

Towards NURBS-Compatible Subdivision

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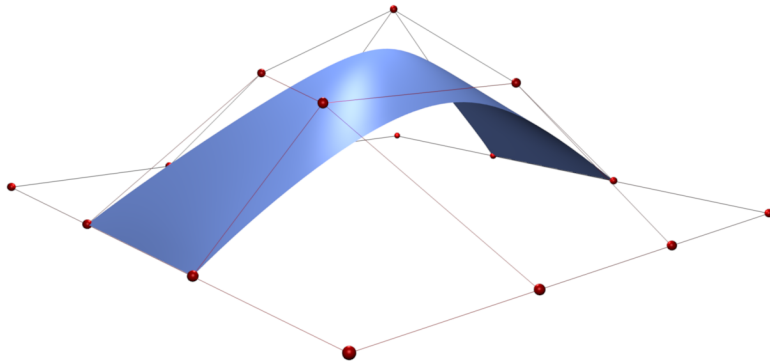
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University of Cambridge

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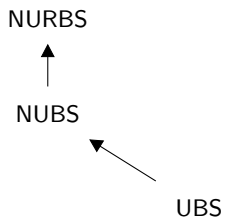
Subdivision and Refinability
May 1 – 4, 2008
Pontignano

NURBS

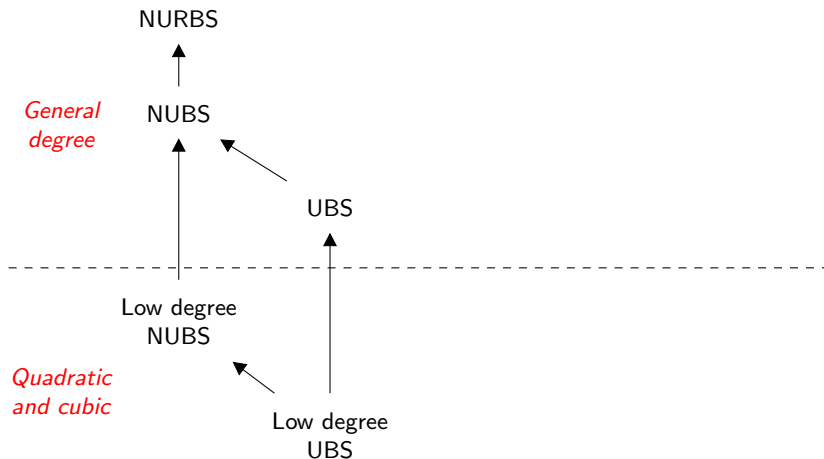
- ▶ Non-Uniform Rational B-Splines
- ▶ NURBS surfaces use a rectangular control grid
- ▶ Industry standard for Computer-Aided Design (CAD)



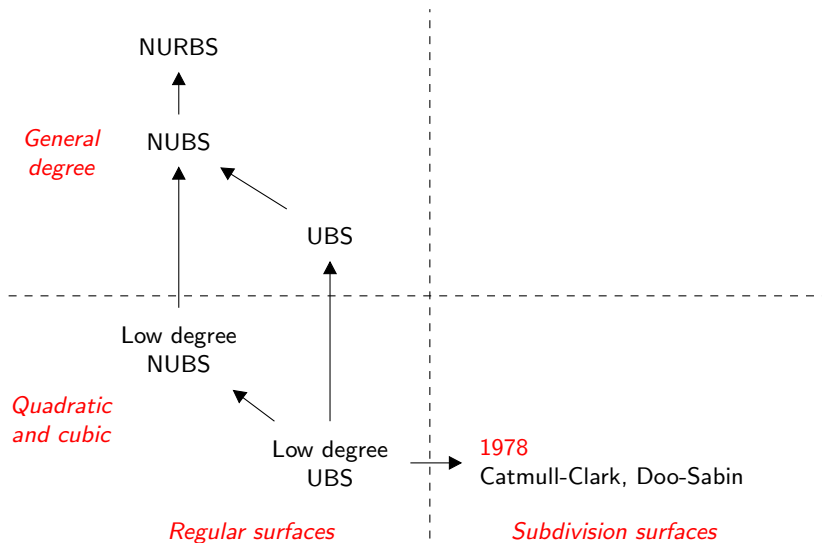
Why non-uniform B-splines?



