



#### Cache Networks with Optimality Guarantees

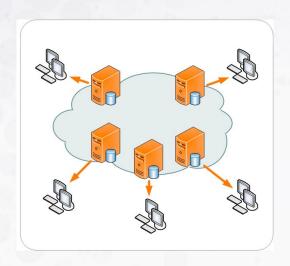
#### Stratis Ioannidis

Department of Electrical and Computer Engineering
Northeastern University

Joint work with Khashayar Kamran, Yuezhou Liu, Yuanyuan Li, Qian Ma, Milad Mahdian, Armin Moharrer, Tareq Si Salem, Giovanni Neglia, and Edmund Yeh

#### **Motivation**

#### Caching and object allocation problems are ubiquitous



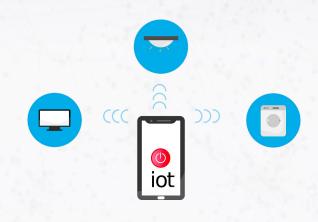
#### **CDNs**

[Traverso et al. CCR 2013] [Leconte et al. ITC 2015] [Leconte et al. SIGMETRICS 2012]



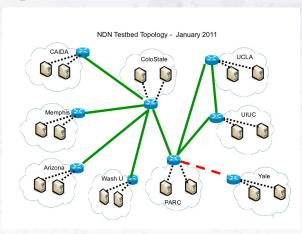
**Cloud Computing** 

[Cara et al. INFOCOM 2019] [Arteaga et al. FAST 2016]



Edge/Wireless IoT

[Deghan et al. INFOCOM 2015] [Leconte et al. ITC 2015] [Leconte et al. SIGMETRICS 2012]

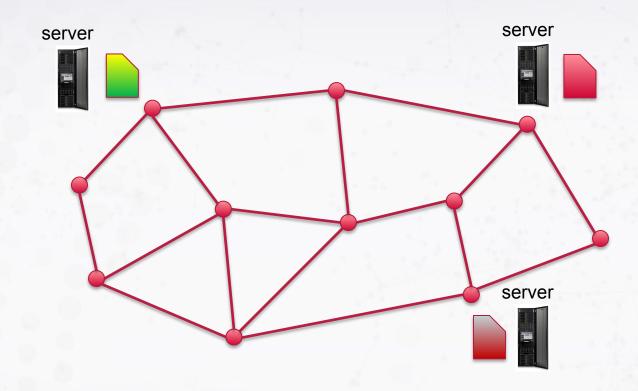


# Content-Centric Networking

[Martina et al. INFOCOM 2014]
[Rosenweig et al. INFOCOM 2013]
[Wang et al. ICNP 2013]
[Tyson et al. ICCCN 2012]
[Yeh et al. ICN 2013]
[Jacobson et al. CONEXT 2009]

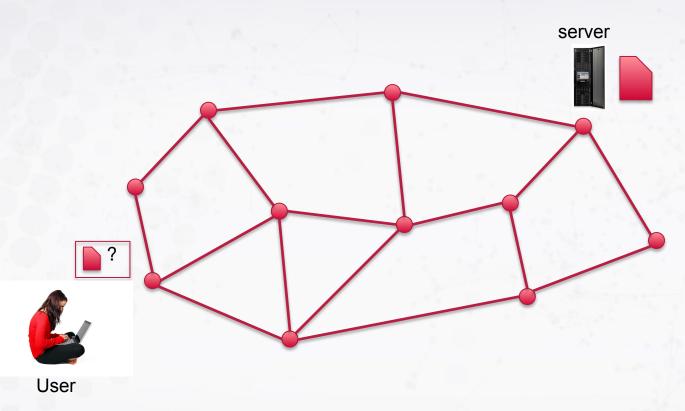


Institute for the Wireless Internet of Things at Northeastern



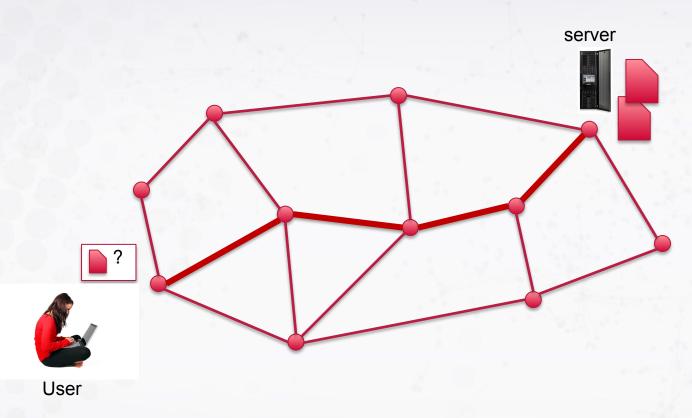
Designated servers in the network store content items (e.g., files, file chunks).





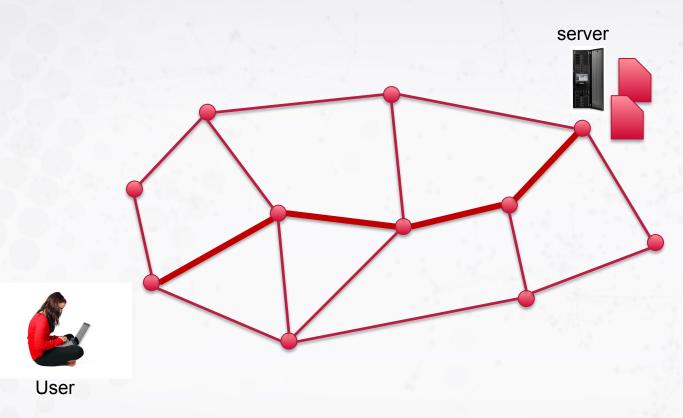
Nodes generate **requests** for content items





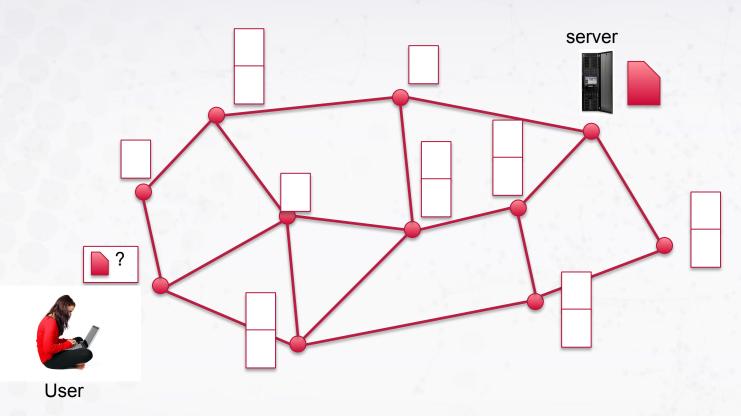
Requests routed towards a designated server





