NURBS with Extraordinary Points

Tom Cashman[1] Neil Dodgson[1] Malcolm Sabin[2]

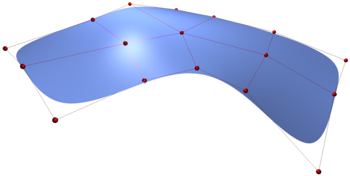
[1]Computer Laboratory
University of Cambridge

[2]Numerical Geometry Ltd

Workshop on Industry Challenges
Darmstadt University of Technology
12th March 2009

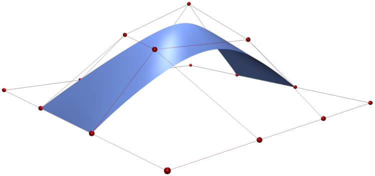
Freeform surfaces

Subdivision surfaces



Greater flexibility

NURBS



Greater control

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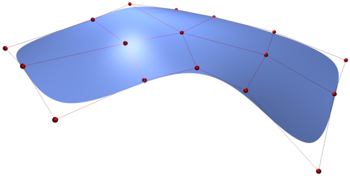
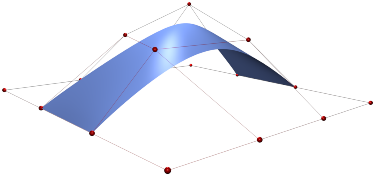


Greater flexibility

Industry standard for animation/entertainment

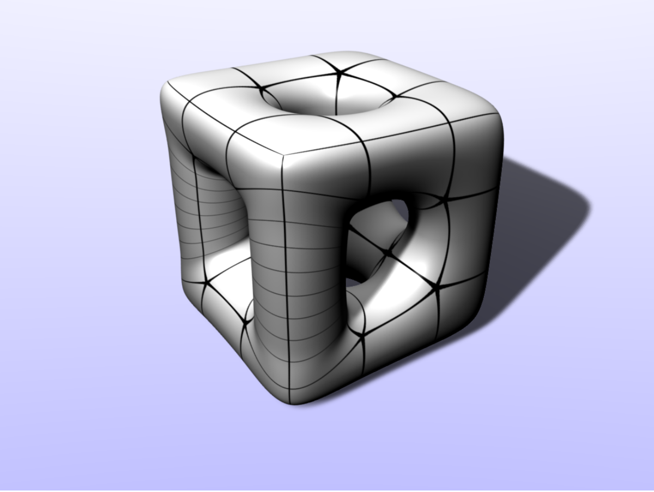
NURBS



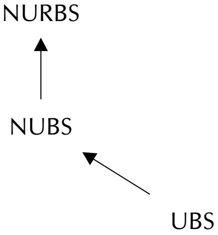
Greater control

Industry standard for Computer Aided Design

A subdivision superset of NURBS



Previous work



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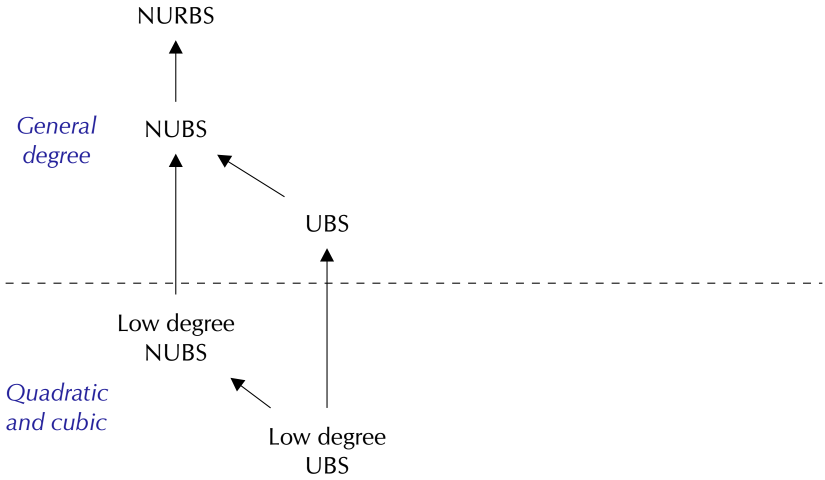

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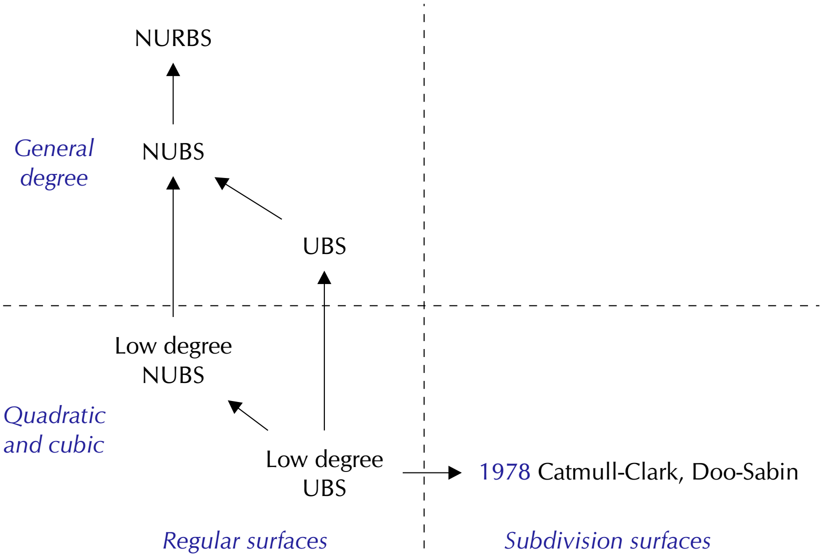

