Contrast sensitivity functions (CSF) characterize the sensitivity of the human visual system at different spatial scales, but little is known as to how contrast sensitivity for achromatic and chromatic stimuli changes from a mesopic to a highly photopic range reflecting outdoor illumination levels. The purpose of our study was to further characterize the CSF by measuring both achromatic and chromatic sensitivities for background luminance levels from 0.02 cd/m$^2$ to 7000 cd/m$^2$. Stimuli consisted of Gabor patches of different spatial frequencies and angular sizes, varying from 0.125 to 6 cpd, which were displayed on a custom high dynamic range (HDR) display with luminance levels up to 15,000 cd/m$^2$. Contrast sensitivity was measured in three directions in colour space, reflecting early post-receptoral processing stages: an achromatic (L+M) direction, a ‘red-green’ (L-M) direction, and a ‘lime-violet’ direction (S-(L+M)). Within each session, observers were fully adapted to the fixed background luminance (0.02, 2, 20, 200, 2000 or 7000 cd/m$^2$). Our main finding is that the background luminance has a differential effect on achromatic contrast sensitivity compared to chromatic contrast sensitivity. The achromatic contrast sensitivity increases with higher background luminance up to 200 cd/m$^2$ and then shows a sharp decline when background luminance is increased further. In contrast, the chromatic sensitivity curves do not show a significant sensitivity drop at higher luminance levels. We present a computational luminance-dependent model that predicts the CSF for achromatic and chromatic stimuli of arbitrary size.

**Keywords:** contrast sensitivity functions, color vision, luminance, high light level, mesopic, photopic
Introduction

Spatial vision refers to the ability to see image intensity variations across space. Early measurements of spatial visual sensitivity have focused on spatial resolution and spatial acuity (e.g., Shlaer, 1937) and summation of signals across space (Ricco’s law; Graham & Margara, 1935). Campbell and Robson (1968) were the first to use principles of Fourier analysis to study spatial sensitivity and introduced the contrast sensitivity function, which is the reciprocal of the threshold contrast over a range of spatial frequencies.

Since the seminal paper by Campbell and Robson (1968), progress has been made in our understanding of how spatial sensitivity varies with eccentricity (Robson & Graham, 1981), pattern size (Rovamo, Luntinen, & Näsinen, 1993; Noorlander, Heuts, & Koenderink, 1980), spatial orientation (Campbell, Kulikowski, & Levinson, 1966) and mean luminance level (Mustonen, Rovamo, & Näsinen, 1993; Van Nes & Bouman, 1967). The majority of these studies have focused on contrast sensitivity for achromatic image variations and a comprehensive model for achromatic spatial detection mechanisms has been proposed by Watson and Ahumada (2005).

The contrast sensitivity function for chromatic modulations has been studied to a lesser degree, with some notable exceptions (Green, 1968; Cropper, 1998; Andrews & Pollen, 1979; Granger & Heurtley, 1973; Horst & Bouman, 1969; Y. Kim, Reynaud, Hess, & Mullen, 2017; McKeefry, Murray, & Kulikowski, 2001; Swanson, 1996; Valero, Nieves, Hernandez-Andrs, & Garca, 2004; Lucassen, Lambooij, Sekulovski, & Vogels, 2018). The most extensive set of chromatic contrast sensitivity measurements come from Mullen (1985) and Anderson, Mullen, and Hess (1991), who have assessed the contrast sensitivity for isoluminant red-green and S-cone isolating (lime-violet) gratings with individually adjusted isoluminance points to isolate chromatic channels and silence the luminance-driven mechanisms. Sekiguchi, Williams, and Brainard (1993) employed interference fringes to measure chromatic and luminance contrast sensitivity, thereby eliminating optical blur in addition to chromatic aberration; their contrast sensitivity data are in agreement with the measurements by Anderson et al. (1991).