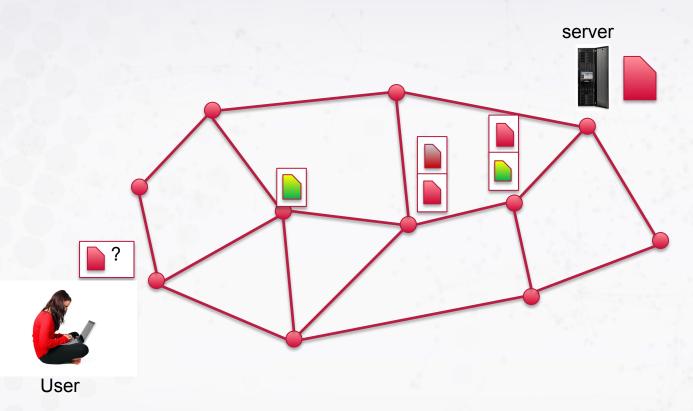
Responses routed over **reverse** path





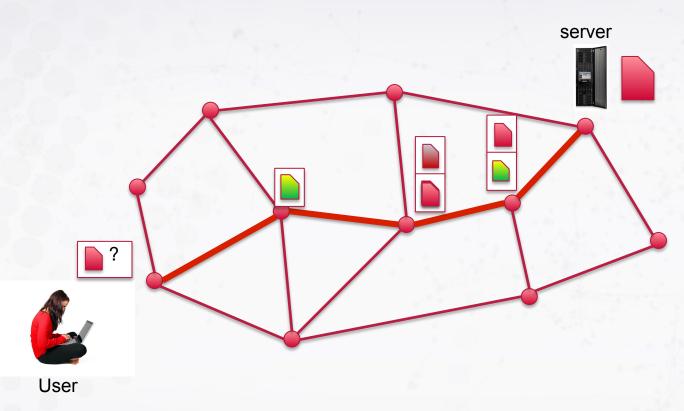
Nodes have **caches** with finite capacities





Nodes have **caches** with finite capacities



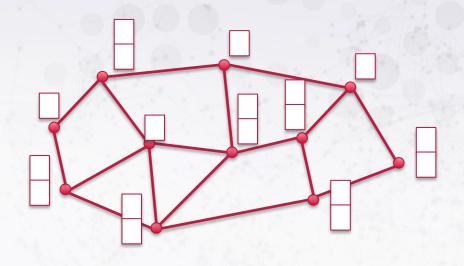


Requests terminate early upon a cache hit



#### **Cache Network Problems**

- ☐ Cache Networks: nodes can store content.
- ☐ Optimize caching decisions
- ☐ …plus:
  - □ Routing
  - ☐ Scheduling/service allocation
  - Admission control
- Minimize delays or transfer costs, maximize throughput or utility, incorporate fairness, study stability ...
- ☐ Distributed, adaptive algorithms



Much, much harder, because caching is combinatorial!!!



#### **Our Research Contributions**

- ☐ Distributed, adaptive, algorithms optimizing **caching** decisions
  - Stochastic requests
  - Adversarial requests/no-regret setting
- Joint optimization of caching and routing
- Queuing Models
  - Kelly cache networks
  - Cache networks with counting queues
  - ☐ Stability/admission control
- Fair caching networks

[l. and Yeh, SIGMETRICS 2016/ToN 2018]

[Li, Si Salem, Neglia, and I., SIGMETRICS 2022]

[I. and Yeh, ICN 2017/JSAC 2018]

[Li, Si Salem, Neglia, and I., SIGMETRICS 2022]

[Mahdian, Moharrer, I., and Yeh, INFOCOM 2019/ToN 2020]

[Li and I., INFOCOM 2020/ToN 2021]

[Kamran, Moharrer, I., and Yeh, INFOCOM 2021]

[Liu, Li, I., and Yeh, Performance 2020]



#### **Overview**

☐ Cache network optimization

☐ Jointly optimizing caching and routing

☐ Introducing queues



#### **Overview**

☐ Cache network optimization

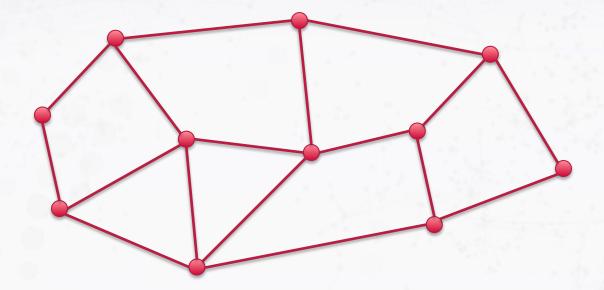
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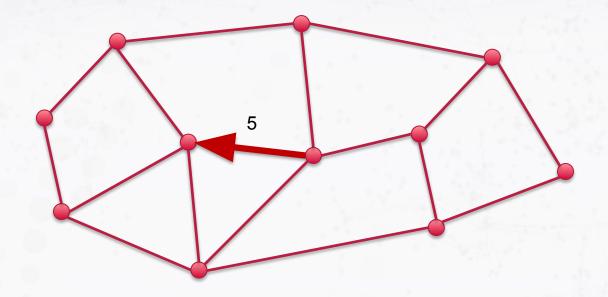
#### **Model: Network**

G(V, E)



Network represented as a directed, bi-directional graph  $\ G(V,E)$ 

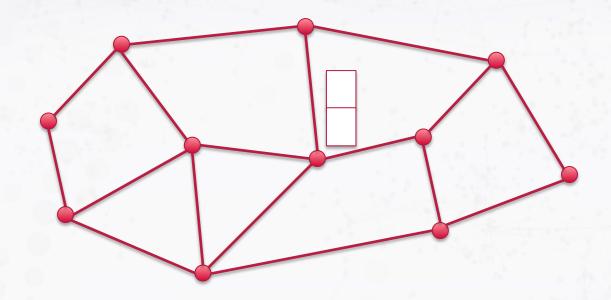




Each edge  $(u,v) \in E$  has a cost/weight  $w_{uv}$ 

Edge costs:  $w_{uv}, (u, v) \in E$ 

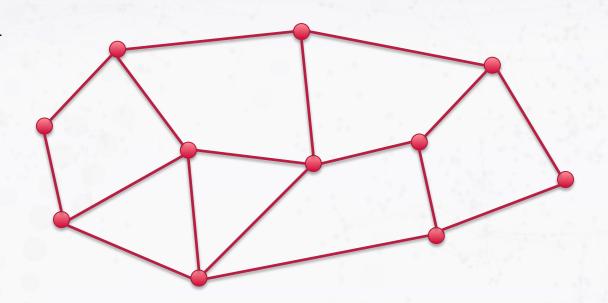




Node  $v \in V$  has a cache with capacity  $c_v \in \mathbb{N}$ 

Edge costs:  $w_{uv}, (u, v) \in E$ Node capacities:  $c_v, v \in V$ 

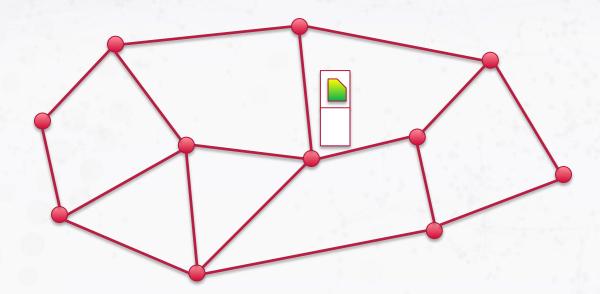
$$\mathcal{C} = \{ \blacksquare \blacksquare \}$$