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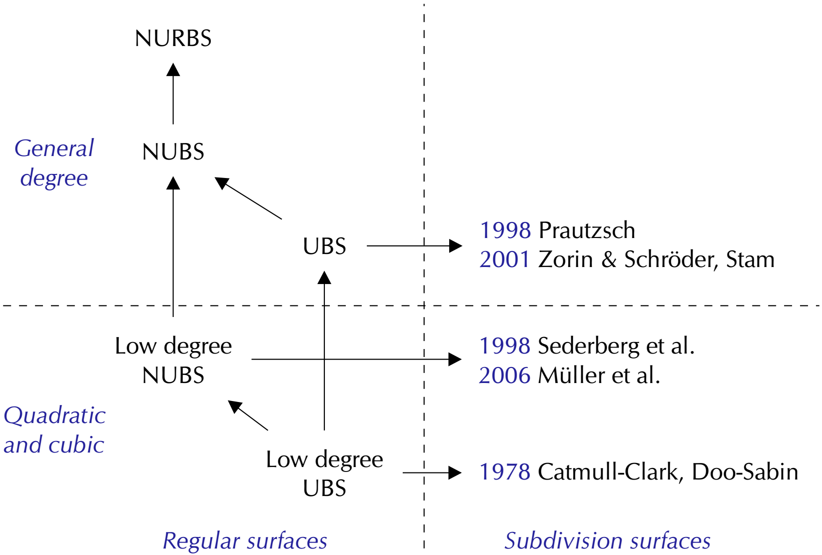

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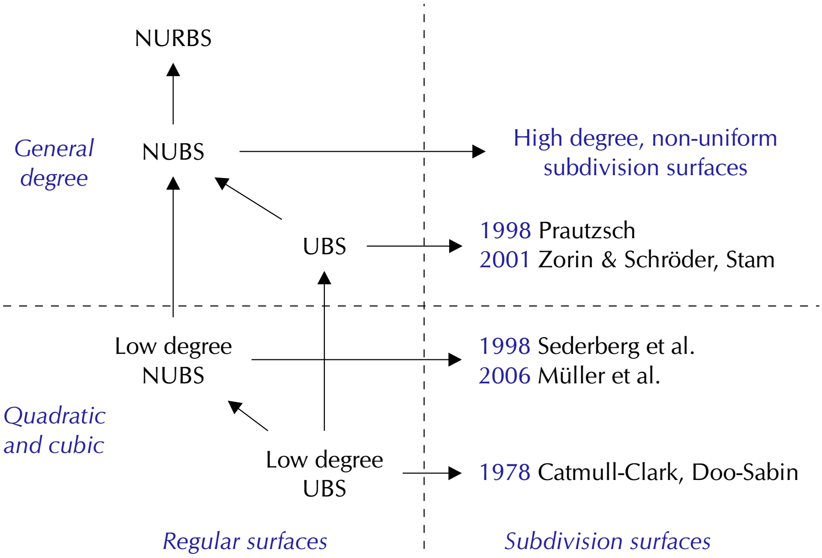

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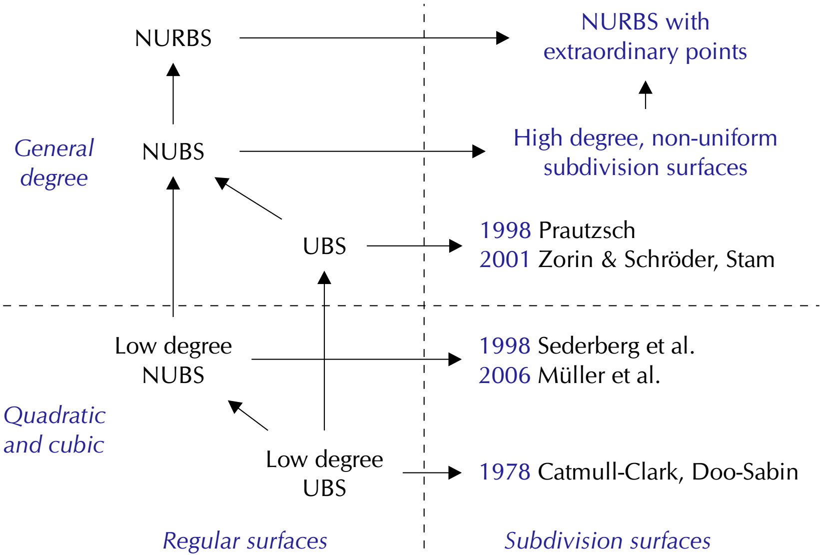

