object-Oriented databases, Nested Relations). XML as a universal data exchange language.

Recommended Reading

Textbooks

UW1997 Ullman, J. and Widom, J. (1997). A first course in database systems. Prentice Hall.

D2004 Date, C.J. (2004). An introduction to database systems. Addison-Wesley (8th ed.).

SL2002 Silberschatz, A., Korth, H.F. and Sudarshan, S. (2002). Database system concepts. McGraw-Hill (4th ed.).

EN2000S Elmasri, R. and Navathe, S.B. (2000). Fundamentals of database systems. Addison-Wesley (3rd ed.).

Reading for the fun of it ...

Research Papers (Google for them)

- **C1970** E.F. Codd, (1970). "A Relational Model of Data for Large Shared Data Banks". Communications of the ACM.
- **F1977** Ronald Fagin (1977) Multivalued dependencies and a new normal form for relational databases. TODS 2 (3).
- **L2003** L. Libkin. Expressive power of SQL. TCS, 296 (2003).
- **C+1996** L. Colby et al. Algorithms for deferred view maintenance. SIGMOD 199.
- **G+1997** J. Gray et al. Data cube: A relational aggregation operator generalizing group-by, cross-tab, and sub-totals (1997) Data Mining and Knowledge Discovery.
- **H2001** A. Halevy. Answering queries using views: A survey. VLDB Journal. December 2001.

Lecture 01: Relations and Tables

Lecture Outline

- Relations, attributes, tuples, and relational schema
- Representation in SQL : Tables, columns, rows (records)
- Important: users should be able to create and manipulate relations (tables) without regard to implementation details!

Edgar F. Codd

pgflastimage

- The problem : in 1970 you could not write a database application without knowing a great deal about the the low-level physical implementation of the data.
- Codd's radical idea [C1970]: give users a model of data and a language for manipulating that data which is completely independent of the details of its physical representation/implementation.
- This decouples development of Database Management Systems (DBMSs) from the development of database applications (at least in a idealized world).

Let's start with mathematical relations

Suppose that S_1 and S_2 are sets. The Cartesian product, $S_1 \times S_2$, is the set

$$S_1 \times S_2 = \{(s_1,\ s_2) \mid s_1 \in S_1,\ s_2 \in S_2\}$$

A (binary) relation over $S_1 \times S_2$ is any set r with

$$r \subseteq S_1 \times S_2.$$

In a similar way, if we have n sets,

$$S_1,\ S_2,\ \ldots, S_n,$$

then an *n*-ary relation r is a set

$$r \subseteq S_1 \times S_2 \times \cdots \times S_n = \{(s_1,\ s_2,\ \ldots, s_n) \mid s_i \in S_i\}$$

Did you notice the prestidigitation?

What do we really mean by this notation?

$$S_1 \times S_2 \times \cdots \times S_n$$

Does it represent $n-1$ applications of a binary operator $\times$? NO!.
If we wanted to be extremely careful we might write something like $\times(S_1,\ S_2, \ldots, S_n)$.
We perform this kind of sleight of hand very often. Here's an example from OCaml:

```
let flatten_left : (('a * 'b) * 'c) -> ('a * 'b * 'c)
     = function p ->
         (fst (fst p), snd (fst p), snd p)
```

Perhaps if we had the option of writing `*('a, 'b, 'c)` it would make this implicit flattening more obvious.

Mathematical vs. database relations

Suppose we have an n-tuple $t \in S_1 \times S_2 \times \cdots \times S_n$. Extracting the i-th component of t, say as $\pi_i(t)$, feels a bit low-level.

- Solution: (1) Associate a name, A_i (called an attribute name) with each domain S_i. (2) Instead of tuples, use records — sets of pairs each associating an attribute name A_i with a value in domain S_i.

A database relation R over the schema $A_1 : S_1 \times A_2 : S_2 \times \cdots \times A_n : S_n$ is a finite set

$$R \subseteq \{\{(A_1, s_1), (A_2, s_2), \ldots, (A_n, s_n)\} \mid s_i \in S_i\}$$

Example

A relational schema

Students(**name**: string, **sid**: string, **age** : integer)

A relational instance of this schema

$$
\begin{array}{lcl}
\textbf{Students} & = & \{ \\
& & \{(\textbf{name},\ \text{Fatima}), (\textbf{sid},\ \text{fm21}), (\textbf{age},\ 20)\}, \\
& & \{(\textbf{name},\ \text{Eva}), (\textbf{sid},\ \text{ev77}), (\textbf{age},\ 18)\}, \\
& & \{(\textbf{name},\ \text{James}), (\textbf{sid},\ \text{jj25}), (\textbf{age},\ 19)\} \\
& & \}
\end{array}
$$

A tabular presentation

name	**sid**	**age**
Fatima	fm21	20
Eva	ev77	18
James	jj25	19

Creating Tables in SQL

```
create table Students
       (sid varchar(10),
        name varchar(50),
        age int);

-- insert record with attribute names
insert into Students set
       name = 'Fatima', age = 20, sid = 'fm21';

-- or insert records with values in same order
-- as in create table
insert into Students values
       ('jj25' , 'James' ,  19),
       ('ev77' , 'Eva' ,  18);
```

Listing a Table in SQL

```
-- list by attribute order of create table
mysql> select * from Students;
+------+--------+------+
| sid  | name   | age  |
+------+--------+------+
| ev77 | Eva    |   18 |
| fm21 | Fatima |   20 |
| jj25 | James  |   19 |
+------+--------+------+
3 rows in set (0.00 sec)
```

Listing a Table in SQL

```
-- list by specified attribute order
mysql> select name, age, sid from Students;
+--------+------+------+
| name   | age  | sid  |
+--------+------+------+
| Eva    |   18 | ev77 |
| Fatima |   20 | fm21 |
| James  |   19 | jj25 |
+--------+------+------+
3 rows in set (0.00 sec)
```

Keys in SQL

A key is a set of attributes that will uniquely identify any record (row) in a table. We will get more precise in Lecture 06.

```
-- with this create table
create table Students
       (sid varchar(10),
        name varchar(50),
        age int,
        primary key (sid));

-- if we try to insert this (fourth) student ...
mysql> insert into Students set
       name = 'Flavia', age = 23, sid = 'fm21';

 ERROR 1062 (23000): Duplicate
       entry 'fm21' for key 'PRIMARY'
```

Put all information in one big table?

Suppose we want to add information about college membership to our Student database. We could add an additional attribute for the college.

```
StudentsWithCollege :
+--------+-------+-------+---------+
| name   | age   | sid   | college|
+--------+-------+-------+---------+
| Eva    |    18 | ev77 | King’s |
| Fatima |    20 | fm21 | Clare  |
| James  |    19 | jj25 | Clare  |
+--------+-------+-------+---------+
```

Put logically independent data in distinct tables?

Students :

name	age	sid	cid
Eva	18	ev77	k
Fatima	20	fm21	cl
James	19	jj25	cl

Colleges :

cid	college_name
k	King's
cl	Clare
sid	Sidney Sussex
q	Queens'
...	.....

But how do we put them back together again?

The main themes of these lectures

- We will focus on databases from the perspective of an application writer.
  - We will not be looking at implementation details.
- The main question is this:
  - What criteria can we use to asses the quality of a database application?
- We will see that there is an inherent tradeoff between query response time and (concurrent) update throughput.
- Understanding this tradeoff will involve a careful analysis of the data redundancy implied by a database schema design.

Outline

Lecture 02: Relational Expressions

Outline

- Database query languages
- The Relational Algebra
- The Relational Calculi (tuple and domain)
- SQL

What is a (relational) database query language?

Input : a collection of relation instances

Output : a single relation instance

$$R_1,\ R_2,\ \cdots,\ R_k \quad \Longrightarrow \quad Q(R_1,\ R_2,\ \cdots,\ R_k)$$

How can we express Q?

In order to meet Codd's goals we want a query language that is high-level and independent of physical data representation.

There are many possibilities ...

The Relational Algebra (RA)

$$
\begin{array}{rcrr}
Q & ::= & R & \text{base relation} \\
 & | & \sigma_p(Q) & \text{selection} \\
 & | & \pi_{\mathbf{X}}(Q) & \text{projection} \\
 & | & Q \times Q & \text{product} \\
 & | & Q - Q & \text{difference} \\
 & | & Q \cup Q & \text{union} \\
 & | & Q \cap Q & \text{intersection} \\
 & | & \rho_M(Q) & \text{renaming}
\end{array}
$$

- p is a simple boolean predicate over attributes values.
- $\mathbf{X} = \{A_1,\ A_2,\ \ldots,\ A_k\}$ is a set of attributes.
- $M = \{A_1 \mapsto B_1,\ \ A_2 \mapsto B_2,\ \ldots,\ A_k \mapsto B_k\}$ is a renaming map.