With the advent of high-dynamic range displays, it is vital to understand how the visual system operates at very high and very low luminance levels. For achromatic contrast modulations, Van Nes and Bouman (1967) and Mustonen et al. (1993) characterized the dependence of the contrast sensitivity on light levels up to 5900 trolands (Van Nes & Bouman, 1967). There are no corresponding measurements for chromatic contrast sensitivity. The purpose of our study is to provide a comprehensive set of measurements and a computational model of contrast sensitivity for achromatic and chromatic modulations as a function of light level, reflecting the contrast sensitivity of an average (standard) observer. CSF models reflecting the visual system of a standard observer afford the generality necessary for practical applications.

Due to the aforementioned purpose, the current study approaches the characterization of chromatic contrast sensitivity slightly differently from Mullen (1985). Truly isoluminant stimuli are difficult to achieve even when using a heterochromatic flicker paradigm (Wagner & Boynton, 1972). There are many possible sources of luminance intrusion, including inter-observer variations in $V(\lambda)$ (Gibson & Tyndall, 1923), retinal illumination (Ikeda & Shimozono, 1981), chromatic aberration (Flitcroft, 1989), and the variation of the isoluminance point across the visual field (Bilodeau & Faubert, 1997). Therefore, rather than experimentally controlling for luminance intrusion, we instead allowed for the possibility that the stimuli are not perfectly isoluminant for each observer, and included luminance intrusion in our model of chromatic channels. Since our aim is to provide a model of chromatic contrast sensitivity for an average (standard) observer which would be applicable to complex spatio-chromatic images (e.g., To & Tolhurst, 2019), it is not useful to optimize stimulus parameters for a small set of individual observers.

In the main experiment (Experiment 1) we measured contrast thresholds for three directions in colour space (achromatic: $\mathcal{L} + \mathcal{M}$, red-green: $\mathcal{L} - \mathcal{M}$, lime-violet: $S - (\mathcal{L} + \mathcal{M})$) as a function of spatial frequency (0.5, 1, 2, 4, 6 cpd) under steady-state adaptation to low mesopic (0.02 cd/m$^2$) and high photopic (7000 cd/m$^2$) light levels. The subsequent experiments served as controls or were necessary to formulate a more general model. In Experiment 2 we tested whether the contrast sensitivity at medium to high luminance levels could be affected by incomplete adaptation, by measuring the contrast sensitivity with the room light on and bright diffuse lights near the stimuli. In Experiment 3, we measured the contrast sensitivity for two additional lower spatial frequencies (0.125 cpd, 0.25 cpd) to evaluate...
whether the chromatic contrast sensitivity has indeed a low-pass shape (Mullen, 1985) or whether, at sufficiently low spatial frequencies, the contrast sensitivity drops as it does for achromatic modulations. In Experiment 4, additional contrast sensitivity data were collected for two more envelope sizes for each spatial frequency to assess spatial summation for the three contrast modulations, which will allow us to generalize our model predictions from the fixed-cycle stimuli to arbitrary stimuli. In Experiment 1, we standardized the width of the Gaussian envelope to the spatial frequency of the underlying sine wave, so that we can treat the width of the Gaussian as a fixed parameter. This is useful for modeling, since we can then treat the width of the Gaussian as a free parameter for predicting contrast sensitivity to stimuli of different sizes.

### Experiment 1: Light Level and Spatial Frequency

In Experiment 1, we tested how contrast sensitivity to both achromatic and chromatic contrast modulations is dependent on the background light level. We measured contrast thresholds for Gabor patches at mean luminances ranging from 0.02 cd/m² (low mesopic range) to 7000 cd/m² (high photopic range).

### Methods

#### Observers

We recruited five observers from the University of Cambridge and 16 observers from the University of Liverpool. Observers provided informed consent prior to participation, in accordance with the ethical approval of respective University Ethics Committees. All naïve observers were reimbursed for their time.