Items stored and requested form the **item catalog**  $\, \mathcal{C} \,$ 

Edge costs:  $w_{uv}, (u, v) \in E$ Node capacities:  $c_v, v \in V$ 

$$\mathcal{C} = \{ \blacksquare \blacksquare \blacksquare \}$$



Edge costs:  $w_{uv}, (u, v) \in E$ 

Node capacities:  $c_v, v \in V$ 

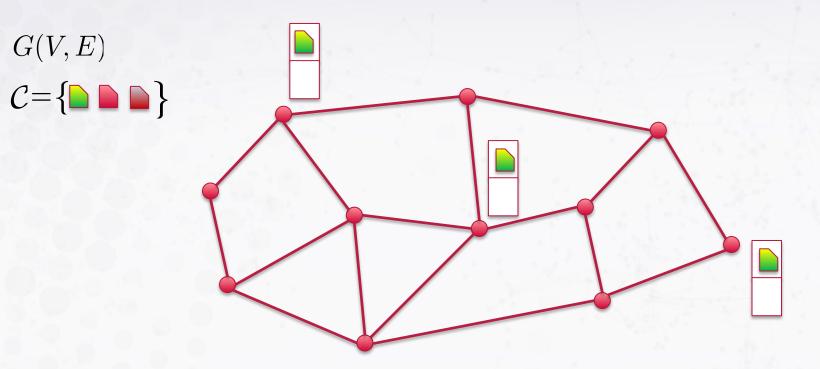
For 
$$v \in V$$
 and  $i \in \mathcal{C}$ , let  $x_{vi} = \begin{cases} 1, & \text{if } v \text{ stores } i \\ 0, & \text{o.w.} \end{cases}$ 

Then, for all  $v \in V$ ,  $\sum_{i \in \mathcal{C}} x_{vi} \le c_v$ 



# **Model: Designated/Permanent Servers**

[I. and Yeh, SIGMETRICS 2016/ToN 2018]



Edge costs:  $w_{uv}, (u, v) \in E$ Node capacities:  $c_v, v \in V$ 

 $\sum_{i \in \mathcal{C}} x_{vi} \leq c_v$ , for all  $v \in V$ 

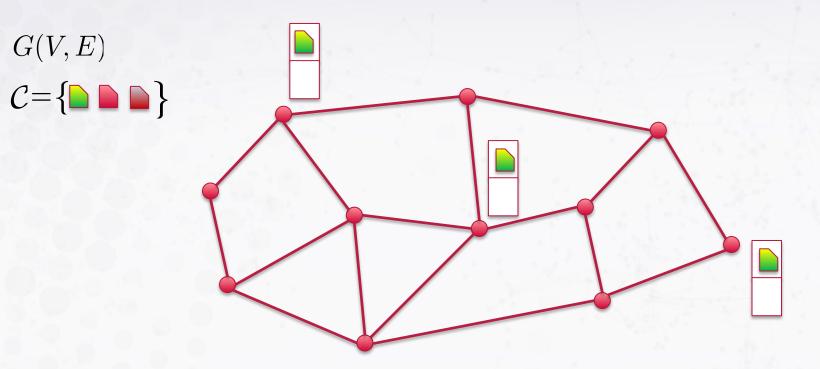
For each and  $i \in \mathcal{C}$ , there exists a set of nodes  $S_i \subset V$  (the **designated servers** of i) that **permanently store** i.

I.e., if  $v \in S_i$  then  $x_{vi} = 1$ 



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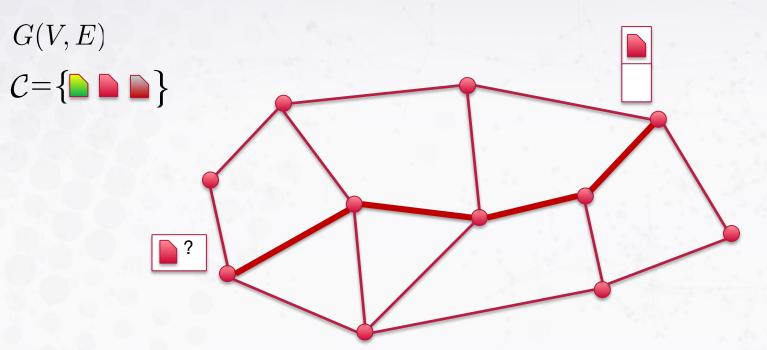
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Edge costs:  $w_{uv}, (u, v) \in E$ 

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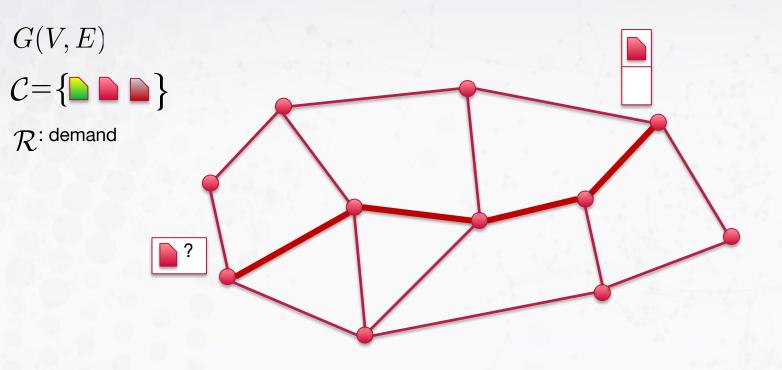
$$\sum_{i \in \mathcal{C}} x_{vi} \leq c_v$$
, for all  $v \in V$ 

Requests are always satisfied!

A **request** is a pair (i, p) such that:

- $oldsymbol{\square}$  i is an item in  ${\cal C}$
- $\square$   $p = \{p_1, \dots, p_K\}$  is a simple path in G such that  $p_K \in S_i$  .





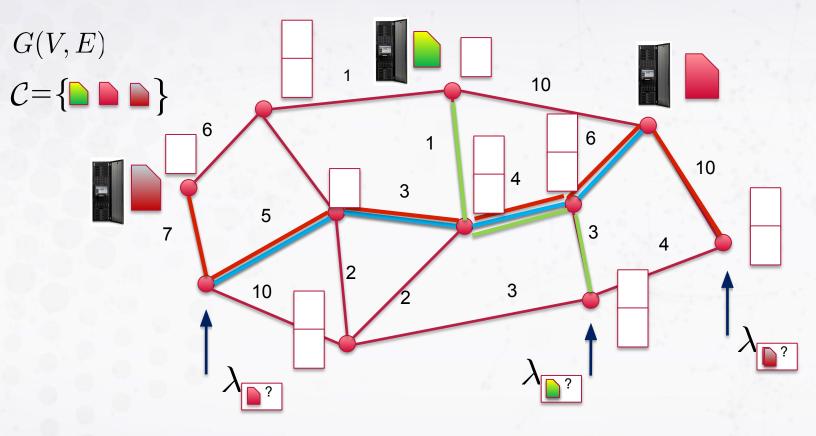
**Demand**  $\mathcal{R}$ : set of all requests (i, p)