Relational Calculi

The Tuple Relational Calculus (TRC)

$$Q = \{t \mid P(t)\}$$

The Domain Relational Calculus (DRC)

$$Q = \{(A_1 = v_1,\ A_2 = v_2, \ldots, A_k = v_k) \mid P(v_1,\ v_2, \cdots,\ v_k)\}$$

The SQL standard

- Origins at IBM in early 1970's.
- SQL has grown and grown through many rounds of standardization :
  - ANSI: SQL-86
  - ANSI and ISO : SQL-89, SQL-92, SQL:1999, SQL:2003, SQL:2006, SQL:2008
- SQL is made up of many sub-languages :
  - Query Language
  - Data Definition Language
  - System Administration Language
  - ...

Selection

R

A	B	C	D
20	10	0	55
11	10	0	7
4	99	17	2
77	25	4	0

$\Longrightarrow$

$Q(R)$

A	B	C	D
20	10	0	55
77	25	4	0

RA $Q = \sigma_{A>12}(R)$

TRC $Q = \{t \mid t \in R \land t.A > 12\}$

DRC $Q = \{\{(A,\ a),\ (B,\ b),\ (C,\ c), (D,\ d)\} \mid$
$\{(A,\ a),\ (B,\ b),\ (C,\ c), (D,\ d)\} \in R \land a > 12\}$

SQL `select * from R where R.A > 12`

Projection

R

A	B	C	D
20	10	0	55
11	10	0	7
4	99	17	2
77	25	4	0

$\Longrightarrow$

$Q(R)$

B	C
10	0
99	17
25	4

RA $Q = \pi_{B,C}(R)$

TRC $Q = \{t \mid \exists u \in R \wedge t.[B,C] = u.[B,C]\}$

DRC $Q = \{\{(B,\ b),\ (C,c)\} \mid$
$\exists\{(A,\ a),\ (B,\ b),\ (C,c),\ (D,\ d)\} \in R\}$

SQL `select distinct B, C from R`

Why the `distinct` in the SQL?

The SQL query

```
select B, C from R
```

will produce a bag (multiset)!

R

A	B	C	D
20	10	0	55
11	10	0	7
4	99	17	2
77	25	4	0

$\Longrightarrow$

$Q(R)$

B	C	
10	0	$\star\star\star$
10	0	$\star\star\star$
99	17	
25	4	

SQL is actually based on multisets, not sets. We will look into this more in Lecture 09.

Renaming

R					$Q(R)$			
A	B	C	D		A	E	C	F
20	10	0	55	$\Longrightarrow$	20	10	0	55
11	10	0	7		11	10	0	7
4	99	17	2		4	99	17	2
77	25	4	0		77	25	4	0

RA $Q = \rho_{\{B \mapsto E,\ D \mapsto F\}}(R)$

TRC $Q = \{t \mid \exists u \in R \wedge t.A = u.A \wedge t.E = u.E \wedge t.C = u.C \wedge t.F = u.D\}$

DRC $Q = \{\{(A,\ a),\ (E,\ b),\ (C,\ c), (F,\ d)\} \mid \exists\{(A,\ a),\ (B,\ b),\ (C,\ c), (D,\ d)\} \in R\}$

SQL `select A, B as E, C, D as F from R`

Product

R

A	B
20	10
11	10
4	99

S

C	D
14	99
77	100

$\Longrightarrow$

$Q(R,\ S)$

A	B	C	D
20	10	14	99
20	10	77	100
11	10	14	99
11	10	77	100
4	99	14	99
4	99	77	100

Note the automatic flattening

RA $Q = R \times S$

TRC $Q = \{t \mid \exists u \in R, v \in S,\ t.[A, B] = u.[A, B] \wedge t.[C, D] = v.[C, D]\}$

DRC $Q = \{\{(A,\ a),\ (B,\ b),\ (C,\ c), (D,\ d)\} \mid \{(A,\ a),\ (B,\ b)\} \in R \wedge \{(C,\ c), (D,\ d)\} \in S\}$

SQL `select A, B, C, D from R, S`

Union

R

A	B
20	10
11	10
4	99

S

A	B
20	10
77	1000

$\Longrightarrow$

$Q(R, S)$

A	B
20	10
11	10
4	99
77	1000

RA $Q = R \cup S$

TRC $Q = \{t \mid t \in R \vee t \in S\}$

DRC $Q = \{\{(A, a), (B, b)\} \mid \{(A, a), (B, b)\} \in R \vee \{(A, a), (B, b)\} \in S\}$

SQL `(select * from R) union (select * from S)`

Intersection

R

A	B
20	10
11	10
4	99

S

A	B
20	10
77	1000

$\Longrightarrow$

$Q(R)$

A	B
20	10

RA $Q = R \cap S$

TRC $Q = \{t \mid t \in R \wedge t \in S\}$

DRC $Q = \{\{(A,\ a),\ (B,\ b)\} \mid \{(A,\ a),\ (B,\ b)\} \in R \wedge \{(A,\ a), (B,\ b)\} \in S\}$

SQL

```
(select * from R) intersect (select * from S)
```

Difference

R

A	B
20	10
11	10
4	99

S

A	B
20	10
77	1000

$\Longrightarrow$

$Q(R)$

A	B
11	10
4	99

RA $Q = R - S$

TRC $Q = \{t \mid t \in R \wedge t \notin S\}$

DRC $Q = \{\{(A,\ a),\ (B,\ b)\} \mid \{(A,\ a),\ (B,\ b)\} \in R \wedge \{(A,\ a), (B,\ b)\} \notin S\}$

SQL `(select * from R) except (select * from S)`

Query Safety

A query like $Q = \{t \mid t \in R \wedge t \notin S\}$ raises some interesting questions. Should we allow the following query?

$$Q = \{t \mid t \notin S\}$$

We want our relations to be finite!

Safety

A (TRC) query

$$Q = \{t \mid P(t)\}$$

is safe if it is always finite for any database instance.

- Problem : query safety is not decidable!
- Solution : define a restricted syntax that guarantees safety.

Safe queries can be represented in the Relational Algebra.

Division

Given $R(\mathbf{X},\ \mathbf{Y})$ and $S(\mathbf{Y})$, the division of R by S, denoted $R \div S$, is the relation over attributes $\mathbf{X}$ defined as (in the TRC)

$$R \div S \equiv \{x \mid \forall s \in S,\ x \cup s \in R\}.$$

name	award
Fatima	writing
Fatima	music
Eva	music
Eva	writing
Eva	dance
James	dance

$\div$

award
music
writing
dance

$=$

name
Eva

Division in the Relational Algebra?

Clearly, $R \div S \subseteq \pi_{\mathbf{X}}(R)$. So $R \div S = \pi_{\mathbf{X}}(R) - C$, where C represents counter examples to the division condition. That is, in the TRC,

$$C = \{x \mid \exists s \in S,\ x \cup s \notin R\}.$$

- $U = \pi_{\mathbf{X}}(R) \times S$ represents all possible $x \cup s$ for $x \in \mathbf{X}(R)$ and $s \in S$,
- so $T = U - R$ represents all those $x \cup s$ that are not in R,
- so $C = \pi_{\mathbf{X}}(T)$ represents those records x that are counter examples.

Division in RA

$$R \div S \equiv \pi_{\mathbf{X}}(R) - \pi_{\mathbf{X}}((\pi_{\mathbf{X}}(R) \times S) - R)$$

Limitations of simple relational query languages

- The expressive power of RA, TRC, and DRC are essentially the same.
  - None can express the transitive closure of a relation.
- We could extend RA to a more powerful languages (like Datalog).
- SQL has been extended with many features beyond the Relational Algebra.
  - stored procedures
  - recursive queries
  - ability to embed SQL in standard procedural languages

Outline

Lecture 03:

Outline

- Joining Tables
- Foreign Keys
- What is `NULL` in SQL?
  - The need for three-valued logic (3VL).
- Views

Product is special!

R

A	B
20	10
4	99

$\Longrightarrow$

$R \times \rho_{A \mapsto C,\ B \mapsto D}(R)$

A	B	C	D
20	10	20	10
20	10	4	99
4	99	20	10
4	99	4	99

- $\times$ is the only operation in the Relational Algebra that created new records (ignoring renaming),
- But $\times$ usually creates too many records!
- Joins are the typical way of using products in a constrained manner.

First, a wee bit of notation

Let $\mathbf{X}$ be a set of k attribute names.