Eleven of the observers were naïve to the purpose of the study (5 female, 11 male, mean age = 26.8 ± 7.7); the rest were the authors (4 female, 1 male, mean age = 40.4 ± 12.6). All observers had normal or corrected-to-normal visual acuity. All observers had normal color vision, verified using the Cambridge Color Test for the CRS ViSaGe System (Mollon & Reffin, 1989) or Ishihara’s Tests for Colour Deficiency, 38-plates edition.

In order to verify that the experimental set-ups in the two locations were calibrated to the same standard, three observers repeated the experiment in both Cambridge and Liverpool. We found that the data from these observers were consistent across location and report only pooled data from these observers.

#### Apparatus

The stimuli were displayed on two custom-built high-dynamic-range (HDR) displays: one in Liverpool (peak luminance: 4,000 cd/m²) and one in Cambridge (peak luminance: 15,000 cd/m²). As the two displays were otherwise identical in construction, we describe the display in Cambridge and flag the differences. The HDR display consisted of an LCD panel (9.7”, 2048 × 1536 px iPad 3/4 retina display; product code: LG LP097QX1) and a DLP projector (Optoma X600 in Cambridge, Acer P1276 in Liverpool; both 1024 × 768 px). The backlight of the LCD was removed and the DLP acted as the replacement backlight (Seetzen et al., 2004); see the schematic diagram (Fig. 1). Because we could modulate both the pixels on the LCD and on the DLP, the maximum contrast we could achieve was a product of the contrast of each display; given 1,000:1 contrast of the LCD and 1,000:1 contrast of the DLP, the maximum contrast of our display was 1,000,000:1. The image on such a display is formed by factorizing the target image, in a linear color space, into the DLP and LCD components, such that their product forms the desired image. The factorization was performed using the original method from Seetzen et al. (2004).

Several steps were taken to improve the light efficiency and therefore the brightness of the display. The DLP had its color wheel removed, increasing its brightness by a factor of 3. The color wheel was unnecessary as the LCD panel was responsible for forming a color image. A Fresnel lens with the focal length of 32 cm was introduced behind the LCD panel to ensure that most of the light was directed towards the observer.
Figure 1: Left: a photograph of the HDR display in Cambridge. Right: the schematic diagram of the HDR display design. The image from the DLP is projected on a diffuser and further modulated by an LCD panel with its backlight removed. To improve the light efficiency of the system, a Fresnel lens with a focal length of 32 cm was introduced next to the diffuser such that the light was directed towards the eyes of the observer.

The display was calibrated and driven by custom-made software, written in MATLAB and relying on Psychtoolbox and MATLAB OpenGL (MOGL) extensions (Kleiner, Brainard, & Pelli, 2007). The calibration involved displaying a series of grids consisting of dots, individually on the LCD and DLP, photographing them with a DSLR camera (Canon 550D) and finding both homographic and mesh-based transformations between DLP and LCD pixel coordinates. This step ensured an accurate alignment between LCD and DLP pixels. To compensate for spatial non-uniformity, a photograph of the display showing a uniform field was taken and used to compensate pixel values on the DLP. Because the resolution of the DLP was lower than that of the LCD, and because DLP image sharpness was further reduced by a diffuser, it was necessary to model a point-spread function (PSF) of the DLP and to use it when factorizing target images into LCD and DLP components. The PSF was modeled by taking multiple exposures of the grid of dots, reconstructing from them an HDR image and fitting a Gaussian function approximating the shape the PSF.

The color calibration was performed by measuring display’s spectral emission, individually for LCD and DLP, using a spectroradiometer (JETI Specbos 1211 in Cambridge, PhotoResearch PR-670 in Liverpool). CIE 2006 cone fundamentals (CIE, 2006) were used to calculate the $L$, $M$ and $S$ cone responses as follows:

$$L = 0.689903 \int_{\lambda} l_{1}(\lambda) E(\lambda) \, d\lambda, \quad M = 0.348322 \int_{\lambda} m_{2}(\lambda) E(\lambda) \, d\lambda, \quad S = 0.0371597 \int_{\lambda} s_{2}(\lambda) E(\lambda) \, d\lambda,$$