Request arrival process is Poisson with rate  $\lambda_{(i,p)}$ 

Edge costs:  $w_{uv}, (u,v) \in E$ Node capacities:  $c_v, v \in V$   $\sum_{i \in \mathcal{C}} x_{vi} \leq c_v \text{, for all } v \in V$ 

Request rates:  $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$ 

#### **Model: Goal**



Edge costs:  $w_{uv}, (u, v) \in E$ 

Node capacities:  $c_v, v \in V$ 

 $\sum x_{vi} \le c_v, \text{ for all } v \in V$ 

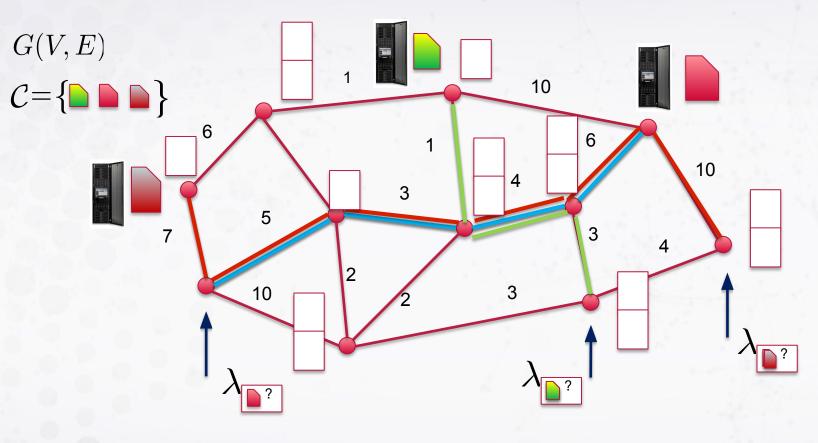
 $i \in \mathcal{C}$ 

Request rates:  $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$ 

Design content allocation so that expected transfer costs are minimized.



#### **Model: Goal**



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Challenge: Caching algorithm should be

- **adaptive**, and
- ☐ distributed.



# A Simple Algorithm: Path Replication + LRU

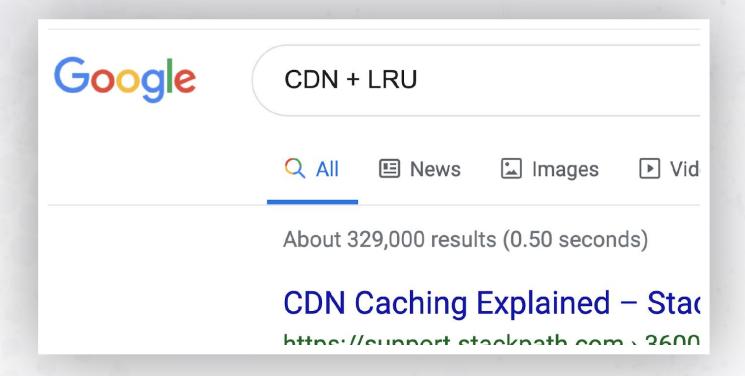


- ✓ Distributed
- Adaptive
- ✓ Extremely Popular

- ☐ Cache item on every node in the reverse path
- ☐ Evict using a simple policy, e.g., LRU, LFU, FIFO etc.
- Many variants: Move-Copy-Down (MCD), Leave-Copy-Down (LCD)...



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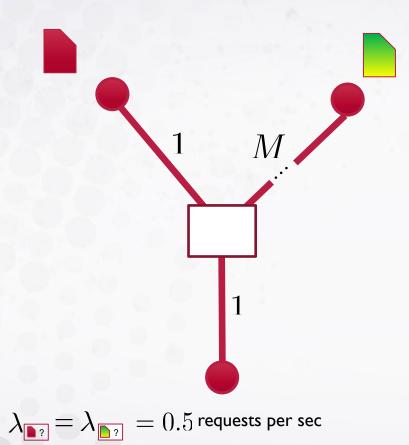


But...

Path Replication + LRU is arbitrarily suboptimal.



# Path Replication + LRU is Arbitrarily Suboptimal



Cost when caching



$$0.5 \times 1 + 0.5 \times 2 = 1.5$$

Cost of PR+LRU:

$$0.25 \times (M+1) + 0.25 \times 1 +$$
  
  $+ 0.25 \times 2 + 0.25 \times 1 = \mathbf{0.25}M + \mathbf{1.25}$ 

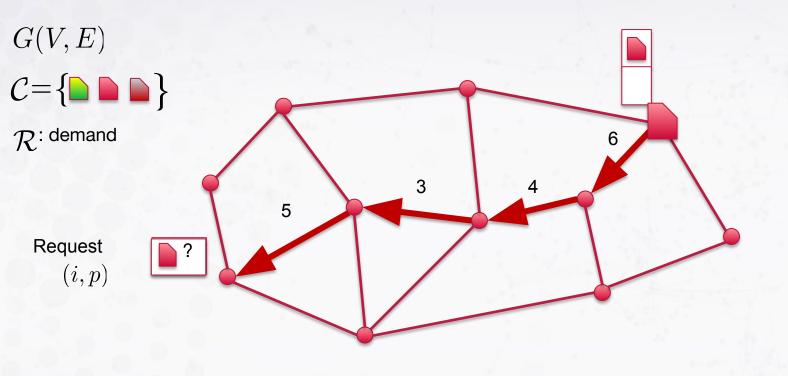
☐ When M is large, PR+LRU is **arbitrarily suboptimal!** 

☐ True for any strategy (LRU,LFU,FIFO,RR+LCD,MCD) that **ignores upstream costs!!** 



# **Model: Routing Costs & Caching Gain**

[l. and Yeh, SIGMETRICS 2016/ToN 2018]



Edge costs:  $w_{uv}, (u, v) \in E$ 

Node capacities:  $c_v, v \in V$ 

$$\sum x_{vi} \le c_v, \text{ for all } v \in V$$

 $i \in C$ 

Request rates:  $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$ 

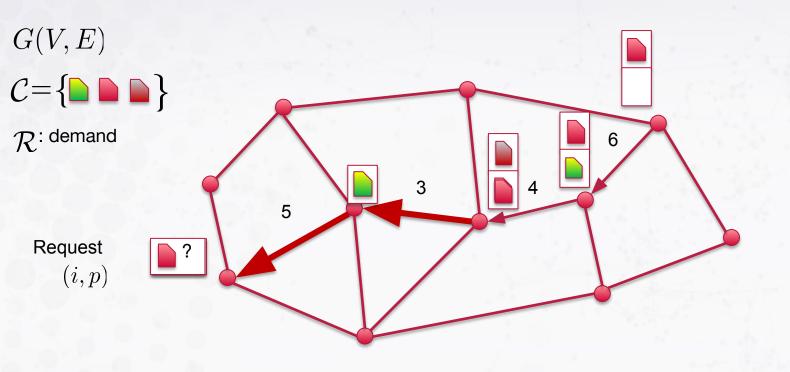
Worst case routing cost:

18



# **Model: Routing Costs & Caching Gain**

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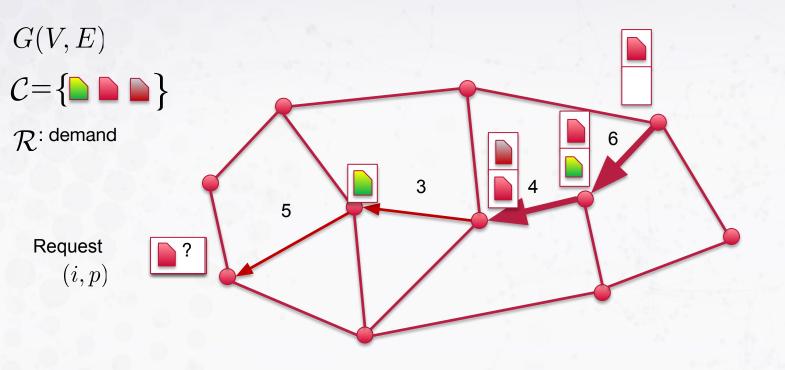
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# **Model: Routing Costs & Caching Gain**

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Worst case routing cost: 18

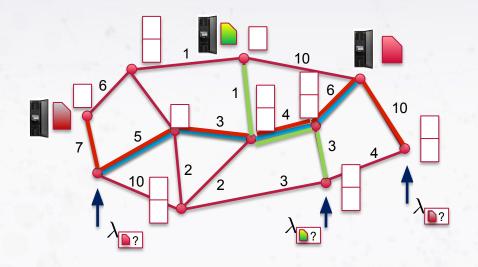
Cost due to intermediate caching: 8

**Caching Gain:** 18-8 = 10



$$\mathcal{C} = \{ \blacksquare \blacksquare \blacksquare \}$$

 $\mathcal{R}$ : demand



Edge costs:  $w_{uv}, (u, v) \in E$ 

Node capacities:  $c_v, v \in V$ 

$$\sum x_{vi} \leq c_v$$
, for all  $v \in V$ 

 $i \in C$ 

Request rates:  $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$ 

#### MaxCG

Maximize:

$$F(X) = \sum_{(i,p)\in\mathcal{R}} \lambda_{(i,p)} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k} \left( 1 - \prod_{k'=1}^k (1 - x_{p_{k'i}}) \right)$$

Subject to:

$$\sum_{i \in \mathcal{C}} x_{vi} = c_v, \qquad \text{for all } v \in V$$

$$x_{vi} = 1$$
,

for all 
$$i \in \mathcal{C}$$
 and  $v \in S_i$ 

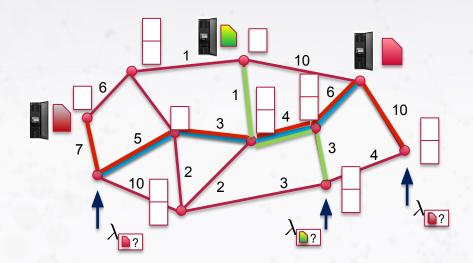
$$x_{vi} \in \{0, 1\},\$$

for all 
$$\,v \in V$$
 and  $\,i \in \mathcal{C}\,$ 



$$\mathcal{C} = \{ \bigcirc \bigcirc \bigcirc \}$$

 $\mathcal{R}$ : demand



- **NP-hard** but...
- ☐ .. Submodular maximization under matroid constraints
- ☐ 1-1/e polytime approximation algorithm

Edge costs:  $w_{uv}, (u, v) \in E$ 

Node capacities:  $c_v, v \in V$ 

$$\sum x_{vi} \le c_v, \text{ for all } v \in V$$

 $i \in C$ 

Request rates:  $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$ 

MaxCG

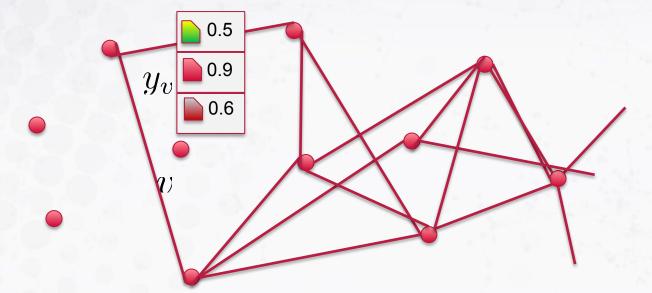
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Subject to:  $\displaystyle \sum_{i \in \mathcal{C}} x_{vi} = c_v$  for all  $v \in V$ 

$$x_{vi}=1,$$
 for all  $i\in\mathcal{C}$  and  $v\in S_i$   $x_{vi}\in\{0,1\},$  for all  $v\in V$  and  $i\in\mathcal{C}$ 

# Distributed, Adaptive Algorithm

$$\mathcal{C} = \{ \mathbf{L} \setminus \mathbf{L} \} \quad Y = [y_v]_{v \in V}$$

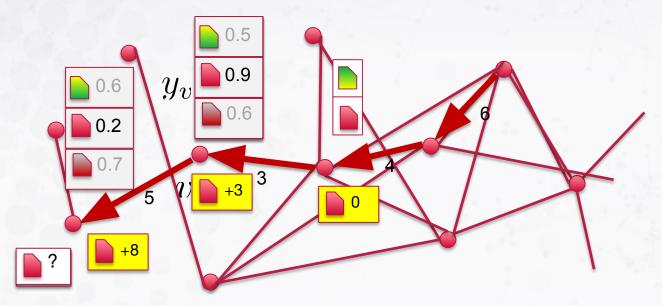


- ☐ Each cache maintains **state**
- ☐ State = probability of caching item



# Distributed, Adaptive Algorithm

$$\mathcal{C} = \{ \mathbf{b} \mid \mathbf{b} \mid \mathbf{b} \} \quad Y = [y_v]_{v \in V}$$



**Theorem:** The proposed algorithm leads to an allocation  $X_k$  such that

$$\lim_{k \to \infty} \mathbb{E}[F(X_k)] \ge (1 - \frac{1}{e})F(X^*)$$

where  $X^*$  an optimal solution to the (NP-hard) offline problem.

