NURBS with Extraordinary Points

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Freeform surfaces

Subdivision surfaces

 Greater flexibility

NURBS

 Greater control
Freeform surfaces

Subdivision surfaces

Greater flexibility
Industry standard for animation/entertainment

NURBS

Greater control
Industry standard for Computer Aided Design
A subdivision superset of NURBS
Previous work

NURBS

UBS

Low degree NURBS

General degree Quadratic and cubic Regular surfaces

1978 Catmull-Clark, Doo-Sabin

1998 Sederberg et al.

2006 Müller et al.

2001 Prautzsch

2001 Zorin & Schröder, Stam

High degree, non-uniform subdivision surfaces

NURBS with extraordinary points
Previous work

NURBS

General degree

UBS

Low degree

Quadratic and cubic

Low degree

NURBS

UBS

1978, Catmull-Clark, Doo-Sabin

1998, Sederberg et al.

2001, Prautzsch

2006, Müller et al.

NURBS with extraorindary points

High degree, non-uniform subdivision surfaces
Previous work

- NURBS
- General degree NUBS
- Low degree NUBS
- Quadratic and cubic
- Low degree UBS
- UBS

Regular surfaces

Subdivision surfaces

1978 Catmull-Clark, Doo-Sabin

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2006 Müller et al.

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High degree, non-uniform subdivision surfaces
Previous work

- **NURBS**
  - General degree
  - Quadratic and cubic
  - Low degree
  - Regular surfaces

- **UBS**
  - High degree, non-uniform subdivision surfaces
  - NURBS with extraordinary points

- **Subdivision surfaces**
  - 1978: Catmull-Clark, Doo-Sabin
  - 1998: Prautzsch
  - 2001: Zorin & Schröder, Stam
  - 1998: Sederberg et al.
  - 2006: Müller et al.
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Regular surfaces

Subdivision surfaces

Quadratic and cubic
Outline

Knot insertion algorithm

NURBS with extraordinary points

Knot insertion strategy

Bounded curvature solutions
Outline

Knot insertion algorithm

Knot insertion strategy

NURBS with extraordinary points

Bounded curvature solutions
Oslo algorithm: Cohen et al. (1980)

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Refine and smooth: Cashman et al. (2009)

cf. Lane-Riesenfeld (1980), Schaefer and Goldman (2009)
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Knot insertion for surfaces

\[ a + b + c = 1 \]
Knot insertion for surfaces

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Local knot vectors

We assign a knot spacing to each strip of quadrilateral faces.

This face has knot spacings \( \{k_1, k_2, k_3, k_4, k_5\} \) in one direction and \( \{k_6, k_7, k_8, k_9, k_{10}\} \) in the other.
Choosing where to insert new knots

We can subdivide each interval at any given position or not at all.

Our strategy is to:
- subdivide large knot intervals first.
- create uniform spacing around extraordinary points.
An example
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NURBS with extraordinary points

Knot insertion algorithm

Knot insertion strategy

Bounded curvature solutions
Bounded curvature at extraordinary points

Tuning for bounded curvature avoids

► flat spots
   \( \text{(where quadratic components shrink too fast)} \)

► divergent curvature
   \( \text{(where quadratic components shrink too slowly)} \)
Bounded curvature at extraordinary points

Tuning for bounded curvature avoids

- flat spots
  
  *(where quadratic components shrink too fast)*

- divergent curvature
  
  *(where quadratic components shrink too slowly)*

As well as

- prescribed positive Gaussian curvature
  
  *(where hyperbolic components shrink too fast)*

- prescribed negative Gaussian curvature
  
  *(where the elliptic component shrinks too fast)*
Bounded curvature at extraordinary points

Tuning for bounded curvature avoids

▶ flat spots
  (where quadratic components shrink too fast)
▶ divergent curvature
  (where quadratic components shrink too slowly)

As well as

▶ prescribed positive Gaussian curvature
  (where hyperbolic components shrink too fast)
▶ prescribed negative Gaussian curvature
  (where the elliptic component shrinks too fast)

In general, our knot insertion strategy means that non-uniform configurations have bounded curvature too
Characteristic rings

Degree 3

Degree 5

Degree 7

cf. Augsdörfer et al. (2006)
Characteristic rings

Degree 3

Degree 5

Degree 7

cf. Augsdörfer et al. (2006)
What is the required elliptic eigenvector?

Degree 5
What is the required elliptic eigenvector?
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Degree 5

Valency 3

Degree 13
Valency 3

Degree 13
Valency 3

Degree 13
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Valency 3

Degree 13
Valency 3

Degree 13
Conclusion

We can offer surfaces with
- all the capabilities of NURBS, and
- the flexibility of subdivision

Our “to do” list:
- Bounded curvature for even degrees

Open questions:
- What about faces with a non-rectangular parameter space?
- Is there a better knot insertion strategy?
- What is the performance of our bounded curvature solutions?