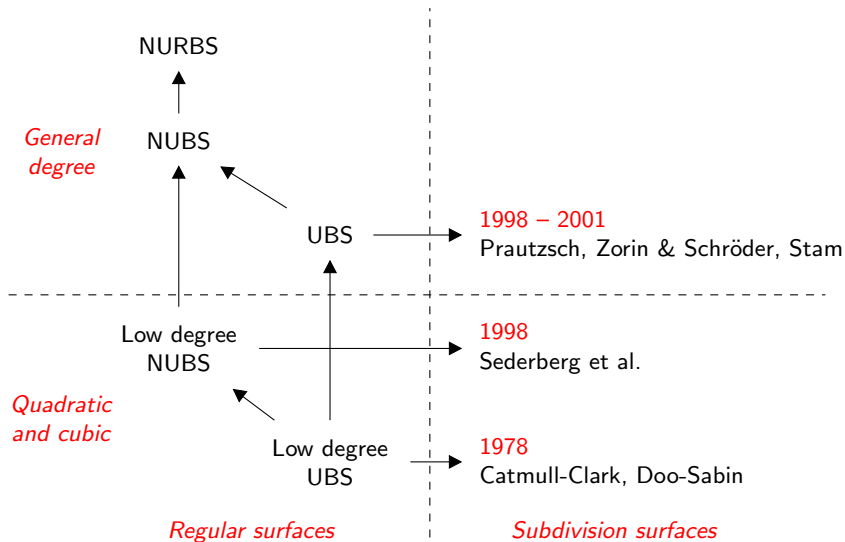
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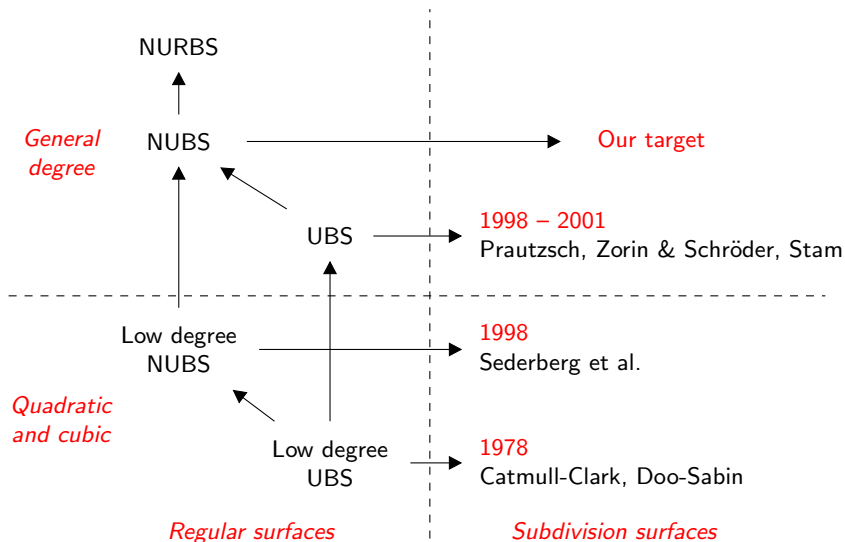
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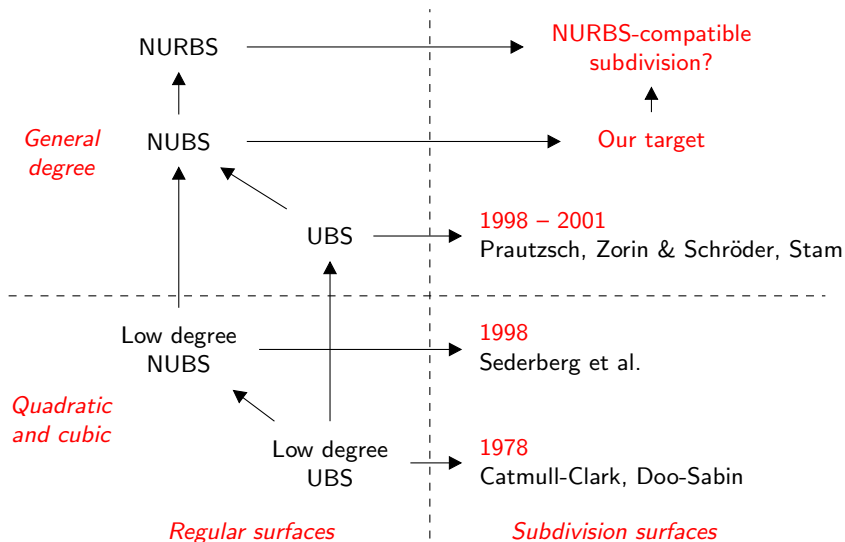
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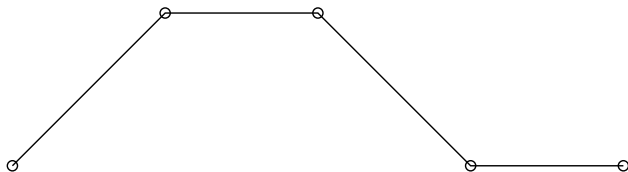


Why non-uniform B-splines?



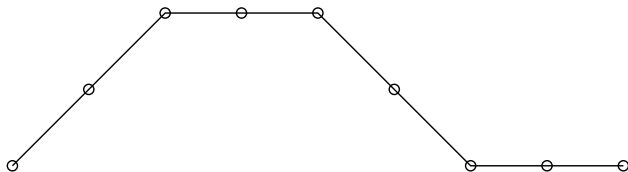
Refine and smooth subdivision

- ▶ Lane-Riesenfeld – uniform B-spline subdivision



Refine and smooth subdivision

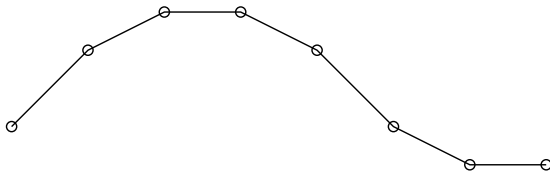
- ▶ Lane-Riesenfeld – uniform B-spline subdivision