- We will often ignore domains (types) and say that $R(\mathbf{X})$ denotes a relational schema.
- When we write $R(\mathbf{Z},\ \mathbf{Y})$ we mean $R(\mathbf{Z} \cup \mathbf{Y})$ and $\mathbf{Z} \cap \mathbf{Y} = \phi$.
- $u.[\mathbf{X}] = v.[\mathbf{X}]$ abbreviates $u.A_1 = v.A_1 \wedge \cdots \wedge u.A_k = v.A_k$.
- $\vec{\mathbf{X}}$ represents some (unspecified) ordering of the attribute names, $A_1,\ A_2,\ \ldots,\ A_k$
- If $\vec{\mathbf{W}} = B_1,\ B_2,\ \ldots,\ B_k$, then $\mathbf{X} \mapsto \mathbf{W}$ abbreviates $A_1 \mapsto B_1,\ \cdots A_k \mapsto B_k$.

Equi-join

Equi-Join

Given $R(\mathbf{X},\ \mathbf{Y})$ and $S(\mathbf{Y},\ \mathbf{Z})$, we define the equi-join, denoted $R \bowtie S$, as a relation over attributes $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ defined as

$$R \bowtie S \equiv \{t \mid \exists u \in R,\ v \in S,\ u.[\mathbf{Y}] = v.[\mathbf{Y}] \wedge t = u.[\mathbf{X}] \cup u.[\mathbf{Y}] \cup v.[\mathbf{Z}]\}$$

In the Relational Algebra:

$$R \bowtie S = \pi_{\mathbf{X},\mathbf{Y},\mathbf{Z}}(\sigma_{\mathbf{Y}=\mathbf{Y}'}(R \times \rho_{\vec{\mathbf{Y}} \mapsto \vec{\mathbf{Y}'}}(S)))$$

Join example

Students

name	sid	age	cid
Fatima	fm21	20	cl
Eva	ev77	18	k
James	jj25	19	cl

Colleges

cid	cname
k	King's
cl	Clare
q	Queens'
⋮	⋮

$$\pi_{\textbf{name},\textbf{cname}}(\text{Students} \bowtie \text{Colleges})$$

$\Longrightarrow$

name	cname
Fatima	Clare
Eva	King's
James	Clare

The same in SQL

```
select name, cname
from Students, Colleges
where Students.cid = Colleges.cid
```

```
+--------+--------+
| name   | cname  |
+--------+--------+
| Eva    | King’s |
| Fatima | Clare  |
| James  | Clare  |
+--------+--------+
```

Keys, again

Relational Key

Suppose $R(\mathbf{X})$ is a relational schema with $\mathbf{Z} \subseteq \mathbf{X}$. If for any records u and v in any instance of R we have

$$u.[\mathbf{Z}] = v.[\mathbf{Z}] \Longrightarrow u.[\mathbf{X}] = v.[\mathbf{X}],$$

then $\mathbf{Z}$ is a superkey for R. If no proper subset of $\mathbf{Z}$ is a superkey, then $\mathbf{Z}$ is a key for R. We write $R(\underline{\mathbf{Z}},\ \mathbf{Y})$ to indicate that $\mathbf{Z}$ is a key for $R(\mathbf{Z} \cup \mathbf{Y})$.

Note that this is a semantic assertion, and that a relation can have multiple keys.

Foreign Keys and Referential Integrity

Foreign Key

Suppose we have $R(\underline{\mathbf{Z}},\ \mathbf{Y})$. Furthermore, let $S(\mathbf{W})$ be a relational schema with $\mathbf{Z} \subseteq \mathbf{W}$. We say that $\mathbf{Z}$ represents a Foreign Key in S for R if for any instance we have $\pi_{\mathbf{Z}}(S) \subseteq \pi_{\mathbf{Z}}(R)$. This is a semantic assertion.

Referential integrity

A database is said to have referential integrity when all foreign key constraints are satisfied.

Foreign Keys in SQL

```
create table Colleges
(   cid varchar(3) not NULL,
    cname varchar(50) not NULL,
    primary key (cid)    )

create table Students
(   sid varchar(10) not NULL,
    name varchar(50) not NULL,
    age int,
    cid varchar(3) not NULL,
    primary key (sid),
    constraint student_college
          foreign key (cid)
          references Colleges(cid)   )
```

An Example : Whatsamatta U

The entities of Whatsamatta U :

Person

name	pid	email
Fatima	fm21	ft@happy.com
Eva	ev77	eva@funny.com
James	jj25	jj@sad.com
Tim	tgg22	tgg@glad.com

College

cid	cname
k	King's
cl	Clare
q	Queens'
⋮	⋮

Course

csid	course_name	part
a1	Algorithms I	IA
a2	Algorithms II	IB
db	databases	IB
ds	Denotational Semantics	II

Term

tid	term_name
lt	Lent
ms	Michaelmas
er	Easter

An Example : Whatsamatta U

The relationships (more about this in Lecture 11) of Whatsamatta U :

InCollege

pid	**cid**
fm21	cl
ev77	k
ev77	q
jj25	cl
tgg22	k

Attends

pid	**csid**
ev77	a2
ev77	db
jj25	a1

OfferedIn

csid	**tid**
a1	er
a2	ms
db	lt
ds	ms

Lectures

csid	**pid**
a1	fm21
a2	fm21
a2	tgg22
db	tgg22

Example query

Query

All records of **name** and **term_name** associated with each lecturer and the terms in which they are lecturing.

$$\pi_{\textbf{name},\textbf{term\_name}}(\text{Person} \bowtie \text{Lectures} \bowtie \text{Course} \bowtie \text{OfferedIn} \bowtie \text{Term})$$

name	**term_name**
Fatima	Michaelmas
Fatima	Easter
Tim	Lent
Tim	Michaelmas

What is `NULL` in SQL?

What if you don't know Kim's age?

```
mysql> select * from students;
   +------+--------+------+
   | sid  | name   | age  |
   +------+--------+------+
   | ev77 | Eva    |   18 |
   | fm21 | Fatima |   20 |
   | jj25 | James  |   19 |
   | ks87 | Kim    | NULL |
   +------+--------+------+
```

What is NULL?

- NULL is a place-holder, not a value!
- NULL is not a member of any domain (type),
- For records with NULL for **age**, an expression like `age > 20` must unknown!
- This means we need (at least) three-valued logic.

Let $\perp$ represent **We don't know!**

$\wedge$	**T**	**F**	$\perp$
T	**T**	**F**	$\perp$
F	**F**	**F**	**F**
$\perp$	$\perp$	**F**	$\perp$

$\vee$	**T**	**F**	$\perp$
T	**T**	**T**	**T**
F	**T**	**F**	$\perp$
$\perp$	**T**	$\perp$	$\perp$

v	$\neg v$
T	**F**
F	**T**
$\perp$	$\perp$

NULL can lead to unexpected results

```
mysql> select * from students;
+------+--------+------+
| sid  | name   | age  |
+------+--------+------+
| ev77 | Eva    |   18 |
| fm21 | Fatima |   20 |
| jj25 | James  |   19 |
| ks87 | Kim    | NULL |
+------+--------+------+

mysql> select * from students where age <> 19;
+------+--------+------+
| sid  | name   | age  |
+------+--------+------+
| ev77 | Eva    |   18 |
| fm21 | Fatima |   20 |
+------+--------+------+
```

The ambiguity of `NULL`

Possible interpretations of `NULL`

- There is a value, but we don't know what it is.
- No value is applicable.
- The value is known, but you are not allowed to see it.
- ...

A great deal of semantic muddle is created by conflating all of these interpretations into one non-value.

On the other hand, introducing distinct NULLs for each possible interpretation leads to very complex logics ...

Not everyone approves of `NULL`

C. J. Date [D2004], Chapter 19

“Before we go any further, we should make it very clear that in our opinion (and in that of many other writers too, we hasten to add), NULLs and 3VL are and always were a serious mistake and have no place in the relational model.”

age is not a good attribute ...

The **age** column is guaranteed to go out of date! Let's record dates of birth instead!

```
create table Students
     (  sid varchar(10) not NULL,
        name varchar(50) not NULL,
        birth_date  date,
        cid varchar(3) not NULL,
        primary key (sid),
        constraint student_college foreign key (cid)
        references Colleges(cid)   )
```

age is not a good attribute ...

```
mysql> select * from Students;
+------+----------+------------+-----+
| sid  | name     | birth_date | cid |
+------+----------+------------+-----+
| ev77 | Eva      | 1990-01-26 | k   |
| fm21 | Fatima   | 1988-07-20 | cl  |
| jj25 | James    | 1989-03-14 | cl  |
+------+----------+------------+-----+
```

Use a view to recover original table

(Note : the age calculation here is not correct!)

```
create view StudentsWithAge as
  select sid, name,
   (year(current_date()) - year(birth_date)) as age,
   cid
  from Students;
```

```
mysql> select * from StudentsWithAge;
+------+----------+------+-----+
| sid  | name     | age  | cid |
+------+----------+------+-----+
| ev77 | Eva      |   19 | k   |
| fm21 | Fatima   |   21 | cl  |
| jj25 | James    |   20 | cl  |
+------+----------+------+-----+
```

Views are simply identifiers that represent a query. The view's name

Contest!! Prizes!! Fame!!