Outline

Knot insertion algorithm

Knot insertion strategy

NURBS with extraordinary points

Bounded curvature solutions

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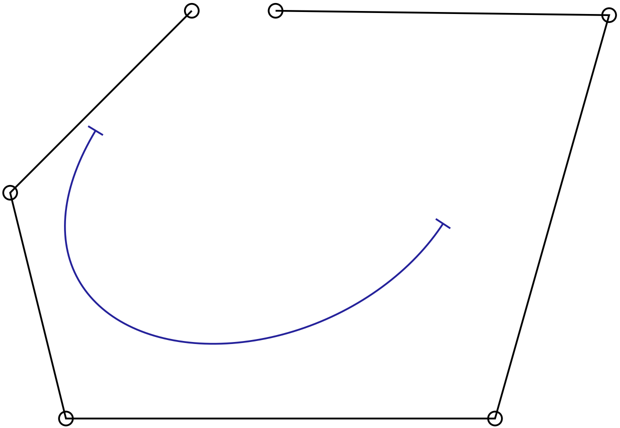
Oslo algorithm: Cohen et al. (1980)



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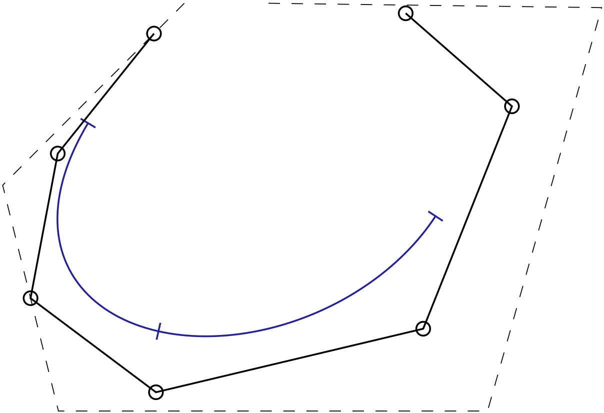

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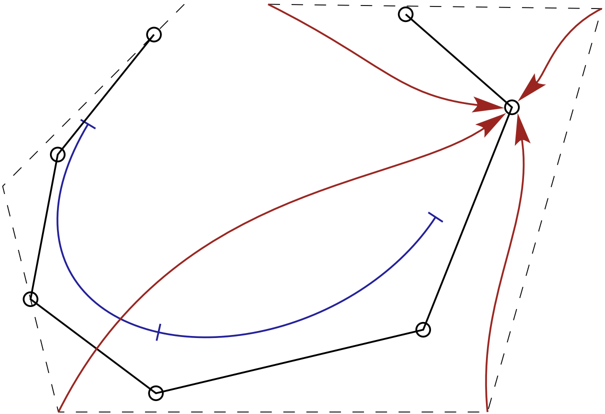

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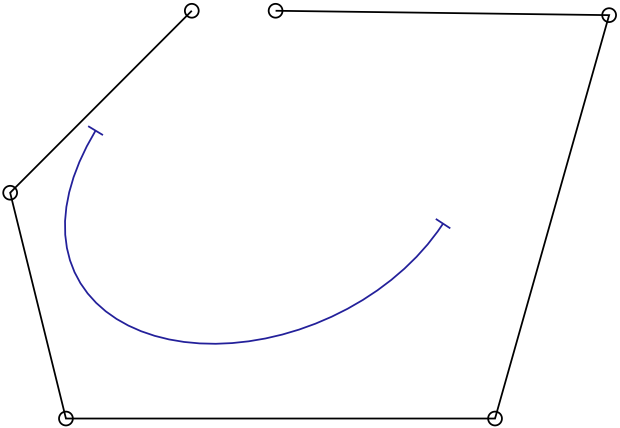
Refine and smooth: Cashman et al. (2009)



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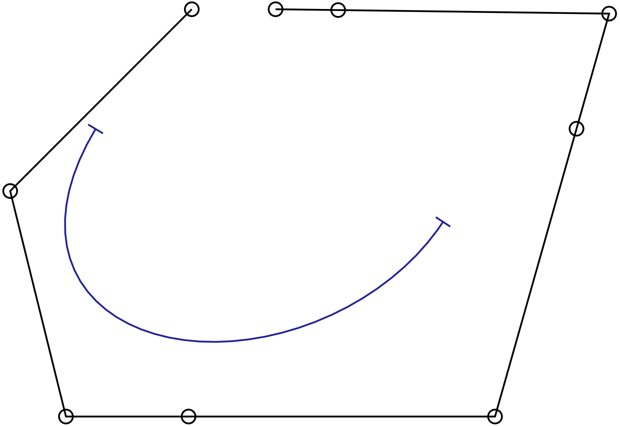

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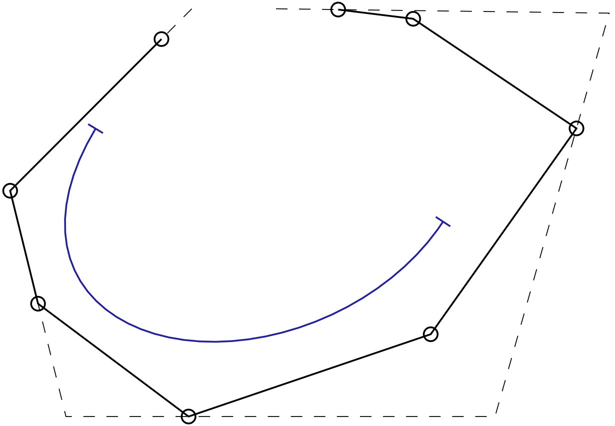