- ▶ **Refine**
 - ▶ polygon lengthened by adding points

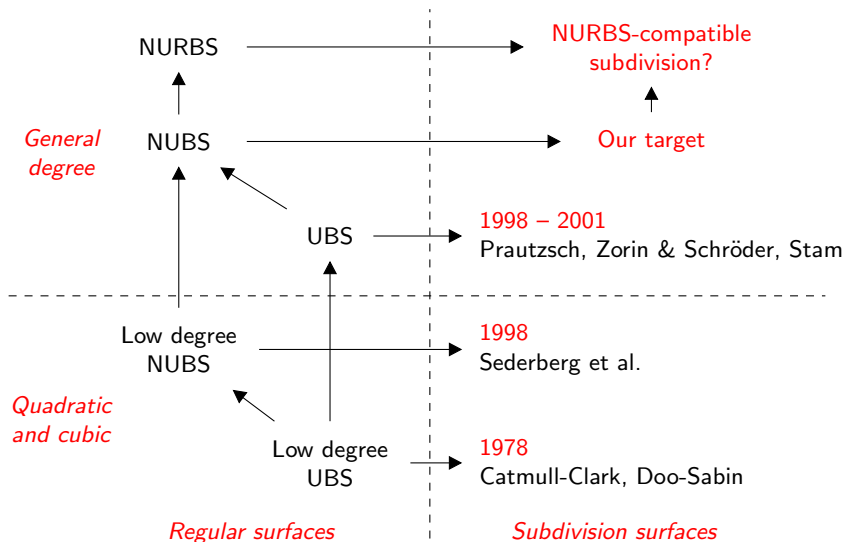
Refine and smooth subdivision

- ▶ Lane-Riesenfeld – uniform B-spline subdivision



- ▶ **Refine**
 - ▶ polygon lengthened by adding points
- ▶ and **Smooth**
 - ▶ each step creates another polygon
 - ▶ points moved using local filters
- ▶ More smoothing steps for higher degree

Why non-uniform B-splines?



Subproblems

Subdivision	Refine and smooth	Blossoming	NURBS
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Subproblems

Non-uniform refine and smooth

Subdivision

Refine and
smooth

Blossoming

NURBS

Subproblems

Non-uniform refine and smooth with multiple knots

Non-uniform refine and smooth

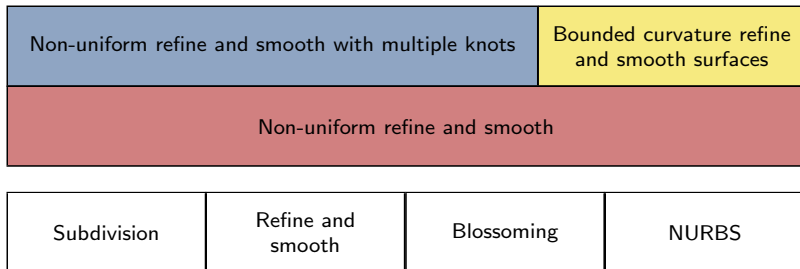
Subdivision

Refine and
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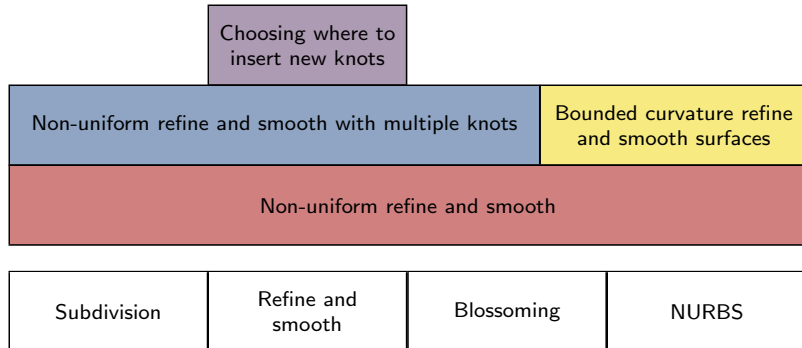
Blossoming