Clearly the calculation of age does not take into account the day and month of year. Two prizes will be awarded in lecture for

SQL Contest

- the cleanest correct solution using standard SQL (no vendor-specific hacks),
- the most obfuscated (yet still correct) solution

Outline

Lecture 05: Functional Dependencies

Outline

- Update anomalies
- Functional Dependencies (FDs)
- Normal Forms, 1NF, 2NF, 3NF, and BCNF

Transactions from an application perspective

Main issues

- Avoid update anomalies
- Minimize locking to improve transaction throughput.
- Maintain integrity constraints.

These issues are related.

Update anomalies

Big Table

sid	name	college	course	part	term_name
yy88	Yoni	New Hall	Algorithms I	IA	Easter
uu99	Uri	King's	Algorithms I	IA	Easter
bb44	Bin	New Hall	Databases	IB	Lent
bb44	Bin	New Hall	Algorithms II	IB	Michaelmas
zz70	Zip	Trinity	Databases	IB	Lent
zz70	Zip	Trinity	Algorithms II	IB	Michaelmas

- How can we tell if an insert record is consistent with current records?
- Can we record data about a course before students enroll?
- Will we wipe out information about a college when last student associated with the college is deleted?

Redundancy implies more locking ...

... at least for correct transactions!

Big Table

sid	name	college	course	part	term_name
yy88	Yoni	New Hall	Algorithms I	IA	Easter
uu99	Uri	King's	Algorithms I	IA	Easter
bb44	Bin	New Hall	Databases	IB	Lent
bb44	Bin	New Hall	Algorithms II	IB	Michaelmas
zz70	Zip	Trinity	Databases	IB	Lent
zz70	Zip	Trinity	Algorithms II	IB	Michaelmas

- Change New Hall to Murray Edwards College
  - Conceptually simple update
  - May require locking entire table.

Redundancy is the root of (almost) all database evils

- It may not be obvious, but redundancy is also the cause of update anomalies.
- By redundancy we do not mean that some values occur many times in the database!
  - A foreign key value may be have millions of copies!
- But then, what do we mean?

Functional Dependency

Functional Dependency (FD)

Let $R(\mathbf{X})$ be a relational schema and $\mathbf{Y} \subseteq \mathbf{X}$, $\mathbf{Z} \subseteq \mathbf{X}$ be two attribute sets. We say $\mathbf{Y}$ functionally determines $\mathbf{Z}$, written $\mathbf{Y} \rightarrow \mathbf{Z}$, if for any two tuples u and v in an instance of $R(\mathbf{X})$ we have

$$u.\mathbf{Y} = v.\mathbf{Y} \rightarrow u.\mathbf{Z} = v.\mathbf{Z}.$$

We call $\mathbf{Y} \rightarrow \mathbf{Z}$ a functional dependency.

A functional dependency is a semantic assertion. It represents a rule that should always hold in any instance of schema $R(\mathbf{X})$.

Example FDs

Big Table

sid	name	college	course	part	term_name
yy88	Yoni	New Hall	Algorithms I	IA	Easter
uu99	Uri	King's	Algorithms I	IA	Easter
bb44	Bin	New Hall	Databases	IB	Lent
bb44	Bin	New Hall	Algorithms II	IB	Michaelmas
zz70	Zip	Trinity	Databases	IB	Lent
zz70	Zip	Trinity	Algorithms II	IB	Michaelmas

- **sid** $\rightarrow$ **name**
- **sid** $\rightarrow$ **college**
- **course** $\rightarrow$ **part**
- **course** $\rightarrow$ **term_name**

Keys, revisited

Candidate Key

Let $R(\mathbf{X})$ be a relational schema and $\mathbf{Y} \subseteq \mathbf{X}$. $\mathbf{Y}$ is a candidate key if

1. The FD $\mathbf{Y} \rightarrow \mathbf{X}$ holds, and
2. for no proper subset $\mathbf{Z} \subset \mathbf{Y}$ does $\mathbf{Z} \rightarrow \mathbf{X}$ hold.

Prime and Non-prime attributes

An attribute A is prime for $R(\mathbf{X})$ if it is a member of some candidate key for R. Otherwise, A is non-prime.

Database redundancy roughly means the existence of non-key functional dependencies!

First Normal Form (1NF)

We will assume every schema is in 1NF.

1NF

A schema $R(A_1 : S_1,\ A_2 : S_2,\ \cdots,\ A_n : S_n)$ is in First Normal Form (1NF) if the domains S_1 are elementary — their values are atomic.

name
Timothy George Griffin

$\Longrightarrow$

first_name	**middle_name**	**last_name**
Timothy	George	Griffin

Second Normal Form (2NF)

Second Normal Form (2CNF)

A relational schema R is in 2NF if for every functional dependency $\mathbf{X} \rightarrow A$ either

- $A \in \mathbf{X}$, or
- $\mathbf{X}$ is a superkey for R, or
- A is a member of some key, or
- $\mathbf{X}$ is not a proper subset of any key.

3NF and BCNF

Third Normal Form (3CNF)

A relational schema R is in 3NF if for every functional dependency $\mathbf{X} \rightarrow A$ either

- $A \in \mathbf{X}$, or
- $\mathbf{X}$ is a superkey for R, or
- A is a member of some key.

Boyce-Codd Normal Form (BCNF)

A relational schema R is in BCNF if for every functional dependency $\mathbf{X} \rightarrow A$ either

- $A \in \mathbf{X}$, or
- $\mathbf{X}$ is a superkey for R.

Inclusions

Clearly BCNF $\subseteq$ 3NF $\subseteq 2NF$. These are proper inclusions:

In 2NF, but not 3NF

$R(A,\ B,\ C)$, with $F = \{A \rightarrow B,\ B \rightarrow C\}$.

In 3NF, but not BCNF

$R(A,\ B,\ C)$, with $F = \{A, B \rightarrow C,\ C \rightarrow B\}$.

- This is in 3NF since AB and AC are keys, so there are no non-prime attributes
- But not in BCNF since C is not a key and we have $C \rightarrow B$.

The Plan

Given a relational schema $R(\mathbf{X})$ with FDs F :

- Reason about FDs
  - Is F missing FDs that are logically implied by those in F?
- Decompose each $R(\mathbf{X})$ into smaller $R_1(\mathbf{X}_1)$, $R_2(\mathbf{X}_2)$, $\cdots R_k(\mathbf{X}_k)$, where each $R_i(\mathbf{X}_i)$ is in the desired Normal Form.

Are some decompositions better than others?

Desired properties of any decomposition

Lossless-join decomposition

A decomposition of schema $R(\mathbf{X})$ to $S(\mathbf{Y} \cup \mathbf{Z})$ and $T(\mathbf{Y} \cup (\mathbf{X} - \mathbf{Z}))$ is a lossless-join decomposition if for every database instances we have $R = S \bowtie T$.

Dependency preserving decomposition

A decomposition of schema $R(\mathbf{X})$ to $S(\mathbf{Y} \cup \mathbf{Z})$ and $T(\mathbf{Y} \cup (\mathbf{X} - \mathbf{Z}))$ is dependency preserving, if enforcing FDs on S and T individually has the same effect as enforcing all FDs on $S \bowtie T$.

We will see that it is not always possible to achieve both of these goals.

Outline

Lecture 06: Reasoning about FDs

Outline

- Implied dependencies (closure)
- Armstrong's Axioms

Semantic Closure

Notation

$$F \models \mathbf{Y} \rightarrow \mathbf{Z}$$

means that any database instance that that satisfies every FD of F, must also satisfy $\mathbf{Y} \rightarrow \mathbf{Z}$.

The semantic closure of F, denoted F^+, is defined to be

$$F^+ = \{\mathbf{Y} \rightarrow \mathbf{Z} \mid \mathbf{Y} \cup \mathbf{Z} \subseteq \text{atts}(F) \textit{and} \wedge F \models \mathbf{Y} \rightarrow \mathbf{Z}\}.$$

The membership problem is to determine if $\mathbf{Y} \rightarrow \mathbf{Z} \in F^+$.

Reasoning about Functional Dependencies

We write $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$ when $\mathbf{Y} \rightarrow \mathbf{Z}$ can be derived from F via the following rules.

Armstrong's Axioms

Reflexivity If $\mathbf{Z} \subseteq \mathbf{Y}$, then $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$.

Augmentation If $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$ then $F \vdash \mathbf{Y}, \mathbf{W} \rightarrow \mathbf{Z}, \mathbf{W}$.

Transitivity If $F \vdash \mathbf{Y} \rightarrow \mathbf{Z}$ and $F \models \mathbf{Z} \rightarrow \mathbf{W}$, then $F \vdash \mathbf{Y} \rightarrow \mathbf{W}$.

