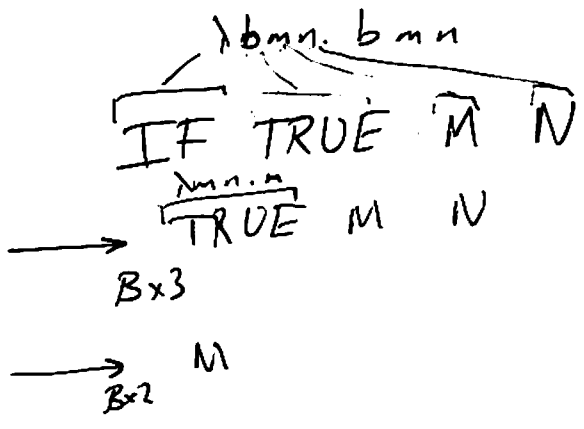


①

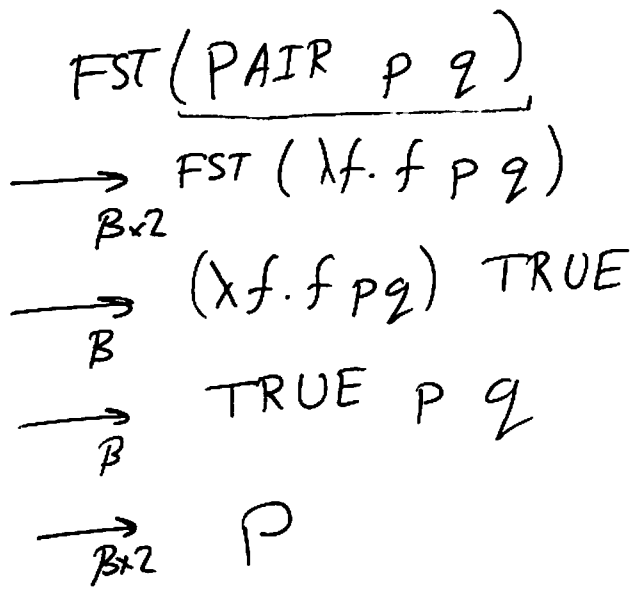


$$\lambda x. M \ x$$

$$\xrightarrow{M} \quad (x \notin FV(M))$$

$$\lambda b m n. b m n$$

$$\lambda b m. \lambda n. (b m) \ n$$

$$SND (PAIR \ p \ q)$$


CASE (INL x) f g

→_B CASE (PAIR TRUE x) f g

→_B IF (FST P) (f (SND P)) (g (SND P))

→_B* f (SND P) →_P f x

CASE (INL x) f g

→_{Bx3} INL x f g

→_{Bx3} f x

ADD 1 1

→_{Bx2} λf x. 1 f (1 f x)

→_{Bx2} λf x. 1 f (f x)

→_{Bx2} λf x. f (f x) = 2

ADD n m

→* λf x. n f (m f x)

→* λf x. n f (m f ... f x)

→* λf x. n f ... f (m f ... f x) = n+m

MULT 2 1

- $\lambda f x. \underline{2} (\underline{1} f) x$
- $\lambda f x. (\underline{1} f) ((\underline{1} f) x)$
- $\lambda f x. (\underline{1} f) (f x)$
- $\lambda f x. f (f x) = \underline{2}$

MULT n m

- * $\lambda f x. \underline{m} (\underline{n} f) x$
- * $\lambda f x. (\underline{n} f) ((\underline{n} f) \dots ((\underline{n} f) x))$
- $\lambda f x. \overbrace{f f}^{\wedge n} (\overbrace{f f}^{\wedge n} (\dots (\overbrace{f f}^{\wedge n} x))) = \underline{n \times m}$

EXP 2 3

- 3 2
- ⇒ $(\lambda f \lambda x. f (f (f x))) \underline{2}$
- $\lambda x. \underline{2} (\underline{2} (\underline{2} x))$
- =_α $\lambda f. \underline{2} (\underline{2} (\underline{2} f))$
- =_β $\lambda f x. \underline{2} (\underline{2} (\underline{2} f)) x$
- $f x (\underline{2} (\underline{2} f)) (\underline{2} (\underline{2} f) x)$
- $(\underline{2} f (\underline{2} f x))$
- $(f (f (f x)))$
- * $\lambda f x. \overbrace{f f f}^{\delta} (f x) = \underline{8}$

ISZERO 5

(4)

→* 5 ($\lambda x. FALSE$) ($TRUE$)

→ ($\lambda x. FALSE$) ($(\lambda x. FALSE) (\lambda x. FALSE) \dots TRUE$)

→ $FALSE$

PRE 3

→* SND (3 $\overbrace{(\lambda p. PAIR (SUCC (fst p)) (fst p))}^F$ $(PAIR\ 0\ 0)$)

→* SND ($F(F(F(PAIR\ 0\ 0)))$)

→* SND ($F(F(PAIR\ 1\ 0))$)

→* SND ($F(PAIR\ 2\ 1)$)

→* SND ($PAIR\ 3\ 2$)

→* 2