NURBS

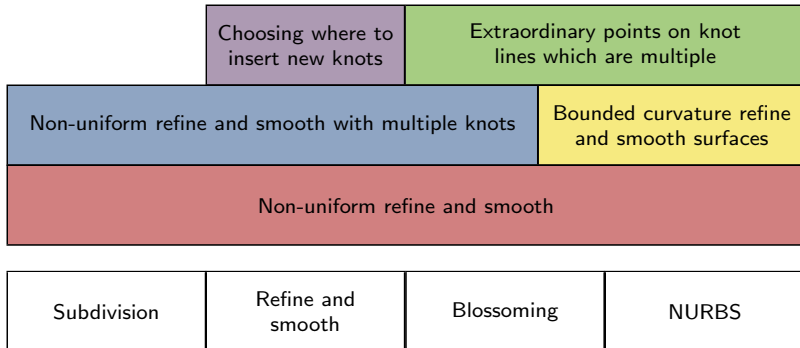
Subproblems



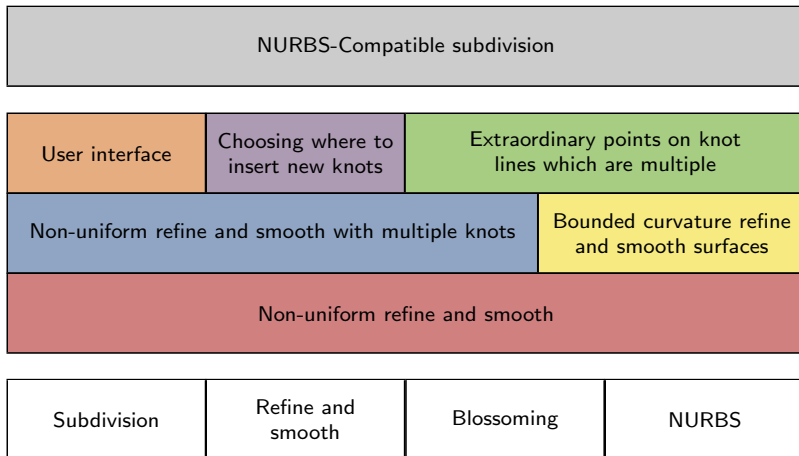
Subproblems



Subproblems



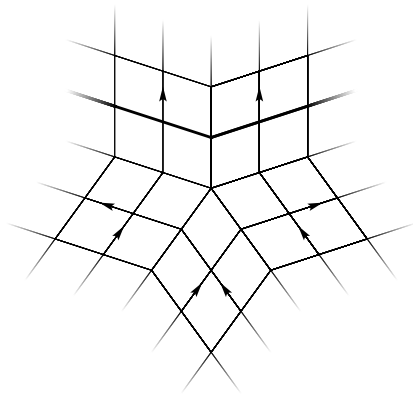
Subproblems

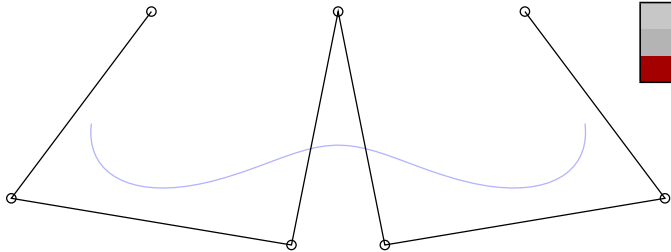




A knot insertion algorithm that

- ▶ uses **refine and smooth**
(affine combinations of two points)
- ▶ is **non-uniform**
- ▶ is **symmetric**



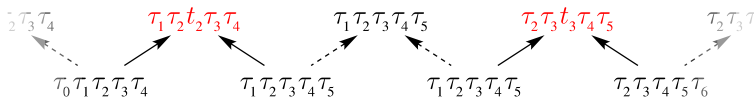
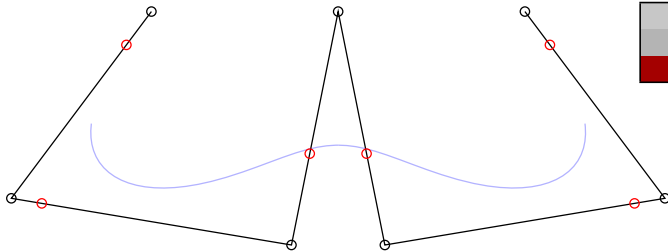


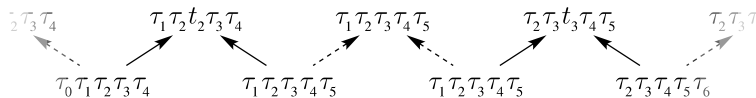
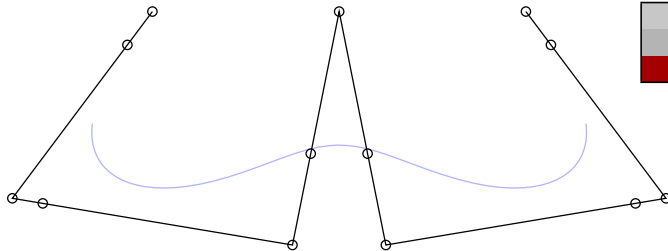
$$\tau_0 \tau_1 \tau_2 \tau_3 \tau_4$$

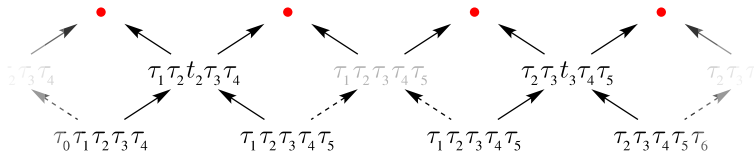
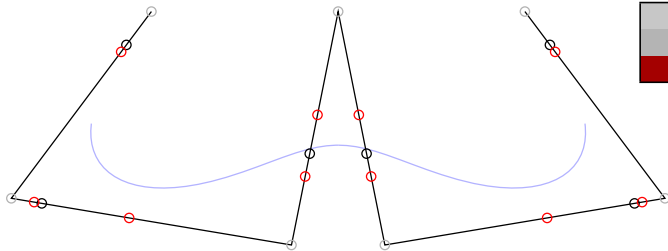
$$\tau_1 \tau_2 \tau_3 \tau_4 \tau_5$$

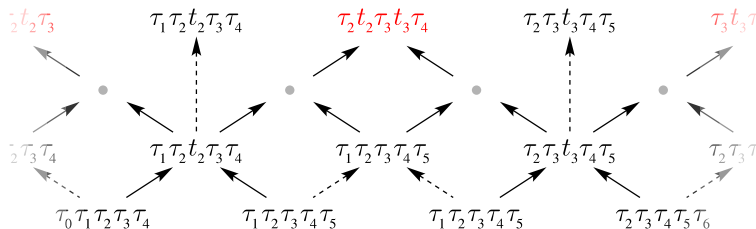
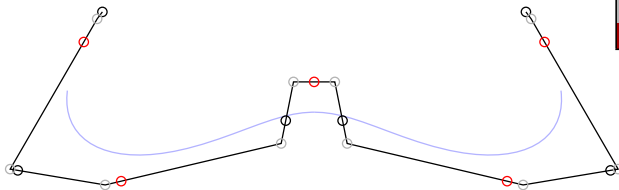
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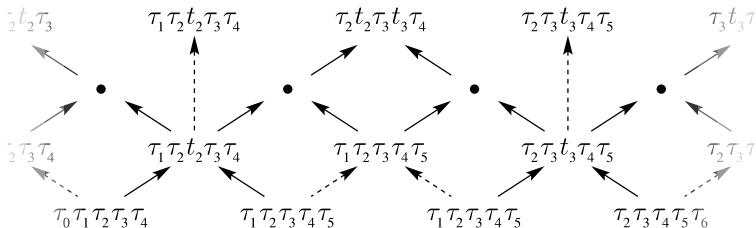
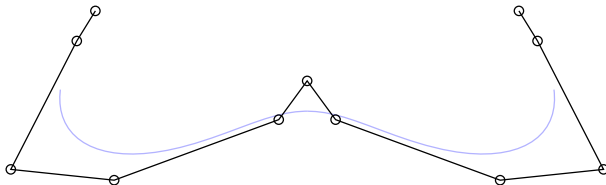
$$\tau_2 \tau_3 \tau_4 \tau_5 \tau_6$$