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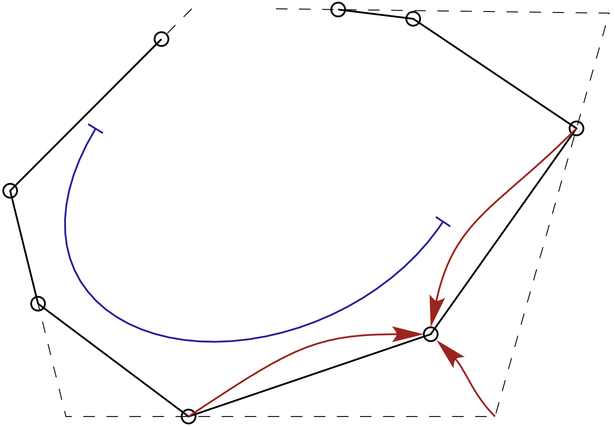

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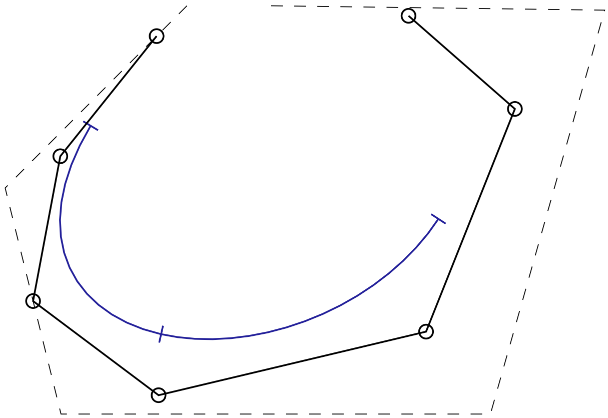

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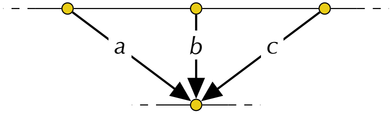
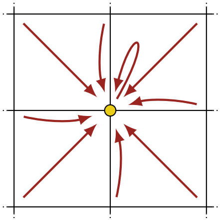
Knot insertion for surfaces



$$a + b + c = 1$$



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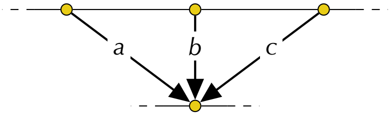

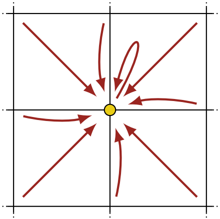

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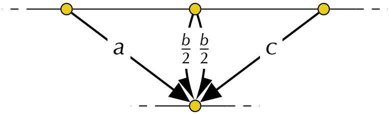
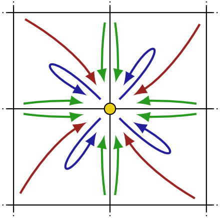




Local knot vectors

We assign a knot spacing to each strip of quadrilateral faces

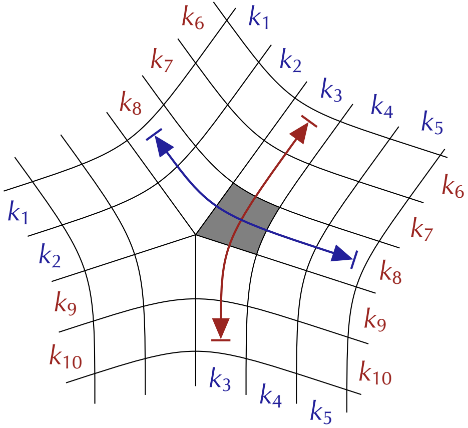
This face has knot spacings

$\{k_1, k_2, k_3, k_4, k_5\}$

in one direction and

$\{k_6, k_7, k_8, k_9, k_{10}\}$

in the other



Outline

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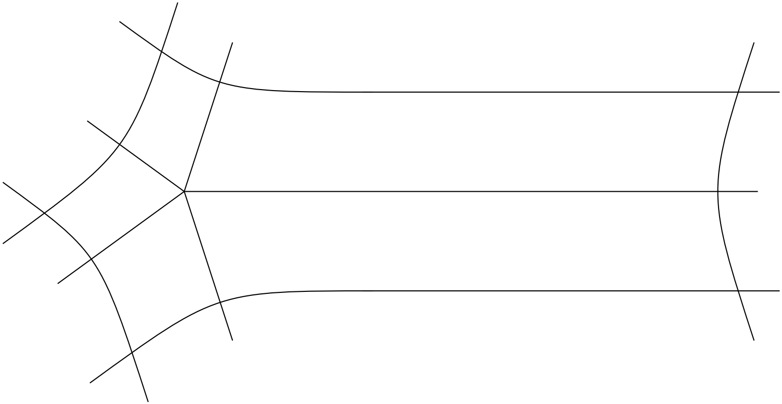
Choosing where to insert new knots

We can subdivide each interval at any given position or not at all

Our strategy is to

- subdivide large knot intervals first
- create uniform spacing around extraordinary points