Logical Closure (of a set of attributes)

Notation

$$\text{closure}(F,\ \mathbf{X}) = \{A \mid F \vdash \mathbf{X} \rightarrow A\}$$

Claim 1

If $\mathbf{Y} \rightarrow \mathbf{W} \in F$ and $\mathbf{Y} \subseteq \text{closure}(F,\ \mathbf{X})$, then $\mathbf{W} \subseteq \text{closure}(F,\ \mathbf{X})$.

Claim 2

$\mathbf{Y} \rightarrow \mathbf{W} \in F^+$ if and only if $\mathbf{W} \subseteq \text{closure}(F,\ \mathbf{Y})$.

Soundness and Completeness

Soundness

$$F \vdash f \implies f \in F^+$$

Completeness

$$f \in F^+ \implies F \vdash f$$

Proof of Completeness (soundness left as an exercise)

Show $\neg(F \vdash f) \implies \neg(F \models f)$:

- Suppose $\neg(F \vdash \mathbf{Y} \rightarrow \mathbf{Z})$ for $R(\mathbf{X})$.
- Let $\mathbf{Y}^{+} = \text{closure}(F,\ \mathbf{Y})$.
- $\exists B \in \mathbf{Z}$, with $B \notin \mathbf{Y}^{+}$.
- Construct an instance of R with just two records, u and v, that agree on $\mathbf{Y}^{+}$ but not on $\mathbf{X} - \mathbf{Y}^{+}$.
- By construction, this instance does not satisfy $\mathbf{Y} \rightarrow \mathbf{Z}$.
- But it does satisfy F! Why?
  - let $\mathbf{S} \rightarrow \mathbf{T}$ be any FD in F, with $u.[\mathbf{S}] = v.[\mathbf{S}]$.
  - So $\mathbf{S} \subseteq \mathbf{Y}+$. and so $\mathbf{T} \subseteq \mathbf{Y}+$ by claim 1,
  - and so $u.[T] = v.[T]$

Consequences of Armstrong's Axioms

Union If $F \models \mathbf{Y} \to \mathbf{Z}$ and $F \models \mathbf{Y} \to \mathbf{W}$, then $F \models \mathbf{Y} \to \mathbf{W}, \mathbf{Z}$.

Pseudo-transitivity If $F \models \mathbf{Y} \to \mathbf{Z}$ and $F \models \mathbf{U}, \mathbf{Z} \to \mathbf{W}$, then $F \models \mathbf{Y}, \mathbf{U} \to \mathbf{W}$.

Decomposition If $F \models \mathbf{Y} \to \mathbf{Z}$ and $\mathbf{W} \subseteq \mathbf{Z}$, then $F \models \mathbf{Y} \to \mathbf{W}$.

Exercise : Prove these using Armstrong's axioms!

Proof of the Union Rule

Suppose we have

$$F \models \mathbf{Y} \rightarrow \mathbf{Z},$$
$$F \models \mathbf{Y} \rightarrow \mathbf{W}.$$

By augmentation we have

$$F \models \mathbf{Y}, \mathbf{Y} \rightarrow \mathbf{Y}, \mathbf{Z},$$

that is,

$$F \models \mathbf{Y} \rightarrow \mathbf{Y}, \mathbf{Z}.$$

Also using augmentation we obtain

$$F \models \mathbf{Y}, \mathbf{Z} \rightarrow \mathbf{W}, \mathbf{Z}.$$

Therefore, by transitivity we obtain

$$F \models \mathbf{Y} \rightarrow \mathbf{W}, \mathbf{Z}.$$

Example application of functional reasoning.

Heath's Rule

Suppose $R(A,\ B,\ C)$ is a relational schema with functional dependency $A \rightarrow B$, then

$$R = \pi_{A,B}(R) \bowtie_A \pi_{A,C}(R).$$

Proof of Heath's Rule

We first show that $R \subseteq \pi_{A,B}(R) \bowtie_A \pi_{A,C}(R)$.

- If $u = (a,\ b,\ c) \in R$, then $u_1 = (a,\ b) \in \pi_{A,B}(R)$ and $u_2 = (a,\ c) \in \pi_{A,C}(R)$.
- Since $\{(a,\ b)\} \bowtie_A \{(a,\ c)\} = \{(a,\ b,\ c)\}$ we know $u \in \pi_{A,B}(R) \bowtie_A \pi_{A,C}(R)$.

In the other direction we must show $R' = \pi_{A,B}(R) \bowtie_A \pi_{A,C}(R) \subseteq R$.

- If $u = (a,\ b,\ c) \in R'$, then there must exist tuples $u_1 = (a,\ b) \in \pi_{A,B}(R)$ and $u_2 = (a,\ c) \in \pi_{A,C}(R)$.
- This means that there must exist a $u' = (a,\ b',\ c) \in R$ such that $u_2 = \pi_{A,C}(\{(a,\ b',\ c)\})$.
- However, the functional dependency tells us that $b = b'$, so $u = (a,\ b,\ c) \in R$.

Closure Example

$R(A,\ B, C, D, D, F)$ with

$$\begin{aligned} A, B &\rightarrow C \\ B, C &\rightarrow D \\ D &\rightarrow E \\ C, F &\rightarrow B \end{aligned}$$

What is the closure of $\{A,\ B\}$?

$$\begin{aligned} \{A,\ B\} &\overset{A,B\rightarrow C}{\Longrightarrow} \{A,\ B,\ C\} \\ &\overset{B,C\rightarrow D}{\Longrightarrow} \{A,\ B,\ C,\ D\} \\ &\overset{D\rightarrow E}{\Longrightarrow} \{A,\ B,\ C,\ D,\ E\} \end{aligned}$$

So $\{A,\ B\}^+ = \{A,\ B,\ C,\ D,\ E\}$ and $A, B \rightarrow C, D, E$.

Outline

Lecture 07: Decomposition to Normal Forms

Outline

- Attribute closure algorithm
- Schema decomposition methods
- Problems with obtaining both dependency preservation and lossless-join property

Closure

By soundness and completeness

$$\text{closure}(F,\ \mathbf{X}) = \{A \mid F \vdash \mathbf{X} \to A\} = \{A \mid \mathbf{X} \to A \in F^+\}$$

Claim 2 (from previous lecture)

$\mathbf{Y} \to \mathbf{W} \in F^+$ if and only if $\mathbf{W} \subseteq \text{closure}(F,\ \mathbf{Y})$.

If we had an algorithm for closure$(F,\ \mathbf{X})$, then we would have a (brute force!) algorithm for enumerating F^+:

F^+

- for every subset $\mathbf{Y} \subseteq \text{atts}(F)$
  - for every subset $\mathbf{Z} \subseteq \text{closure}(F,\ \mathbf{Y})$,
    - output $\mathbf{Y} \to \mathbf{Z}$

Attribute Closure Algorithm

- Input : a set of FDs F and a set of attributes $\mathbf{X}$.
- Output : $\mathbf{Y} = \text{closure}(F,\ \mathbf{X})$

1. $\mathbf{Y} := \mathbf{X}$
2. while there is some $\mathbf{S} \rightarrow \mathbf{T} \in F$ with $\mathbf{S} \subseteq \mathbf{Y}$ and $\mathbf{T} \not\subseteq \mathbf{Y}$, then $\mathbf{Y} := \mathbf{Y} \cup \mathbf{T}$.

An Example (UW1997, Exercise 3.6.1)

$R(A, B, C, D)$ with F made up of the FDs

$$\begin{array}{l} A, B \rightarrow C \\ C \rightarrow D \\ D \rightarrow A \end{array}$$

What is F^+?

Brute force!

Let's just consider all possible nonempty sets **X** — there are only 15...

Example (cont.)

$$F = \{A, B \rightarrow C, \quad C \rightarrow D, \quad D \rightarrow A\}$$

For the single attributes we have

- $\{A\}^+ = \{A\}$,
- $\{B\}^+ = \{B\}$,
- $\{C\}^+ = \{A,\ C,\ D\}$,
  - $\{C\} \stackrel{C \rightarrow D}{\Longrightarrow} \{C,\ D\} \stackrel{D \rightarrow A}{\Longrightarrow} \{A,\ C,\ D\}$
- $\{D\}^+ = \{A,\ D\}$
  - $\{D\} \stackrel{D \rightarrow A}{\Longrightarrow} \{A,\ D\}$

The only new dependency we get with a single attribute on the left is $C \rightarrow A$.

Example (cont.)

$$F = \{A, B \rightarrow C, \quad C \rightarrow D, \quad D \rightarrow A\}$$

Now consider pairs of attributes.