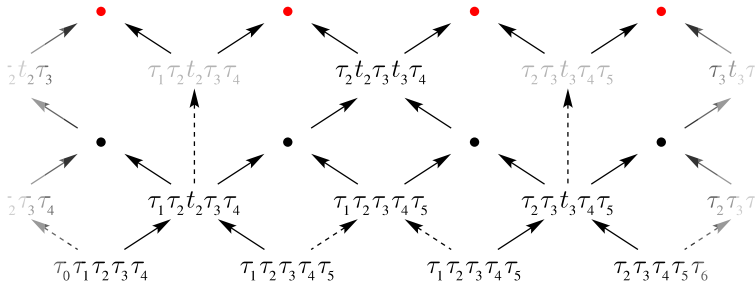
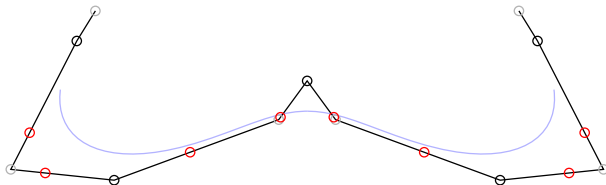


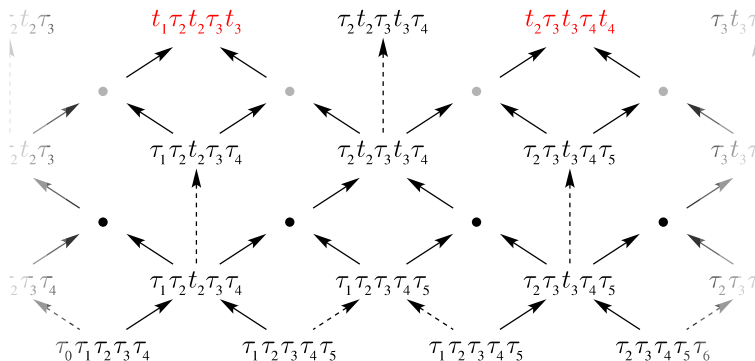
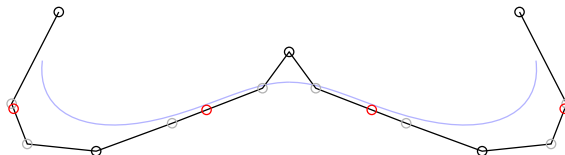


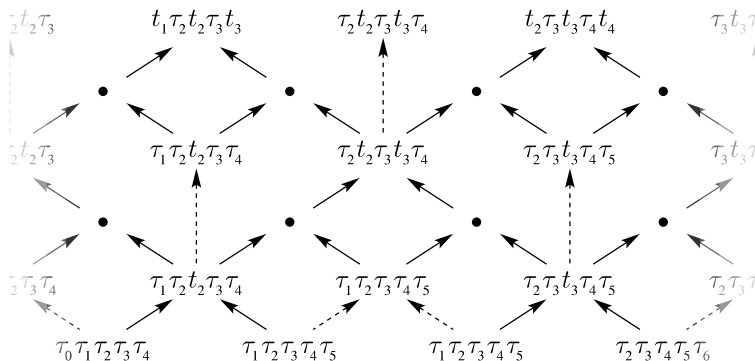
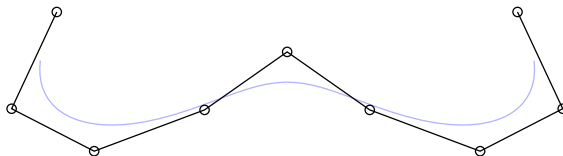












Zero parameter intervals need special handling

Multiple knots



- We don't want to increase the multiplicity of multiple knots



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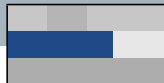


- We don't want to increase the multiplicity of multiple knots



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- It might also be useful to be able to leave some non-zero intervals unchanged



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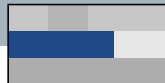


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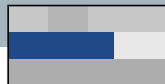


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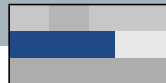
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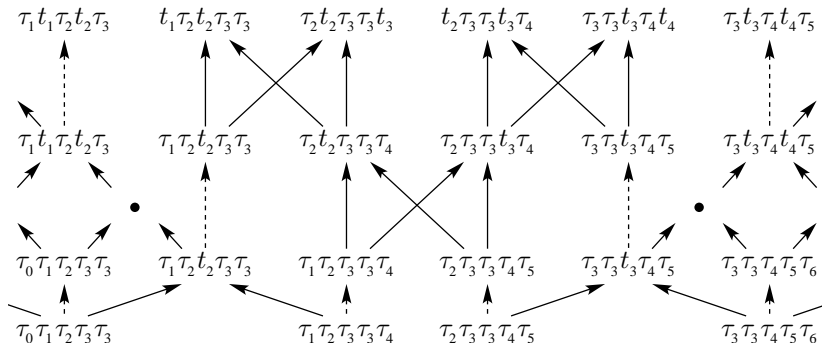
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Multiple knots fit in a common framework



- The refine and smooth algorithm has a natural extension for handling unchanged knot intervals