Figure 2: Spectral power distributions of the HDR displays.
where \( l_2, m_2 \) and \( s_2 \) are 2° cone fundamentals\(^1\) and \( E \) is the measured spectral radiance emitted from the display. The \( l_2 \) and \( m_2 \) spectra were scaled such that the sum corresponded to luminance and the sensitivity of the S cones was set so that \( s_2(\lambda)/V(\lambda) \) peaks at 1 (CIE, 2006). All our calculations were based on photopic luminance, including the lowest luminance levels of 0.02 cd/m\(^2\) which was at the lower end of the mesopic range (Barbur & Stockman, 2010).

The responses were fitted to the gain-offset-gamma display model (Berns, 1996) for the LCD and a 1-dimensional look-up table was used for the DLP (since it was achromatic after removing the color wheel); see Figure 2 for the spectral emission of the two HDR displays.

Both LCD and DLP were natively driven by 8-bit signals. To prevent banding artifacts from quantization, we used spatio-temporal dithering for LCD and bit-stealing for DLP to extend the effective bit-depth to 10-bits per color channel. The display driver was written in the OpenGL shading language (GLSL) to factorize and render images in real-time.

**Stimuli**

The stimuli were Gabor patches created by multiplying a sinusoidal grating with a Gaussian envelope (Fig. 4). The phase was adjusted so that the zero-crossing was exactly in the center of the stimulus. Each grating was modulated along one of the three cardinal colour axes in Derrington-Krauskopf-Lennie (DKL) space: achromatic (C1), red-green (C2), and lime-violet (C3) (Derrington, Krauskopf, & Lennie, 1984). Modulations in this colour space can either be described by the stimulus properties (achromatic, red-green, lime-violet) or by the response properties of a set of hypothesized mechanisms (Brainard, 1996) that are isolated by these stimulus modulations (\( L + M, L - M, S - (L + M) \)).

In terms of the stimulus properties (Fig. 3), changes along the achromatic direction (C1) resulted in all three cone classes being modulated such that the cone contrasts are identical; modulations along the red-green axis (C2) leave the excitation of the S cones constant and the excitation of the L and M cones co-varies as to keep their sum constant. Along the third, the lime-violet direction (C3), only the S cones are modulated. These modulations in colour space (C1, C2, C3) are designed to isolate a set of hypothesized mechanisms: a luminance mechanism (\( L + M \)), and two cone-opponent colour mechanisms (\( L - M, S - (L + M) \)). The responses of these mechanisms are as follows:

\[
R_{Ach} = L + M \\
R_{RG} = L - \frac{L_{D65}}{M_{D65}} M \\
R_{YV} = \frac{L_{D65} + M_{D65}}{S_{D65}} \cdot S - (L + M) ,
\]

where \( L_{D65}, M_{D65} \) and \( S_{D65} \) are the cone responses corresponding to D65 grey (the background chromaticity used in all experiments).

For the clarity of notation, we will use \( Ach \) to denote \( L + M \) mechanism, \( RG \) to denote \( L - M \) mechanism and \( YV \) to denote \( S - (L + M) \) mechanism. To achieve comparable response units in these three mechanisms, the responses can be scaled such that the response for each mechanism is unity for a stimulus of unit pooled cone contrast. Equations 2 to 4 define the matrix that maps L, M, S cone excitations into the three mechanisms. The inverse of that matrix then defines the stimulus modulations in LMS space that are required to achieve selective stimulation of the hypothesized mechanisms. This inverse matrix can then be converted to linearized RGB (see appendix for the matlab files). For a tutorial on how to implement the DKL space, the reader should consult Brainard (1996).