- Each cache maintains stateState = probability of caching item
- Upon request, control message collects information about upstream costs
  - = gradient of concave relaxation of objective (in expectation)
- During slot of length T, average upstream costs
- At end of slot, adapt state and refresh contents by randomly sampling from distribution/state, independently across nodes.
- "value" of item is

 $\lambda_{\blacksquare?} \times \mathbb{E}[\text{upstream cost upon} \blacksquare \text{ miss}]$ 



# **No-Regret Algorithms**

- ☐ Theorem assumes:
- ☐ Stationary, stochastic request arrivals
- Negligible costs for updates



## **No-Regret Algorithms**

- ☐ Arbitrary, adversarial request arrivals per time-slot
- □ Account for update costs

Theorem: A distributed, online algorithm that attains regret

$$R(T) = (1 - \frac{1}{e}) \sum_{t=1}^{T} F_t(X^*) - \left(\sum_{t=1}^{T} F_t(X_t) - \sum_{t=1}^{T} UC(X_t, X_{t-1})\right) = O(\sqrt{T})$$

optimal offline static policy

caching gain of online policy

penalty for update costs



#### **Overview**

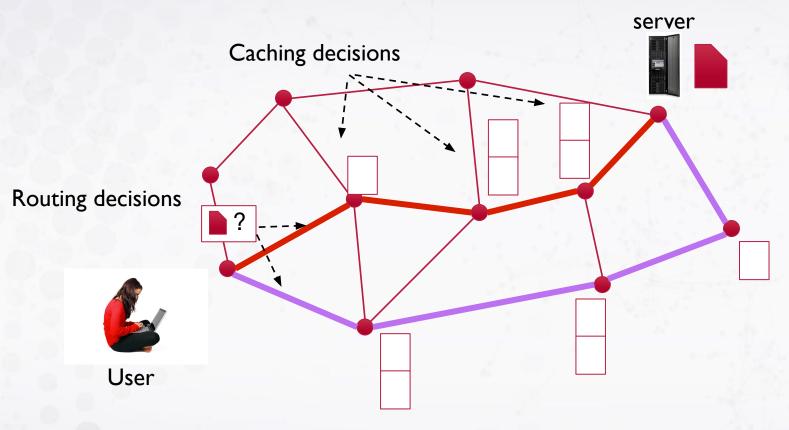
☐ Cache network optimization

☐ Jointly optimizing caching and routing

☐ Introducing queues



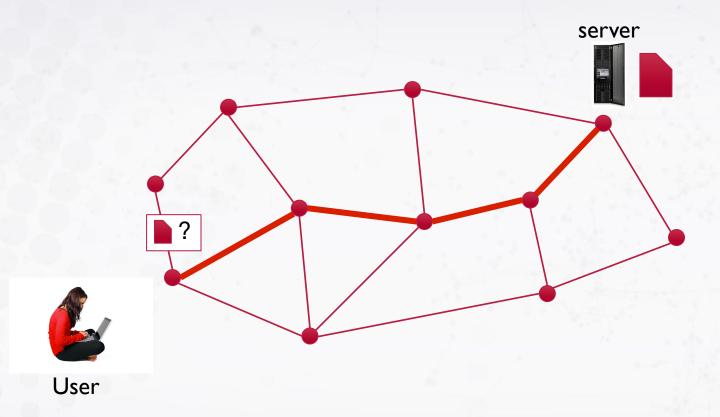
## **Joint Optimization**



☐ Both caching and routing decisions are part of optimization



#### Is Joint Optimization Really Necessary?

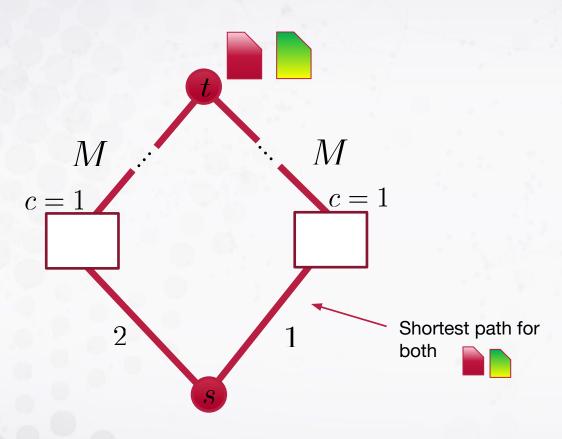


Shortest Weight Path

☐ Why not just use **shortest weight path** routing towards **nearest designated server**?



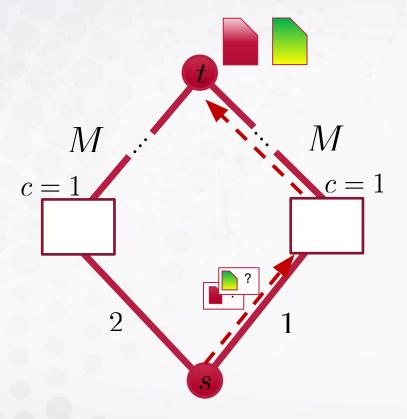
# **Shortest Path Routing is Arbitrarily Suboptimal**



$$\lambda_{\text{\tiny{\tiny{\tiny{\tiny{1}}}}}?} = \lambda_{\text{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{\tiny{1}}}}}}?}}} = 0.5$$
 requests per sec



# **Shortest Path Routing is Arbitrarily Suboptimal**

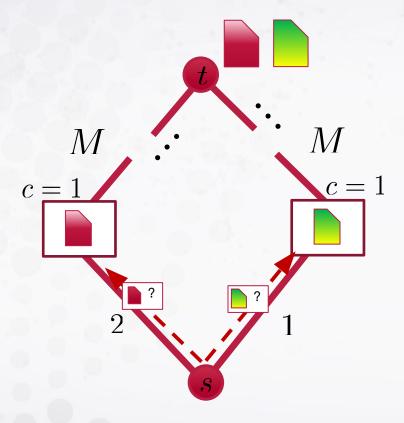


Irrespective of caching algorithm used, cost under shortest path routing is  $\Theta(M)$ 

$$\lambda_{
ho_!} = \lambda_{
ho_!} = 0.5$$
 requests per sec



# **Shortest Path Routing is Arbitrarily Suboptimal**



Cost under "split" routing strategy is  ${\cal O}(1)$ .

Shortest path routing to nearest server is arbitrarily suboptimal.

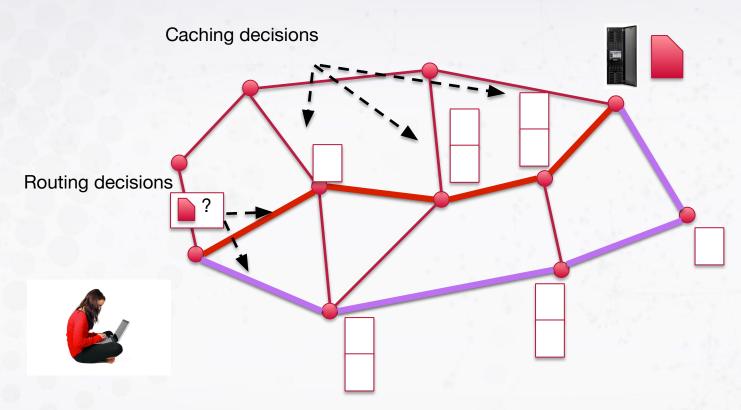
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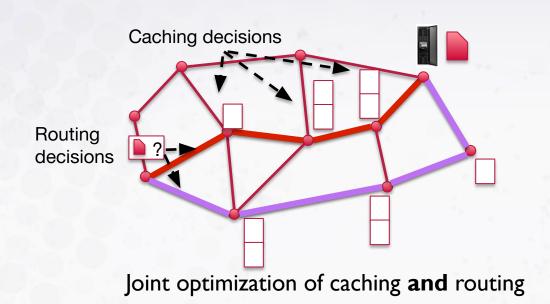
## **Key Intuition**



Increasing path diversity creates more caching opportunities.