- $\{A,\ B\}^+ = \{A,\ B,\ C,\ D\}$,
  - so $A, B \rightarrow D$ is a new dependency
- $\{A,\ C\}^+ = \{A,\ C,\ D\}$,
  - so $A, C \rightarrow D$ is a new dependency
- $\{A,\ D\}^+ = \{A,\ D\}$,
  - so nothing new.
- $\{B,\ C\}^+ = \{A,\ B,\ C,\ D\}$,
  - so $B, C \rightarrow A, D$ is a new dependency
- $\{B,\ D\}^+ = \{A,\ B,\ C,\ D\}$,
  - so $B, D \rightarrow A, C$ is a new dependency
- $\{C,\ D\}^+ = \{A,\ C,\ D\}$,
  - so $C, D \rightarrow A$ is a new dependency

Example (cont.)

$$F = \{A, B \to C, \;\; C \to D, \;\; D \to A\}$$

For the triples of attributes:

- $\{A, \; C, \; D\}^+ = \{A, \; C, \; D\}$,
- $\{A, \; B, \; D\}^+ = \{A, \; B, \; C, \; D\}$,
  - so $A, B, D \to C$ is a new dependency
- $\{A, \; B, \; C\}^+ = \{A, \; B, \; C, \; D\}$,
  - so $A, B, C \to D$ is a new dependency
- $\{B, \; C, \; D\}^+ = \{A, \; B, \; C, \; D\}$,
  - so $B, C, D \to A$ is a new dependency

And since $\{A, \; B, \; C, \; D\}+ = \{A, \; B, \; C, \; D\}$, we get no new dependencies with four attributes.

Example (cont.)

We generated 11 new FDs:

$$
\begin{array}{lcl}
C & \rightarrow & A \\
A, C & \rightarrow & D \\
B, C & \rightarrow & D \\
B, D & \rightarrow & C \\
A, B, C & \rightarrow & D \\
B, C, D & \rightarrow & A
\end{array}
\qquad
\begin{array}{lcl}
A, B & \rightarrow & D \\
B, C & \rightarrow & A \\
B, D & \rightarrow & A \\
C, D & \rightarrow & A \\
A, B, D & \rightarrow & C
\end{array}
$$

Can you see the Key?

$\{A,\ B\}$, $\{B,\ C\}$, and $\{B,\ D\}$ are keys.

Note: this schema is already in 3NF! Why?

General Decomposition Method (GDM)

GDM

1. Understand your FDs F (compute F^+),
2. find $R(\mathbf{X}) = R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ (sets **Z**, **W** and **Y** are disjoint) with FD $\mathbf{Z} \rightarrow \mathbf{W} \in F^+$ violating a condition of desired NF,
3. split R into two tables $R_1(\mathbf{Z}, \mathbf{W})$ and $R_2(\mathbf{Z}, \mathbf{Y})$
4. wash, rinse, repeat

Reminder

For $\mathbf{Z} \rightarrow \mathbf{W}$, if we assume $\mathbf{Z} \cap \mathbf{W} = \{\}$, then the conditions are

1. **Z** is a superkey for R (2NF, 3NF, BCNF)
2. **W** is a subset of some key (2NF, 3NF)
3. **Z** is not a proper subset of any key (2NF)

The lossless-join condition is guaranteed by GDM

- This method will produce a lossless-join decomposition because of (repeated applications of) Heath's Rule!
- That is, each time we replace an S by S_1 and S_2, we will always be able to recover S as $S_1 \bowtie S_2$.
- Note that in GDM step 3, the FD $\mathbf{Z} \rightarrow \mathbf{W}$ may represent a key constraint for R_1.

But does the method always terminate? Please think about this ....

Return to Example — Decompose to BCNF

$R(A,\ B, C, D)$

$$F = \{A, B \to C,\ \ C \to D,\ \ D \to A\}$$

Which FDs in F^+ violate BCNF?

$$\begin{array}{ccc} C & \to & A \\ C & \to & D \\ D & \to & A \\ A, C & \to & D \\ C, D & \to & A \end{array}$$

Return to Example — Decompose to BCNF

Decompose $R(A, B, C, D)$ to BCNF

Use $C \rightarrow D$ to obtain

- $R_1(C, D)$. This is in BCNF. Done.
- $R_2(A, B, C)$ This is not in BCNF. Why? A, B and B, C are the only keys, and $C \rightarrow A$ is a FD for R_1. So use $C \rightarrow A$ to obtain
  - $R_{2.1}(A, C)$. This is in BCNF. Done.
  - $R_{2.2}(B, C)$. This is in BCNF. Done.

Exercise : Try starting with any of the other BCNF violations and see where you end up.

The GDM does not always preserve dependencies!

$R(A,\ B,\ C,\ D,\ E)$

$$\begin{array}{rcl} A, B & \rightarrow & C \\ D, E & \rightarrow & C \\ B & \rightarrow & D \end{array}$$

- $\{A,\ B\}^+ = \{A,\ B,\ C,\ D\}$,
- so $A, B \rightarrow C, D$,
- and $\{A,\ B,\ E\}$ is a key.

- $\{B,\ E\}^+ = \{B,\ C,\ D,\ E\}$,
- so $B, E \rightarrow C, D$,
- and $\{A,\ B,\ E\}$ is a key (again)

Let's try for a BCNF decomposition ...

Decomposition 1

Decompose $R(A,\ B,\ C,\ D,\ E)$ using $A, B \rightarrow C, D$:

- $R_1(A,\ B,\ C,\ D)$. Decompose this using $B \rightarrow D$:
    - $R_{1.1}(B,\ D)$. Done.
    - $R_{1.2}(A,\ B,\ C)$. Done.
- $R_2(A,\ B,\ E)$. Done.

But in this decomposition, how will we enforce this dependency?

$$D, E \rightarrow C$$

Decomposition 2

Decompose $R(A,\ B,\ C,\ D,\ E)$ using $B, E \rightarrow C, D$:

- $R_3(B,\ C,\ D,\ E)$. Decompose this using $D, E \rightarrow C$
  - $R_{3.1}(C,\ D,\ E)$. Done.
  - $R_{3.2}(B,\ D,\ E)$. Decompose this using $B \rightarrow D$:
    - $R_{3.2.1}(B,\ D)$. Done.
    - $R_{3.2.2}(B,\ E)$. Done.
- $R_4(A,\ B,\ E)$. Done.

But in this decomposition, how will we enforce this dependency?

$$A, B \rightarrow C$$

Summary

- It always is possible to obtain BCNF that has the lossless-join property (using GDM)
  - But the result may not preserve all dependencies.
- It is always possible to obtain 3NF that preserves dependencies and has the lossless-join property.
  - Using methods based on "minimal covers" (for example, see EN2000).

Outline

Lecture 08: Multivalued Dependencies

Outline

- Multivalued Dependencies
- Fourth Normal Form (4NF)
- General integrity Constraints

Another look at Heath's Rule

Given $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ with FDs F

If $\mathbf{Z} \rightarrow \mathbf{W} \in F^+$, the

$$R = \pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R)$$

What about an implication in the other direction? That is, suppose we have

$$R = \pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R).$$

Q Can we conclude anything about FDs on R? In particular, is it true that $\mathbf{Z} \rightarrow \mathbf{W}$ holds?

A No!

We just need one counter example ...

$$R \quad = \quad \pi_{A,B}(R) \quad \bowtie \quad \pi_{A,C}(R)$$

A	B	C
a	b_1	c_1
a	b_2	c_2
a	b_1	c_2
a	b_2	c_1

A	B
a	b_1
a	b_2

A	C
a	c_1
a	c_2

Clearly $A \rightarrow B$ is not an FD of R.

A concrete example

course_name	lecturer	text
Databases	Tim	Ullman and Widom
Databases	Fatima	Date
Databases	Tim	Date
Databases	Fatima	Ullman and Widom

Assuming that texts and lecturers are assigned to courses independently, then a better representation would in two tables:

course_name	lecturer
Databases	Tim
Databases	Fatima

course_name	text
Databases	Ullman and Widom
Databases	Date

Time for a definition!

Multivalued Dependencies (MVDs)

Let $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ be a relational schema. A multivalued dependency, denoted $\mathbf{Z} \twoheadrightarrow \mathbf{W}$, holds if whenever t and u are two records that agree on the attributes of $\mathbf{Z}$, then there must be some tuple v such that

1. v agrees with both t and u on the attributes of $\mathbf{Z}$,
2. v agrees with t on the attributes of $\mathbf{W}$,
3. v agrees with u on the attributes of $\mathbf{Y}$.

A few observations

Note 1

Every functional dependency is multivalued dependency,

$$(\mathbf{Z} \rightarrow \mathbf{W}) \implies (\mathbf{Z} \twoheadrightarrow \mathbf{W}).$$

To see this, just let $v = u$ in the above definition.

Note 2

Let $R(\mathbf{Z},\ \mathbf{W},\ \mathbf{Y})$ be a relational schema, then

$$(\mathbf{Z} \twoheadrightarrow \mathbf{W}) \iff (\mathbf{Z} \twoheadrightarrow \mathbf{Y}),$$

by symmetry of the definition.