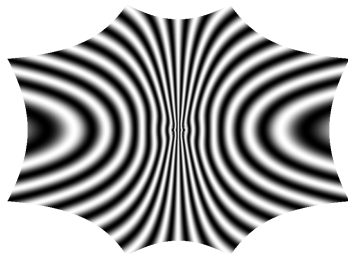
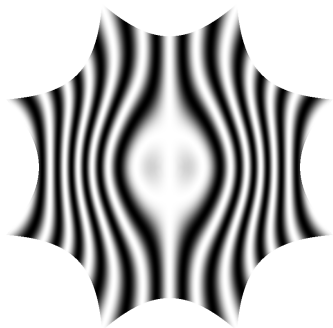


Refine and smooth schemes haven't been tuned

Bounded curvature



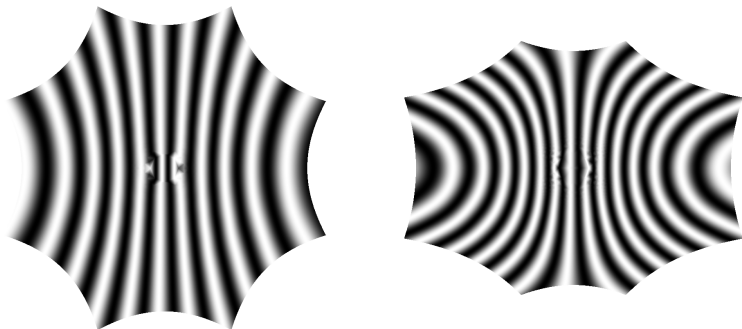
- ▶ Existing general degree schemes have **zero** or **unbounded** curvature at extraordinary points



Degree 9, $\lambda = 0.6612$, $\lambda^2 = 0.4372$, $\mu_0 = 0.25$, $\mu_2 = 0.5$

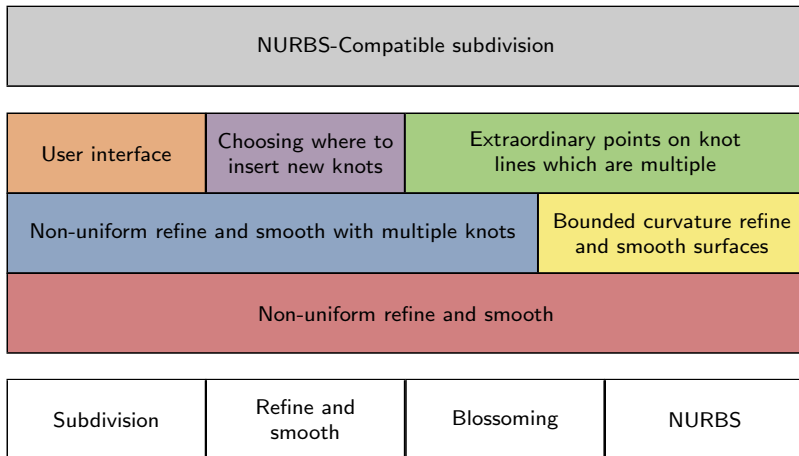


- ▶ For **bounded** curvature, need $\lambda^2 = \mu_0 = \mu_2$
- ▶ We want to tune a **whole family** of uniform schemes



Degree 9, $\lambda = 0.7308$, $\lambda^2 = 0.5340$, $\mu_0 = 0.5340$, $\mu_2 = 0.5340$

Subproblems



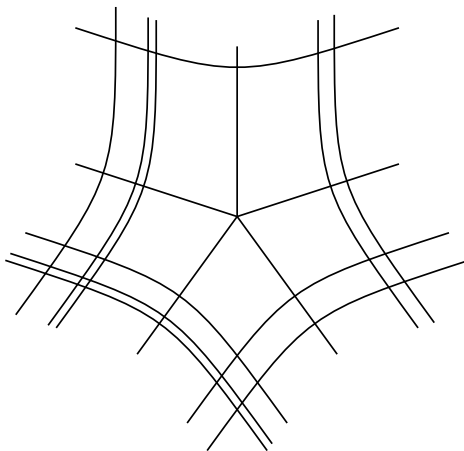
Progress so far

NURBS-Compatible subdivision			
User interface	Choosing where to insert new knots	Extraordinary points on knot lines which are multiple	
Non-uniform refine and smooth with multiple knots			Bounded curvature refine and smooth surfaces
Non-uniform refine and smooth			
Subdivision	Refine and smooth	Blossoming	NURBS

We can choose new knots



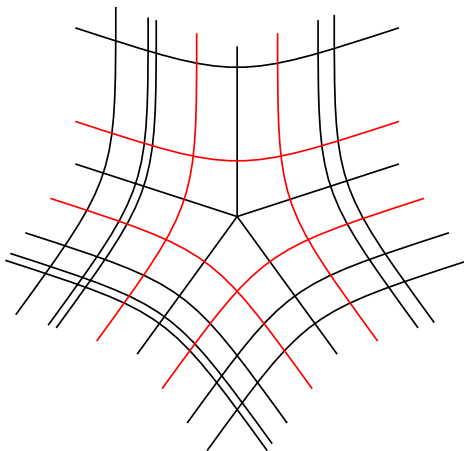
- To give a locally uniform parameterization around extraordinary points