## **Algorithms with Guarantees**



☐ Stochastic requests

[I. and Yeh, ICN 2017/JSAC 2018]

- Distributed, adaptive algorithm withinI-I/e from the optimal
- ☐ Adversarial requests [Li, Si Salem, Neglia, and I., SIGMETRICS 2022]
  - lacksquare Distributed, online algorithm with  $O(\sqrt{T})$  regret w.r.t. I-I/e from the optimal offline solution



# **Experiments**

#### **Graph Topologies**

Graph		V	E	$ \mathcal{C} $	$ \mathcal{R} $	$c_v$	$ \mathcal{P}_{(i,s)} $
cycle		30	60	10	100	2	2
grid-2d		100	360	300	1K	3	30
hypercube		128	896	300	1 <b>K</b>	3	30
expander		100	716	300	1K	3	30
erdos-reny	i	100	1042	300	1 <b>K</b>	3	30
regular		100	300	300	1 <b>K</b>	3	30
watts-stro	gatz	100	400	300	1 <b>K</b>	3	2
small-worl	d	100	491	300	1K	3	30
barabasi-a	lbert	100	768	300	1 <b>K</b>	3	30
geant		22	66	10	100	2	10
abilene		9	26	10	90	2	10
dtelekom		68	546	300	1 <b>K</b>	3	30

#### **Routing Algorithms**

	Uniform
_	Official

Dynamic routing: PGA on L for
routes alone

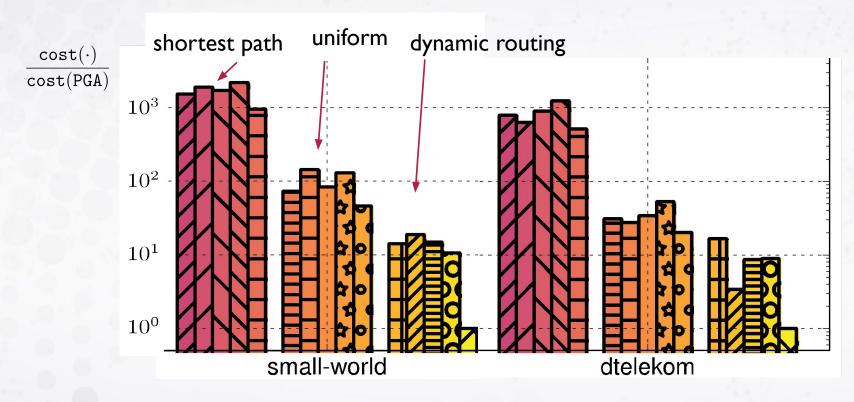
#### **Caching Algorithms**

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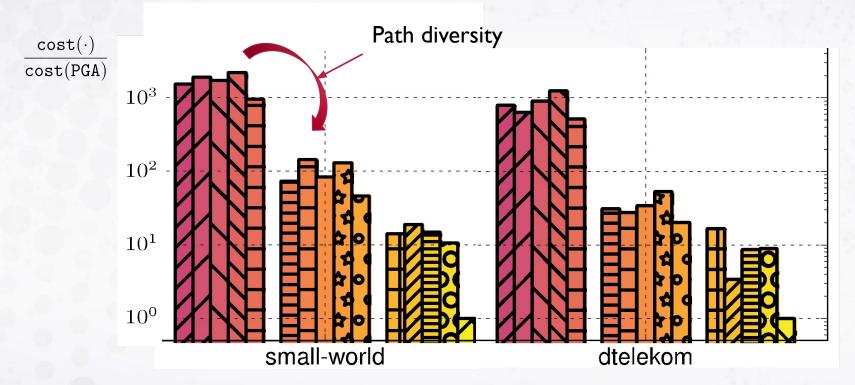
Ratio of expected routing cost to routing cost under our algorithm







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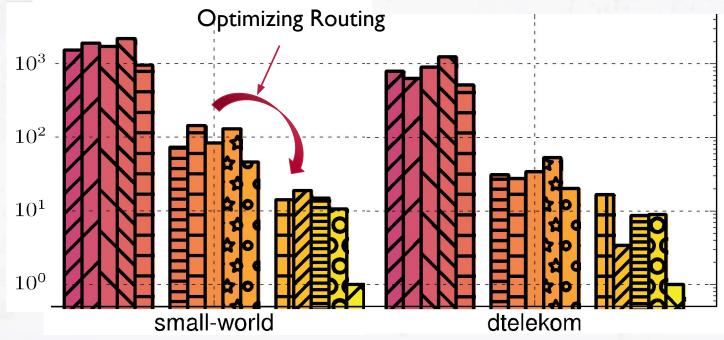