MVDs and lossless-join decompositions

Fun Fun Fact

Let $R(\mathbf{Z},\ \mathbf{W},\ \mathbf{Y})$ be a relational schema. The decomposition $R_1(\mathbf{Z},\ \mathbf{W})$, $R_2(\mathbf{Z},\ \mathbf{Y})$ is a lossless-join decomposition of R if and only if the MVD $\mathbf{Z} \twoheadrightarrow \mathbf{W}$ holds.

Proof of Fun Fun Fact

Proof of $(\mathbf{Z} \twoheadrightarrow \mathbf{W}) \implies R = \pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R)$

- Suppose $\mathbf{Z} \twoheadrightarrow \mathbf{W}$.
- We know (from proof of Heath's rule) that $R \subseteq \pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R)$. So we only need to show $\pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R) \subseteq R$.
- Suppose $r \in \pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R)$.
- So there must be a $t \in R$ and $u \in R$ with $\{r\} = \pi_{\mathbf{Z},\mathbf{W}}(\{t\}) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(\{u\})$.
- In other words, there must be a $t \in R$ and $u \in R$ with $t.\mathbf{Z} = u.\mathbf{Z}$.
- So the MVD tells us that then there must be some tuple $v \in R$ such that
  1. v agrees with both t and u on the attributes of $\mathbf{Z}$,
  2. v agrees with t on the attributes of $\mathbf{W}$,
  3. v agrees with u on the attributes of $\mathbf{Y}$.
- This v must be the same as r, so $r \in R$.

Proof of Fun Fun Fact (cont.)

Proof of $R = \pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R) \implies (\mathbf{Z} \twoheadrightarrow \mathbf{W})$

- Suppose $R = \pi_{\mathbf{Z},\mathbf{W}}(R) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(R)$.
- Let t and u be any records in R with $t.\mathbf{Z} = u.\mathbf{Z}$.
- Let v be defined by $\{v\} = \pi_{\mathbf{Z},\mathbf{W}}(\{t\}) \bowtie \pi_{\mathbf{Z},\mathbf{Y}}(\{u\})$ (and we know $v \in R$ by the assumption).
- Note that by construction we have
  1. $v.\mathbf{Z} = t.\mathbf{Z} = u.\mathbf{Z}$,
  2. $v.\mathbf{W} = t.\mathbf{W}$,
  3. $v.\mathbf{Y} = u.\mathbf{Y}$.
- Therefore, $\mathbf{Z} \twoheadrightarrow \mathbf{W}$ holds.

Fourth Normal Form

Trivial MVD

The MVD $\mathbf{Z} \twoheadrightarrow \mathbf{W}$ is trivial for relational schema $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ if

1. $\mathbf{Z} \cap \mathbf{W} \neq \{\}$, or
2. $\mathbf{Y} = \{\}$.

4NF

A relational schema $R(\mathbf{Z}, \mathbf{W}, \mathbf{Y})$ is in 4NF if for every MVD $\mathbf{Z} \twoheadrightarrow \mathbf{W}$ either

- $\mathbf{Z} \twoheadrightarrow \mathbf{W}$ is a trivial MVD, or
- $\mathbf{Z}$ is a superkey for R.

Note : 4NF $\subset$ BCNF $\subset$ 3NF $\subset$ 2NF

General Decomposition Method Revisited

GDM++

1. Understand your FDs and MVDs F (compute F^+),
2. find $R(\mathbf{X}) = R(\mathbf{Z},\ \mathbf{W},\ \mathbf{Y})$ (sets $\mathbf{Z}$, $\mathbf{W}$ and $\mathbf{Y}$ are disjoint) with either FD $\mathbf{Z} \rightarrow \mathbf{W} \in F^+$ or MVD $\mathbf{Z} \twoheadrightarrow \mathbf{W} \in F^+$ violating a condition of desired NF,
3. split R into two tables $R_1(\mathbf{Z},\ \mathbf{W})$ and $R_2(\mathbf{Z},\ \mathbf{Y})$
4. wash, rinse, repeat

Summary

We always want the lossless-join property. What are our options?

	3NF	BCNF	4NF
Preserves FDs	Yes	Maybe	Maybe
Preserves MVDs	Maybe	Maybe	Maybe
Eliminates FD-redundancy	Maybe	Yes	Yes
Eliminates MVD-redundancy	No	No	Yes

General integrity constraints

- Suppose that C is some constraint we would like to enforce on our database.
- Let $Q_{\neg C}$ be a query that captures all violations of C.
- Enforce (somehow) that the assertion that is always $Q_{\neg C}$ empty.

Example

- $C = \mathbf{Z} \rightarrow \mathbf{W}$, and FD that was not preserved for relation $R(\mathbf{X})$,
- Let Q_R be a join that reconstructs R,
- Let Q'_R be this query with $\mathbf{X} \mapsto \mathbf{X}'$ and
- $Q_{\neg C} = \sigma_{\mathbf{W} \neq \mathbf{W}'}(\sigma_{\mathbf{Z} = \mathbf{Z}'}(Q_R \times Q'_R))$

Assertions in SQL

```
create view C_violations as ....

create assertion check_C
       check not (exists C_violations)
```

Outline

Lecture 04: Database Updates

Outline

- Transactions
- Short review of ACID requirements

Transactions — ACID properties

Should be review from Concurrent Systems and Applications

Atomicity
: Either all actions are carried out, or none are
  - logs needed to undo operations, if needed

Consistency
: If each transaction is consistent, and the database is initially consistent, then it is left consistent
  - This is very much a part of applications design.

Isolation
: Transactions are isolated, or protected, from the effects of other scheduled transactions
  - Serializability, 2-phase commit protocol

Durability
: If a transactions completes successfully, then its effects persist
  - Logging and crash recovery

Lecture 09 and 10

Two Themes ...

- Redundancy can be a GOOD thing!
- Duplicates, aggregates, and group by in SQL, and evolution to "Data Cube"

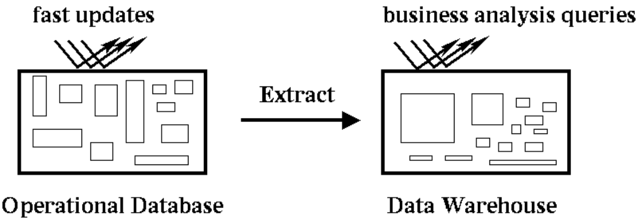
.... come together in OLAP

- OLTP : Online Transaction Processing (traditional databases)
  - Data is normalized for the sake of updates.
- OLAP : Online Analytic Processing
  - These are (almost) read-only databases.
  - Data is de-normalized for the sake of queries!
  - Multi-dimensional data cube emerging as common data model.
    - This can be seen as a generalization of SQL's group by

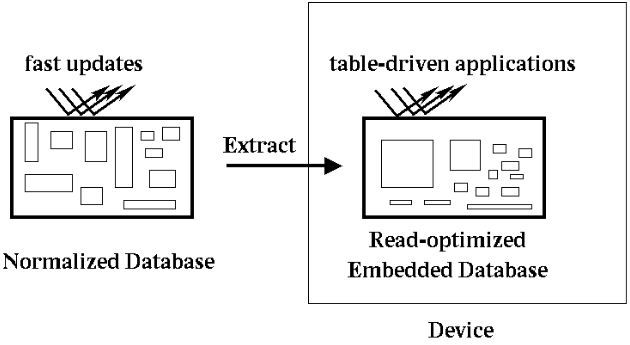
Materialized Views

- Suppose Q is a very expensive, and very frequent query.
- Why not de-normalize some data to speed up the evaluation of Q?
  - This might be a reasonable thing to do, or ...
  - ... it might be the first step to destroying the integrity of your data design.
- Why not store the value of Q in a table?
  - This is called a materialized view.
  - But now there is a problem: How often should this view be refreshed?