The responses of these mechanisms (Eq. 2-4) are scaled to a large extent arbitrarily (Capilla, Malo, Luque, & Artigas, 1998). Therefore, rather than using these responses to define stimulus contrast, we used the length in cone contrast space (Eq. 5) since it would allow comparison across different colour directions. The rationale for measuring contrast sensitivity along these three modulation directions in color space was twofold. First, these modulations were likely to preferentially stimulate early post-receptoral mechanisms. While it was

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\(^1\)Tabulated cone fundamentals can be found at [http://cvrl.ucl.ac.uk/](http://cvrl.ucl.ac.uk/)
unlikely that cortical mechanisms could be isolated with these colour modulations (Shapley & Hawken, 2011), it still allowed us to characterize the contrast sensitivity for salient and, to some degree, independent mechanisms. Second, it constituted a device-independent definition of the chromatic stimulus modulations and allowed comparisons with previously obtained CSF measurements.

Stimuli were modulated around a neutral grey (white) background of a D65 metamer (CIE 1931 x, y = 0.3127, 0.3290). The contrast varied from trial to trial, as determined by an adaptive QUEST procedure (Watson & Pelli, 1983).

The standard deviation of the Gaussian envelope was set to be half of the wavelength (σ = 0.5/f [deg]). The Gabor stimuli were modulated around a neutral grey (white) background of a D65 metamer (CIE 1931 x, y = 0.3127, 0.3290). The contrast varied from trial to trial, as determined by an adaptive QUEST procedure (Watson & Pelli, 1983).

The standard deviation of the Gaussian envelope was set to be half of the wavelength (σ = 0.5/f [deg]). The Gabor stimuli were of spatial frequencies 0.5, 1, 2, 4, or 6 cycles per degree of visual angle (cpd). Thus, the ±2σ region of the Gabor patches subtended 4° × 4°, 2° × 2°, 1° × 1°, 0.5° × 0.5°, and 0.33° × 0.33°, respectively. Using these Gabor stimuli with a fixed number of visible cycles allowed us to treat the width of the Gaussian as a fixed parameter. This was useful for modeling, since we could then treat the width of the Gaussian envelope as a free parameter for predicting contrast sensitivity to stimuli of different sizes.
**Procedure**

The experiment was grouped into multiple sessions by mean luminance level to ensure that observers were fully adapted to the display luminance during data collection. The mean luminance was one of 0.02, 0.2, 2, 20, 200, 2000, or 7000 cd/m²; assuming Watson’s (2012) unified pupillary model, these luminances were equivalent to 0.86, 7.83, 62.87, 416.80, 2335.85, 13245.57, 36560.55 trolands, respectively. For sessions at 0.02 and 0.2 cd/m², observers adapted to the darkness for 5 to 10 minutes prior to starting the study, and remained in the experiment room until the end of the session. Sessions at 7000 cd/m² were conducted exclusively in Cambridge.

At the beginning of each session, we obtained an initial estimate of the contrast threshold using a method of adjustment task. This was used as an initial estimate for the QUEST procedure.

The main task was a 4AFC detection task, in which observers indicated which quadrant of the display contained a Gabor patch. The stimulus was positioned 3.77° from the center of the display: upper left, upper right, lower left, or lower right. The stimulus was displayed until observer response. Between trials, a mask was presented over the 4AFC stimulus region for 500 ms to neutralize adaptation to the previously seen Gabor. To create the mask, we sampled a matrix of random numbers from \( U(-1, 1) \) per color channel, then blurred the resulting image with a Gaussian kernel (\( \sigma = 4 \) px).

The stimulus contrast was determined using a QUEST procedure (Watson & Pelli, 1983). There was one QUEST staircase per spatial frequency and color modulation combination, for a total of 21 staircases per session. Each staircase lasted for a minimum of 25 and a maximum of 35 trials.

Within a session, observers saw Gabor patches of different spatial frequencies and color modulation interleaved in a random order. Since the Gabor orientation was not a stimulus dimension of interest, we randomly chose a vertical or horizontal orientation for each trial. Observers had no information as to the spatial frequency, color modulation, or orientation of the target Gabor patch.

Each session lasted approximately 40 to 50 minutes. Some observers chose to omit sessions at 7000 cd/m², as the high luminance could be uncomfortable to view for an extended period of time.