An example



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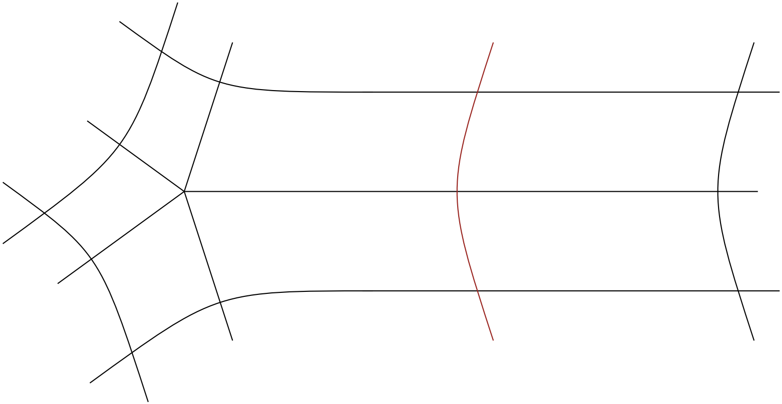

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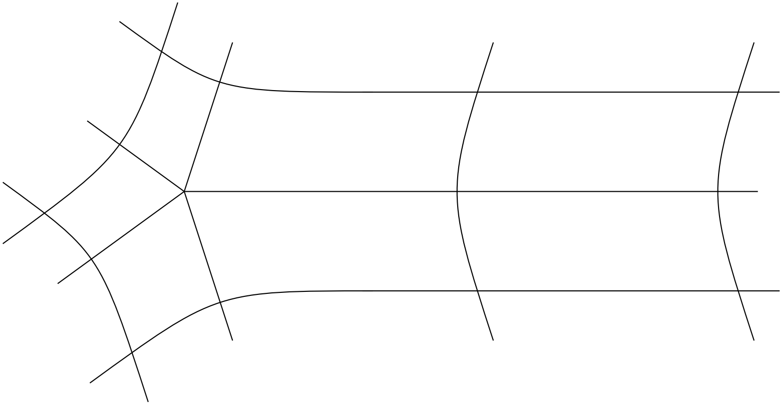

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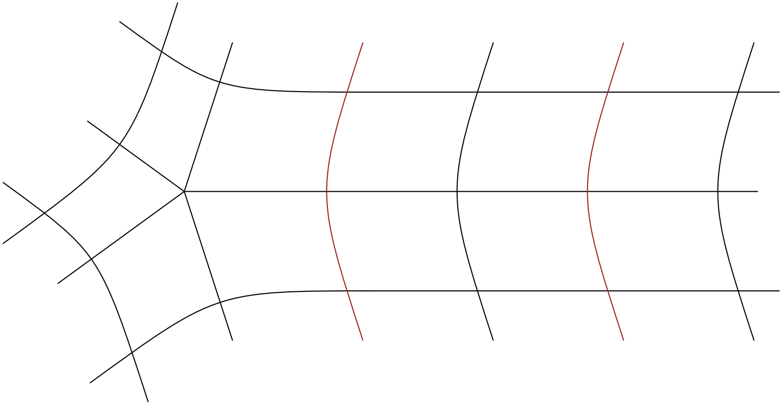

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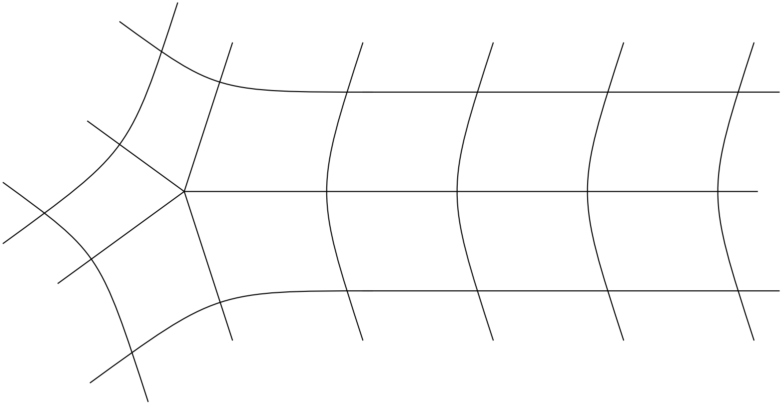

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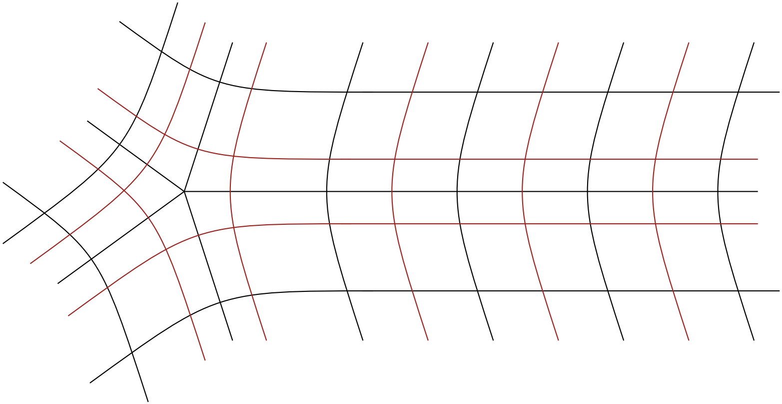