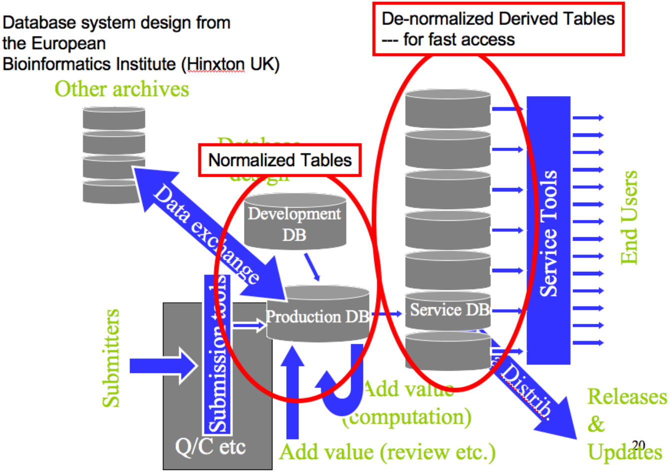
FIDO = Fetch Intensive Data Organization



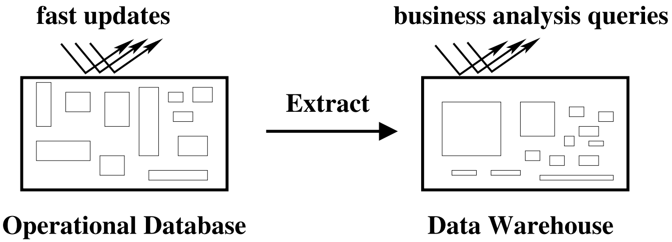
Example : Embedded databases



Example : Hinxton Bioinformatics



Example : Data Warehouse (Decision support)



OLAP vs. OLTP

OLTP Online Transaction Processing

OLAP Online Analytical Processing

- Commonly associated with terms like Decision Support, Data Warehousing, etc.

	OLAP	OLTP
Supports	analysis	day-to-day operations
Data is	historical	current
Transactions mostly	reads	updates
optimized for	query processing	updates
Normal Forms	not important	important

OLAP Databases : Data Models and Design

The big question

Is the relational model and its associated query language (SQL) well suited for OLAP databases?

- Aggregation (sums, averages, totals, ...) are very common in OLAP queries
  - Problem : SQL aggregation quickly runs out of steam.
  - Solution : Data Cube and associated operations (spreadsheets on steroids)
- Relational design is obsessed with normalization
  - Problem : Need to organize data well since all analysis queries cannot be anticipated in advance.
  - Solution : Multi-dimensional fact tables, with hierarchy in dimensions, star-schema design.

Let's start by looking at aggregate queries in SQL ...

An Example ...

```
mysql> select * from marks;
   +-------+-----------+------+
   | sid   | course    | mark |
   +-------+-----------+------+
   | ev77  | databases |   92 |
   | ev77  | spelling  |   99 |
   | tgg22 | spelling  |    3 |
   | tgg22 | databases |  100 |
   | fm21  | databases |   92 |
   | fm21  | spelling  |  100 |
   | jj25  | databases |   88 |
   | jj25  | spelling  |   92 |
   +-------+-----------+------+
```

... of duplicates

```
mysql> select mark from marks;
+-------+
| mark  |
+-------+
|    92 |
|    99 |
|     3 |
|   100 |
|    92 |
|   100 |
|    88 |
|    92 |
+-------+
```

Why Multisets?

Duplicates are important for aggregate functions.

```
mysql> select min(mark),
              max(mark),
              sum(mark),
              avg(mark)
       from marks;
+-----------+-----------+-----------+-----------+
| min(mark) | max(mark) | sum(mark) | avg(mark) |
+-----------+-----------+-----------+-----------+
|         3 |       100 |       666 |   83.2500 |
+-----------+-----------+-----------+-----------+
```

The group by clause

```
mysql> select course,
              min(mark),
              max(mark),
              avg(mark)
       from marks
       group by course;
+-----------+-----------+-----------+-----------+
| course    | min(mark) | max(mark) | avg(mark) |
+-----------+-----------+-----------+-----------+
| databases |        88 |       100 |   93.0000 |
| spelling  |         3 |       100 |   73.5000 |
+-----------+-----------+-----------+-----------+
```

Visualizing group by

sid	course	mark
ev77	databases	92
ev77	spelling	99
tgg22	spelling	3
tgg22	databases	100
fm21	databases	92
fm21	spelling	100
jj25	databases	88
jj25	spelling	92

$\stackrel{\texttt{group by}}{\Longrightarrow}$

course	mark
spelling	99
spelling	3
spelling	100
spelling	92

course	mark
databases	92
databases	100
databases	92
databases	88

Visualizing group by

course	mark
spelling	99
spelling	3
spelling	100
spelling	92

course	mark
databases	92
databases	100
databases	92
databases	88

$\xrightarrow{\text{min}(\textbf{mark})}$

course	min(**mark**)
spelling	3
databases	88

The `having` clause

How can we select on the aggregated columns?

```
mysql> select course,
              min(mark),
              max(mark),
              avg(mark)
       from marks
       group by course
       having min(mark) > 60;
+-----------+-----------+-----------+-----------+
| course    | min(mark) | max(mark) | avg(mark) |
+-----------+-----------+-----------+-----------+
| databases |        88 |       100 |   93.0000 |
+-----------+-----------+-----------+-----------+
```

Use renaming to make things nicer ...

```
mysql> select course,
              min(mark) as minimum,
              max(mark) as maximum,
              avg(mark) as average
       from marks
       group by course
       having minimum > 60;
+-----------+---------+---------+---------+
| course    | minimum | maximum | average |
+-----------+---------+---------+---------+
| databases |      88 |     100 | 93.0000 |
+-----------+---------+---------+---------+
```

Limits of SQL aggregation

sale	prodId	storeId	amt
	p1	c1	12
	p2	c1	11
	p1	c3	50
	p2	c2	8

⟷

	c1	c2	c3
p1	12		50
p2	11	8	

- Flat tables are great for processing, but hard for people to read and understand.
- Pivot tables and cross tabulations (spreadsheet terminology) are very useful for presenting data in ways that people can understand.
- SQL does not handle pivot tables and cross tabulations well.

A very influential paper [G+1997]

Data Mining and Knowledge Discovery 1, 29–53 (1997)

Data Cube: A Relational Aggregation Operator Generalizing Group-By, Cross-Tab, and Sub-Totals*

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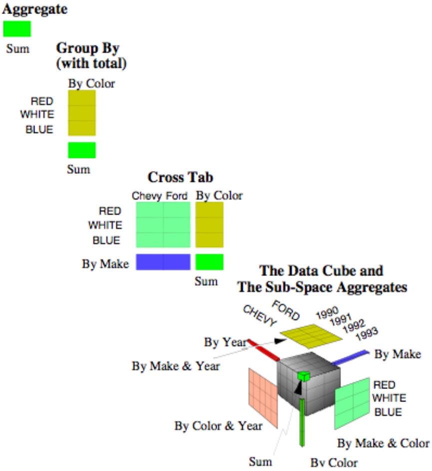
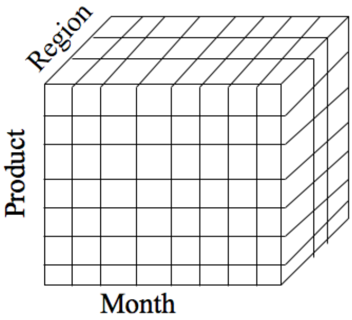
From aggregates to data cubes



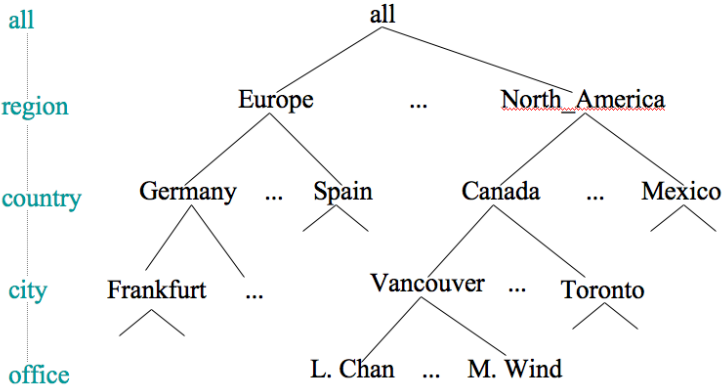
The Data Cube



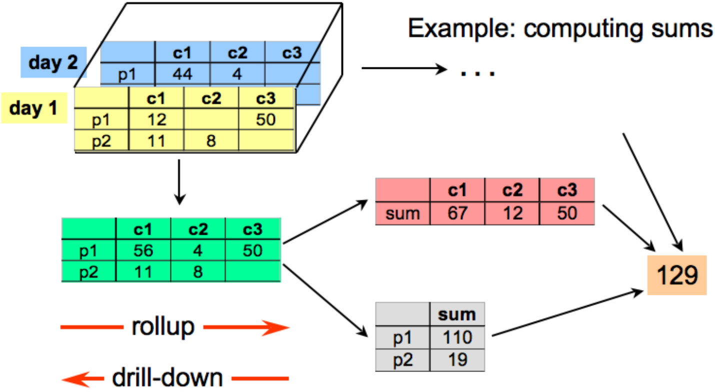
Dimensions:
Product,
Location,
Time

- Data modeled as an *n*-dimensional (hyper-) cube
- Each dimension is associated with a hierarchy
- Each “point” records facts
- Aggregation and cross-tabulation possible along all dimensions

Hierarchy for **Location** Dimension



Cube Operations



The Star Schema as a design tool