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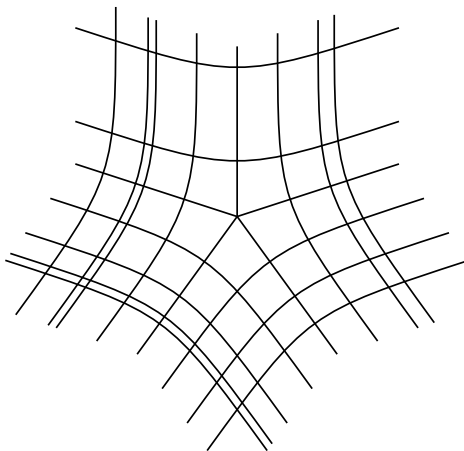
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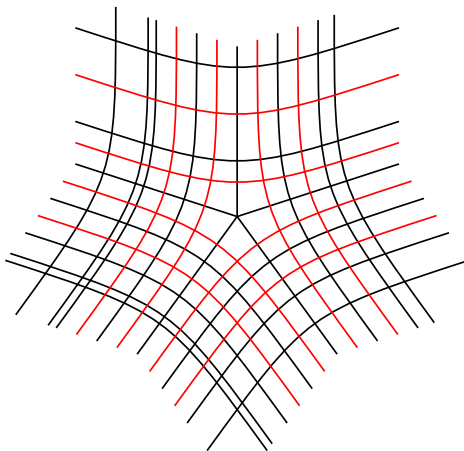
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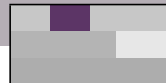
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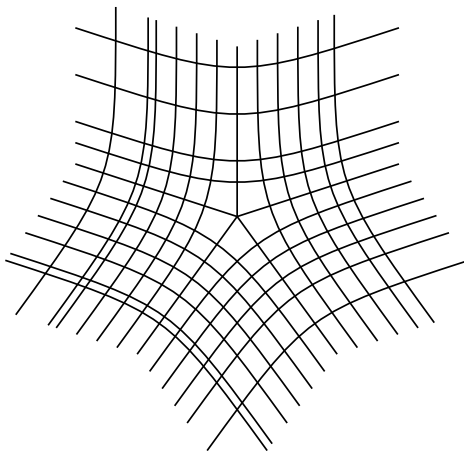
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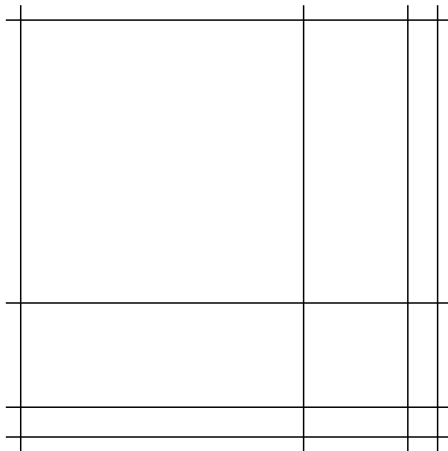
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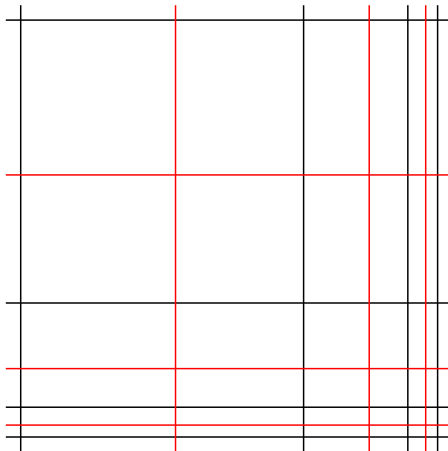
- So that knot vectors tend to uniform almost everywhere



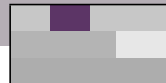
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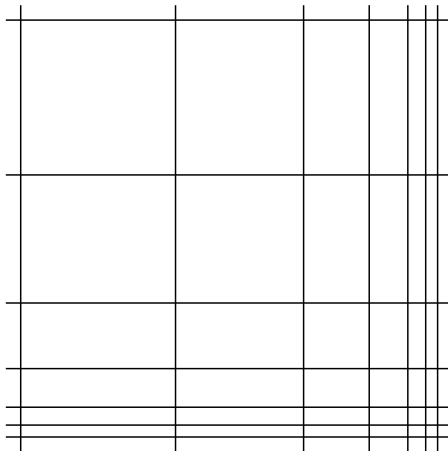
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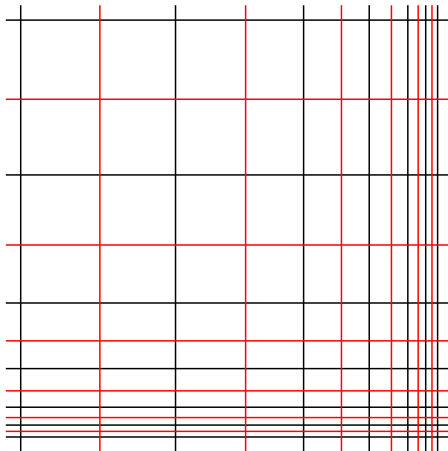
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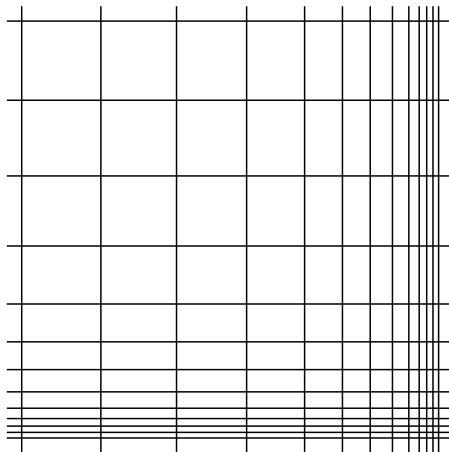
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We can choose new knots