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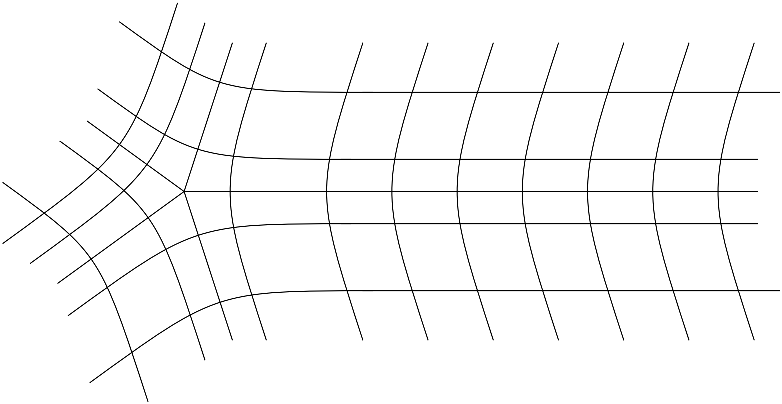

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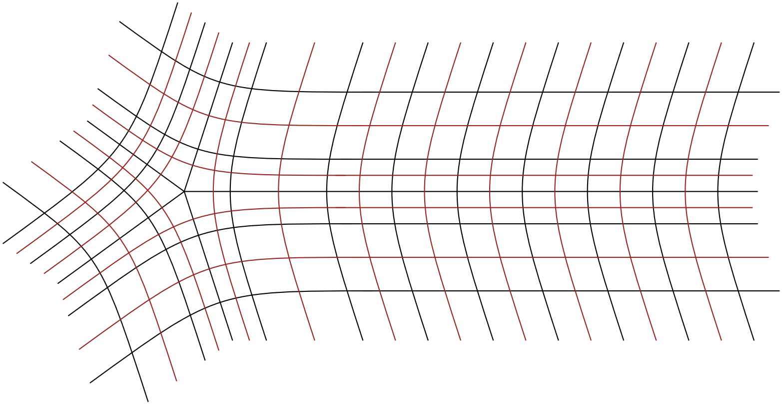

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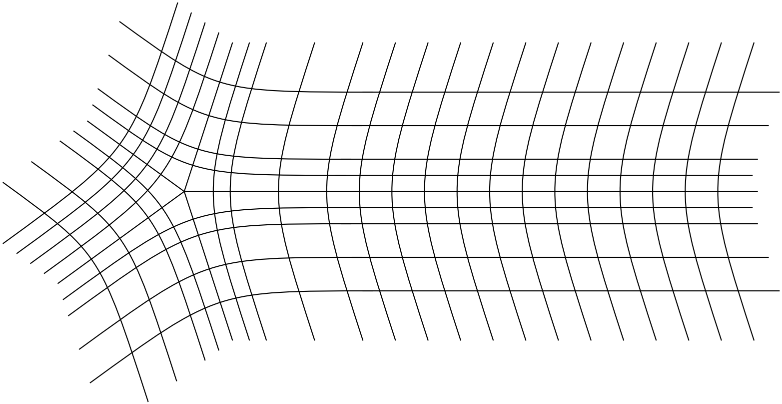

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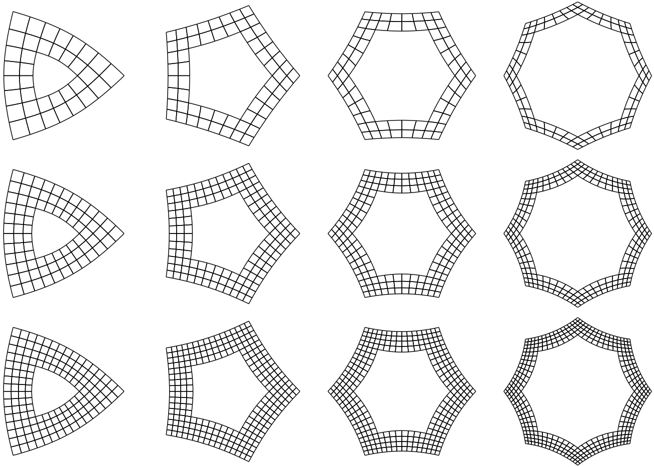
In general, our knot insertion strategy means that non-uniform configurations have bounded curvature too

Characteristic rings

Degree 3

Degree 5

Degree 7



cf. Augsdörfer et al. (2006)

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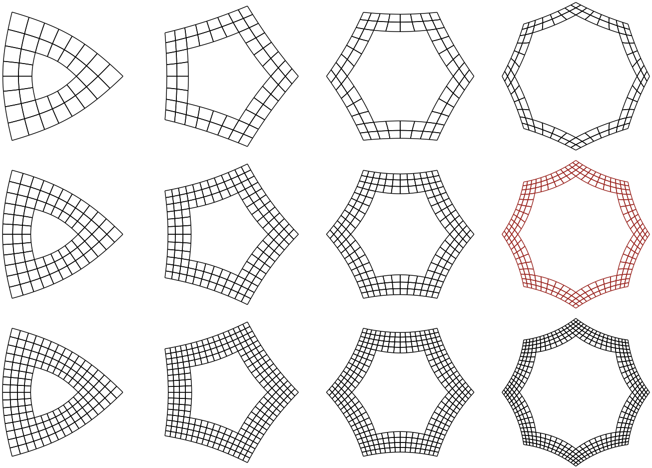

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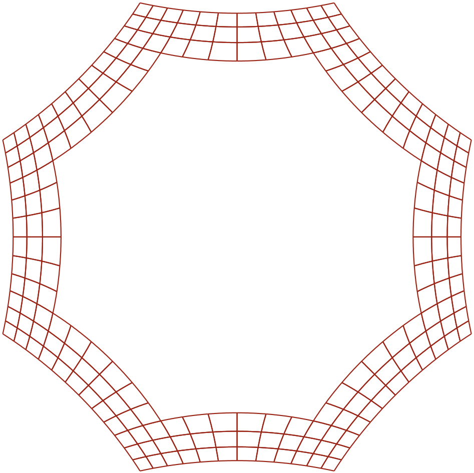
What is the required elliptic eigenvector?