Ratio of expected routing cost to routing cost under our algorithm

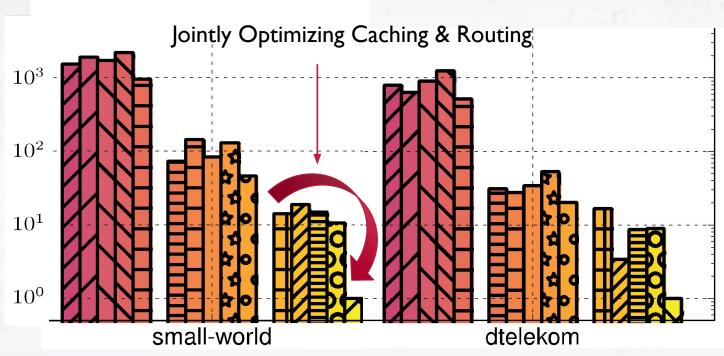






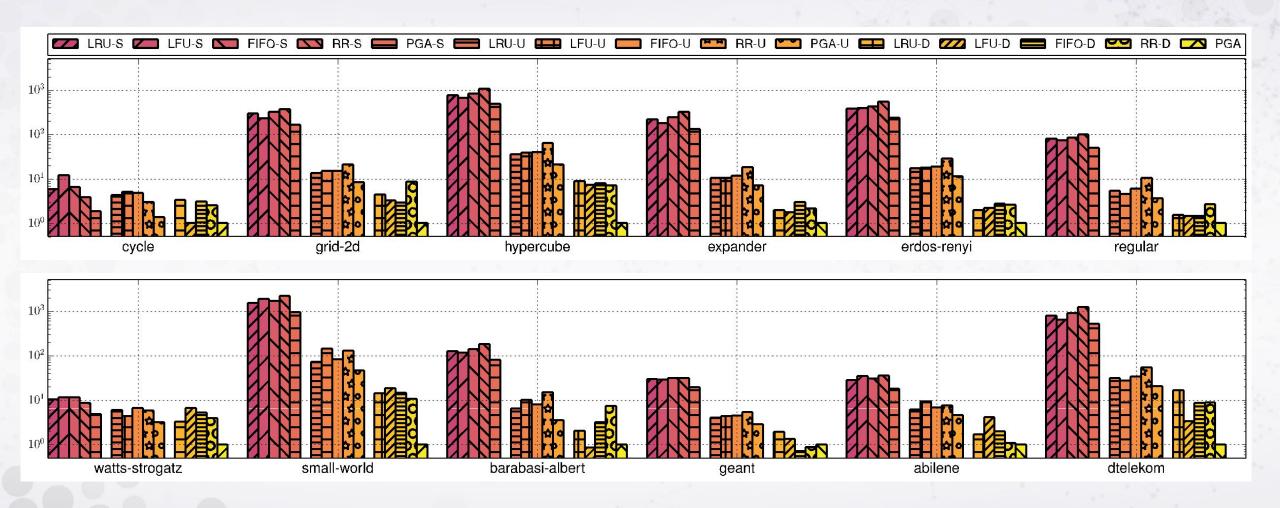














#### **Overview**

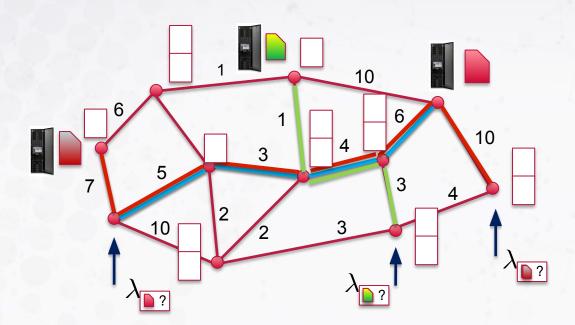
☐ Cache network optimization

☐ Jointly optimizing caching and routing

☐ Introducing queues

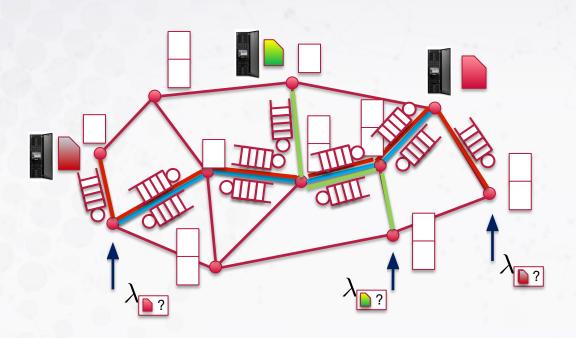


# **Introducing Queues**





## **Introducing Queues**



- Downward edges are associated with M/M/1 queues
- Determine cache contents so that steady state queuing costs are minimized
- $lue{}$  Size of queue at edge  $e: n_e \in \mathbb{N}$
- $f Cost: \ c_e(n_e)$  where  $c_e: \mathbb{R}_+ 
  ightarrow \mathbb{R}_+$  is non-decreasing
  - Queue size, its moments, queuing delay, occupancy probability...
- ☐ Aggregate expected cost:

$$C(\mathbf{x}, \boldsymbol{\lambda}) \equiv \sum_{e \in E} \mathbb{E}_{\mathbf{x}, \boldsymbol{\lambda}}[c(n_e)]$$

□ Caching gain:

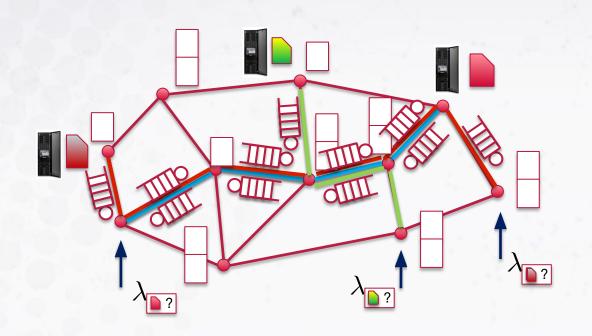
$$F(\mathbf{x}, \boldsymbol{\lambda}) = C(\mathbf{x}_0, \boldsymbol{\lambda}) - C(\mathbf{x}, \boldsymbol{\lambda})$$

caching allocation under which system is stable

Theorem: Maximizing caching gain is a submodular maximization problem subject to matroid constraints.



## **Stability**



☐ Caching gain:

$$F(\mathbf{x}, \boldsymbol{\lambda}) = C(\mathbf{x}_0, \boldsymbol{\lambda}) - C(\mathbf{x}, \boldsymbol{\lambda})$$

caching allocation under which system is stable

☐ How does one find this?

- $\square$  Optimize caching strategy (x) and jointly do admission control ( $\lambda$ ) subject to stability constraints.
  - Much weaker optimality guarantees.

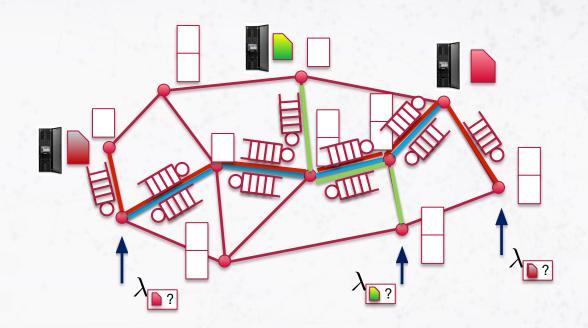


[Kamran, Moharrer, I., and Yeh, INFOCOM 2021]



Catalog  $C = \{ b \mid b \mid b \}$  is finite!







$$\mathcal{C} = \{ \bigcirc \bigcirc \bigcirc \bigcirc \}$$

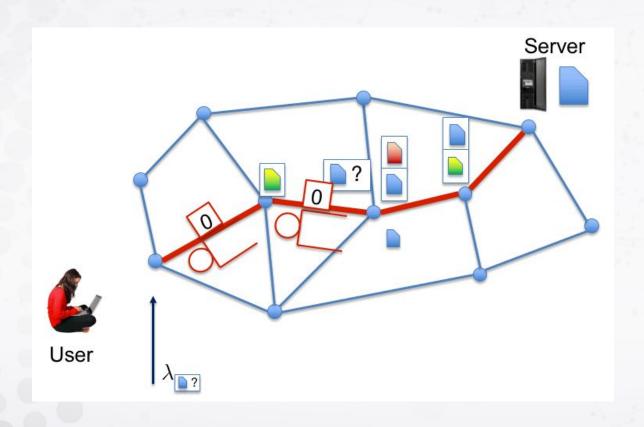


Identical responses merge when collocated

O

M/M/1c queue

## A More Elegant Solution: Counting Queues



- Network with counting queues
- □ Not reversible, steady-state queue distribution has no closed form
- **□** Well-approximated by M/M/∞ queues
- ☐ Theorem: Under this approximation, there exists an algorithm jointly optimizing of caching and service rate allocations within 1-1/e of the optimal.



## **Open Directions**

- No-regret algorithms
- ☐ Merging requests/queries, not responses
- Joint optimization tasks
  - Caching
  - Routing
  - ☐ Service assignment
  - □ Admission control

- Departure from submodularity
- Distributed algorithms





# **Institute for the Wireless Internet of Things**

at Northeastern University



Adaptive Caching Networks with Optimality Guarantees S. Ioannidis and E.Yeh, SIGMETRICS 2016/ToN 2018.

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Kelly Cache Networks

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Cache Networks with Counting Queues, Y. Li and S. Ioannidis, INFOCOM 2020/ToN 2021.

Online Caching Networks with Adversarial Guarantees
Y. Li, T. Si Salem, G. Neglia, and S. Ioannidis, SIGMETRICS/PERFORMANCE 2022.

# Thank You!