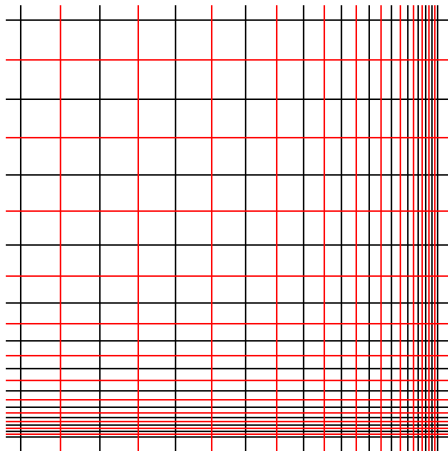
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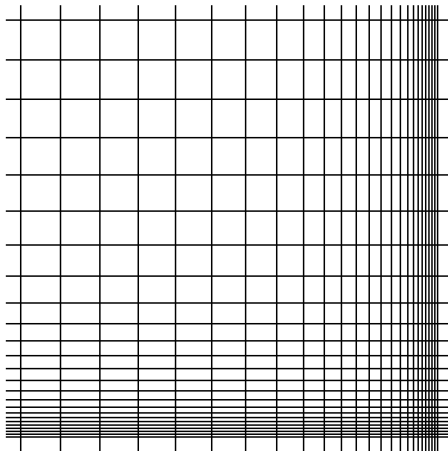
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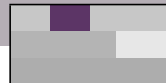
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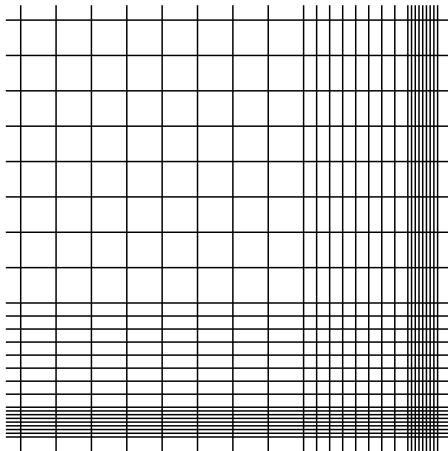
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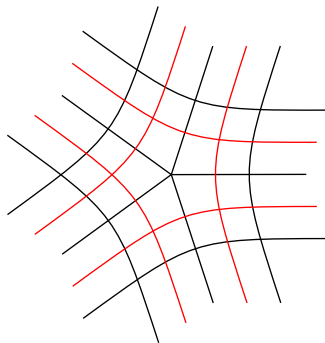
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Multiple knots can cause problems



- ▶ Knot insertion can't create a locally uniform parameterization around multiple knots

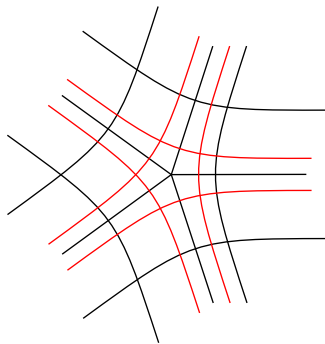


- ▶ We don't know how to solve this one...

Multiple knots can cause problems



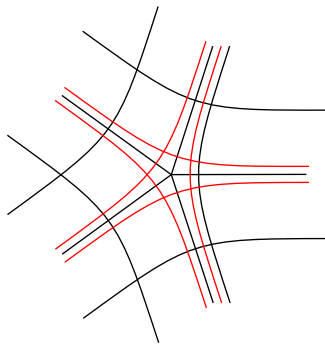
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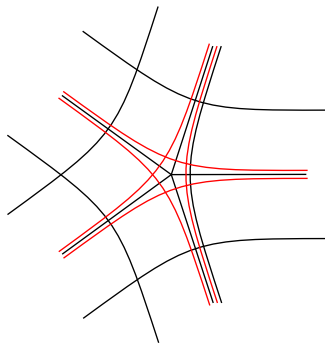


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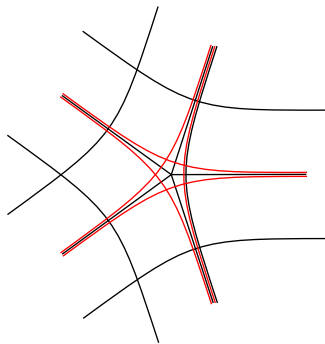
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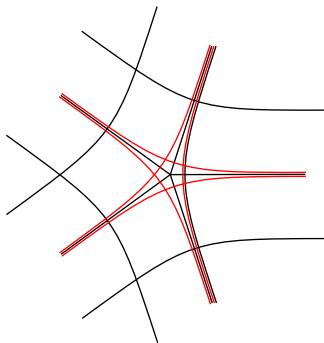


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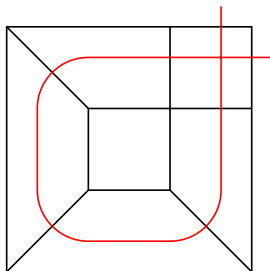
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- ▶ We don't know how to solve this one...
so maybe we should just exclude it?

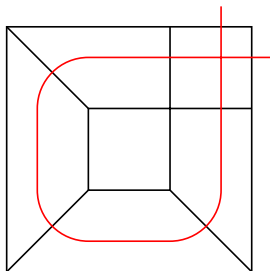


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- ▶ Extraordinary points may cause parameter intervals to propagate in surprising ways





- ▶ Regular regions will have only the flexibility of tensor product B-splines
- ▶ Extraordinary points may cause parameter intervals to propagate in surprising ways



- ▶ How can we support a designer in making these annotations?
- ▶ Is the propagation acceptable?

Conclusion

- ▶ We are close to having subdivision surfaces that contain **NURBS as a subset**
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Conclusion

- ▶ We are close to having subdivision surfaces that contain **NURBS as a subset**
- ▶ Will they be useful?
 - ▶ General enough?
 - ▶ Too complicated?
- ▶ Interesting challenges ahead
 - ▶ Bounded curvature for arbitrary even degree
 - ▶ A knot insertion strategy that works for all surfaces