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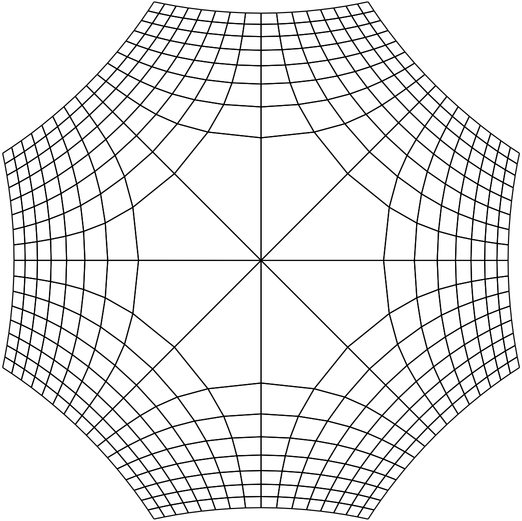

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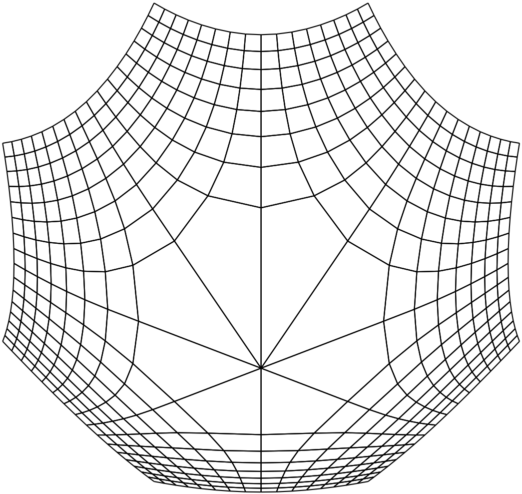

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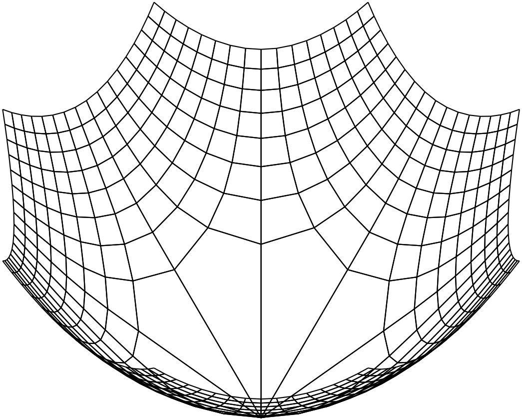

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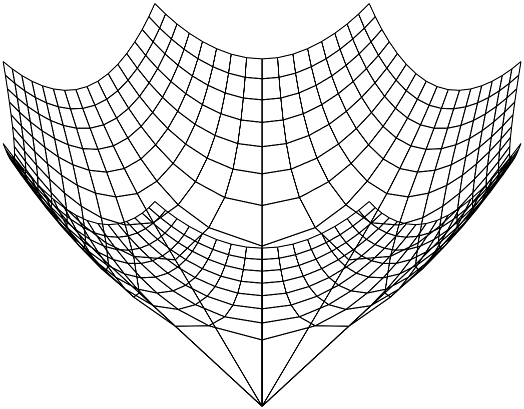

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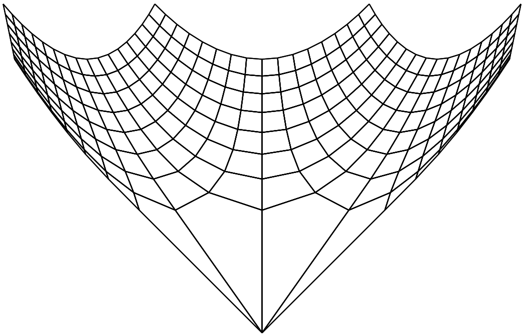

