# Quantum Computing Lecture 1

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## Bits and Qubits

## What is Quantum Computing?

*Aim to use quantum mechanical phenomena that have no classical counterpart for computational purposes.*

Central research tasks include:

- *Building devices* — with a specified behaviour.
- *Designing algorithms* — to use the behaviour.

Mediating these two are models of computation.

## Bird’s eye view

*A computer scientist looks at Quantum Computing:*

Algorithmic Languages

Theory/complexity

System Architecture

Specified Behaviour

Physics

𝔇ragons

## Why look at Quantum Computing?

- *The world is quantum*
  - classical models of computation provide a level of abstraction
  - discrete state systems
- *Devices are getting smaller*
  - Moore’s law
  - the only descriptions that work on the very small scale are quantum
- *Exploit quantum phenomena*
  - using quantum phenomena may allow us to perform computational tasks that are not otherwise possible/efficient
  - understand capabilities/resources

## Course Outline

A total of eight lecturers.

1. *Bits and Qubits* (this lecture).
2. *Linear Algebra*
3. *Quantum Mechanics*
4. *Models of Computation*
5. *Some Applications*
6. *Search Algorithms*
7. *Factorisation*
8. *Complexity*

## Useful Information

Some useful books:

- Nielsen, M.A. and Chuang, I.L. (2000). *Quantum Computation and Quantum Information.* Cambridge University Press.
- Gruska, J. (1999). *Quantum Computing.* McGraw Hill.
- Kitaev, A.Y., Shen, A.H. and Vyalyi, M.N. (2002). *Classical and Quantum Computation.* AMS.
- Hirvensalo, M. (2004). *Quantum Computing.* Springer.

Course website:
`http://www.cl.cam.ac.uk/Teaching/current/QuantComp/`

## Bits

A building block of classical computational devices is a two-state system.

$$0 \quad \longleftrightarrow \quad 1$$

Indeed, any system with a finite set of *discrete, stable* states, with controlled transitions between them will do.

## Qubits

Quantum mechanics tells us that any such system can exist in a *superposition* of states.

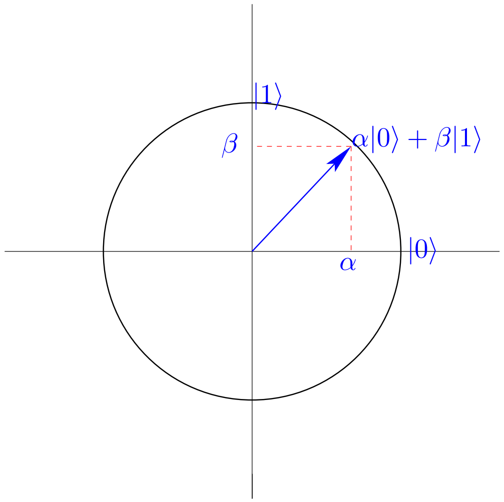
In general, the state of a *quantum bit* (or *qubit* for short) is described by:

$$\alpha|0\rangle + \beta|1\rangle$$

where, $\alpha$ and $\beta$ are complex numbers, satisfying

$$|\alpha|^2 + |\beta|^2 = 1$$

## Qubits



A qubit may be visualised as a unit vector on the plane.

In general, however, $\alpha$ and $\beta$ are *complex* numbers.

## Measurement

Any attempt to measure the state

$$\alpha|0\rangle + \beta|1\rangle$$

results in $|0\rangle$ with probability $|\alpha|^2$, and $|1\rangle$ with probability $|\beta|^2$.

*After the measurement, the system is in the measured state!*

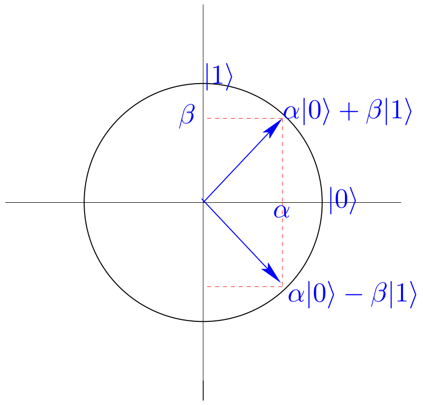
That is, further measurements will always yield the same value.

We can only extract one bit of information from the state of a qubit.

## Measurement

$\alpha|0\rangle + \beta|1\rangle$ and $\alpha|0\rangle - \beta|1\rangle$ have the same probabilities for their measurement

However, they are *distinct* states which behave differently in terms of how they evolve.


$|1\rangle$
$\beta$
$\alpha|0\rangle + \beta|1\rangle$
$|0\rangle$
$\alpha$
$\alpha|0\rangle - \beta|1\rangle$

## Vectors

Formally, the state of a qubit is a unit vector in $\mathbb{C}^2$—the 2-dimensional complex *vector space*.

The vector $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ can be written as

$$\alpha|0\rangle + \beta|1\rangle$$

where, $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$|\phi\rangle$— a *ket*, Dirac notation for vectors.

## Basis

Any pair of vectors $|\phi\rangle, |\psi\rangle \in \mathbb{C}^2$ that are linearly independent could serve as a basis.

$$\alpha|0\rangle + \beta|1\rangle = \alpha'|\phi\rangle + \beta'|\psi\rangle$$

The basis is determined by the measurement process or device.

Most of the time, we assume a standard (orthonormal) basis $|0\rangle$ and $|1\rangle$ is given.

This will be called the *computational basis*

## Example

The vector $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ measured in the computational basis gives either outcome with probability $1/2$.

Measured in the basis

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

it gives the first outcome with probability 1.

## Entanglement

An $n$-qubit system can exist in any superposition of the $2^n$ *basis* states.

$$\alpha_0|000000\rangle + \alpha_1|000001\rangle + \cdots + \alpha_{2^n-1}|111111\rangle$$

with $\sum_{i=0}^{2^n-1} |\alpha_i|^2 = 1$

Sometimes such a state can be decomposed into the states of individual bits

$$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

## Entanglement

Compare the two (2-qubit) states:

$$\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) \quad \text{and} \quad \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

If we measure the first qubit in the first case, we see $|0\rangle$ with probability 1 and the state remains unchanged.

In the second case (*an EPR pair*), measuring the first bit gives $|0\rangle$ or $|1\rangle$ with equal probability. After this, the second qubit is also determined.