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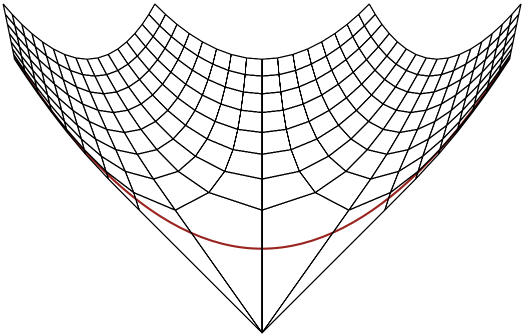
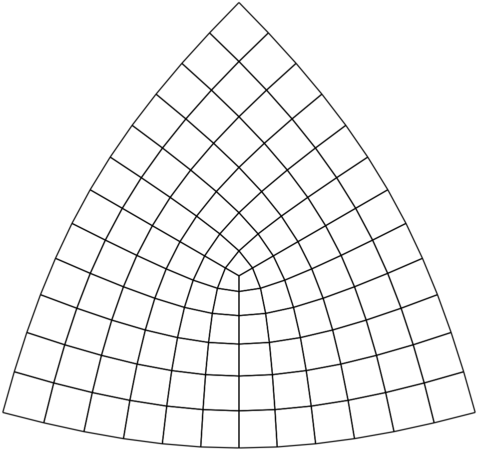


Degree 5

cf. Barthe and Kobbelt (2004)

Valency 3

Degree 13



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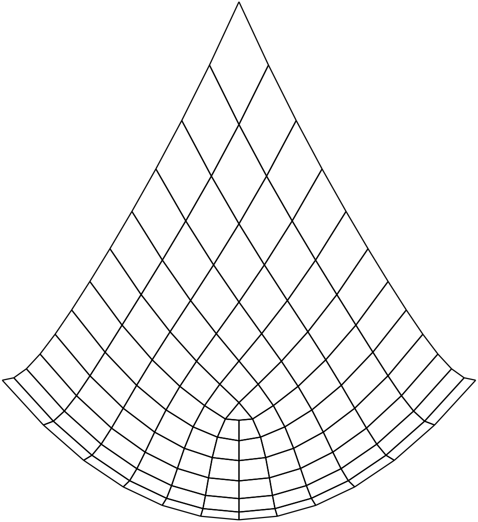

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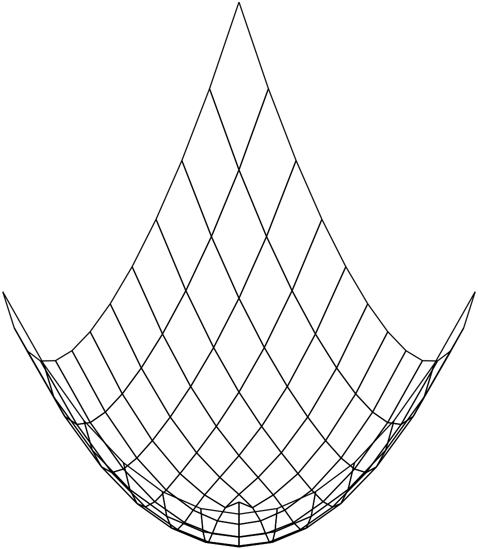

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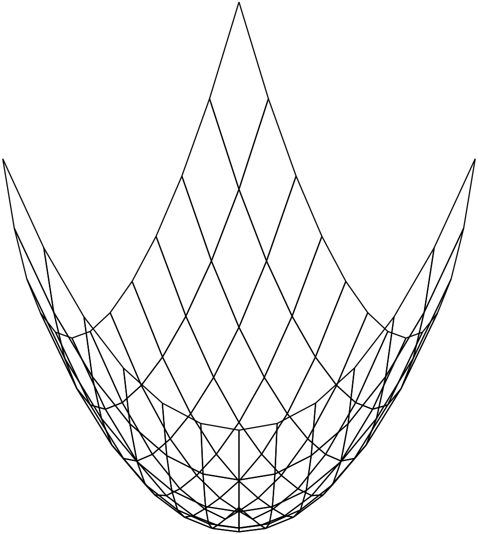

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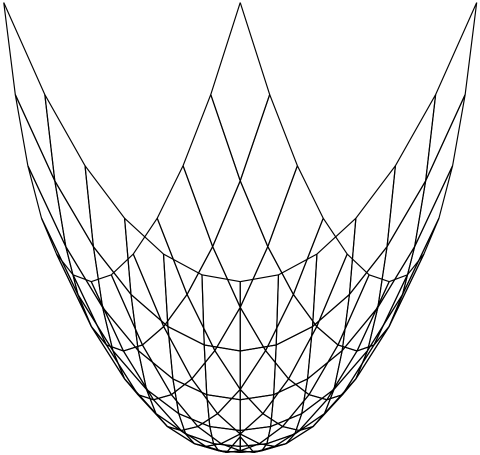

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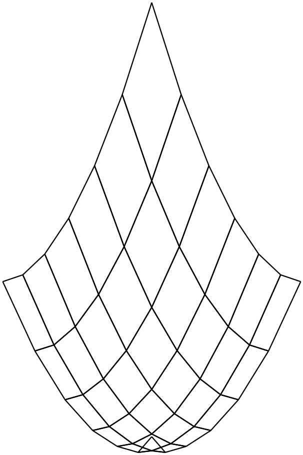
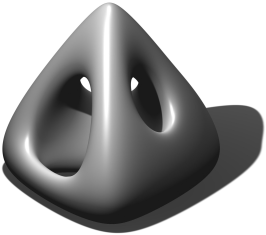


Conclusion

We can offer surfaces with

- all the capabilities of NURBS, and
- the flexibility of subdivision



Our “to do” list:

- Bounded curvature for even degrees

Open questions:

- What about faces with a non-rectangular parameter space?
- Is there a better knot insertion strategy?
- What is the performance of our bounded curvature solutions?