Observers were seated 91 cm from the HDR display such that the display subtended 12.5° × 9.4°. The head position was fixed with a chin rest to the horizontal and vertical center of the display. Observers were allowed to move their eyes in order to examine stimuli. All viewing was binocular.

**Results**

For each condition, we computed the maximum-likelihood estimate of the contrast sensitivity. Each threshold estimate is typically based on between 25 to 35 trials. Threshold contrast is defined as the normalised length in cone contrast space (Eq. 5):

\[
C_t = \frac{1}{\sqrt{3}} \left( \frac{\Delta \mathcal{L}}{L_0} \right)^2 + \left( \frac{\Delta \mathcal{M}}{M_0} \right)^2 + \left( \frac{\Delta \mathcal{S}}{S_0} \right)^2 \tag{5}
\]

\( C_t \) = Threshold cone contrast

\( \Delta \mathcal{L}, \Delta \mathcal{M}, \Delta \mathcal{S} = \) Incremental L,M,S cone absorptions

\( L_0, M_0, S_0 = \) L,M,S absorptions of the display background

The advantage of this contrast measure is that it allows device-independent comparisons between different directions in colour space and is identical to the standard Michelson contrast for achromatic modulations.

Figure 5 shows the contrast sensitivities as a function of frequency for light levels ranging from 0.02 cd/m² to 7000 cd/m². The achromatic modulations resulted in a classic band-pass response for medium to high luminance levels (from 2 cd/m² onwards), with a peak response at medium spatial frequencies (ranging from 1 to 2 cpd). The gradual change from a low-pass shape at very low luminance levels (0.02 cd/m²) to the typical band-pass shape in higher luminance levels is similar to the results of Van Nes and Bouman (1967).
However, it is likely that measurements at even lower spatial frequencies would have revealed a band-pass shape even for the lowest light level. Red-green and yellow-violet modulations, on the other hand, resulted in a low-pass contrast sensitivity curves at all light levels, with the peak sensitivity occurring at the lowest spatial frequency measured (0.5 cpd). Sensitivity was higher for the red-green stimuli than for the achromatic modulation when expressed as the inverse of the cone contrast, which is consistent with Y. Kim et al. (2017).

When contrast sensitivity is plotted as a function of light level (Figure 6), sensitivity was not a monotonic function of luminance for achromatic modulations ($L+M$); rather, contrast sensitivity was lowest at 0.02 cd/m$^2$ and rose steadily with increasing mean luminance till it reached a peak at 20-200 cd/m$^2$ for low to medium frequencies, then decreased again beyond 200 cd/m$^2$. This luminance dependence interacted with spatial frequency, such that the overall maximum sensitivity occurred between 20-200 cd/m$^2$ for 1-2 cpd where observers could reliably detect a Gabor patch of 2-3% contrast. For red-green and yellow-violet modulations, contrast sensitivity rose steadily as a function of luminance, reaching a maximum at around 200 cd/m$^2$. Only for the lowest frequency, a decrease in peak sensitivity was observed.

In Figure 7, thresholds were plotted as a function of retinal illuminance (trolands). For chromatic stimuli ($L-M$, $S-(L+M)$), contrast thresholds were independent of the retinal illuminance beyond about 2000 trolands, hence consistent with Webers’ law, whereas for achromatic stimuli ($L+M$) thresholds rose again for very high light levels. This failure of Weber-law behaviour in the high photopic range has not been reported by Van Nes and Bouman (1967), probably due to the fact that they only investigated contrast sensitivity up to 5900 trolands and our data show that Weber law only fails at retinal illuminances above 10,000 trolands.

For all three modulation directions, log threshold contrast decreased approximately linearly with log retinal illuminance for low and intermediate light levels, with slopes systematically a bit less than -0.5, (DeVries-Rose law Rose, 1948; De Vries, 1943). Mean slopes were -0.42 and -0.36 for $L-M$ and $S-(L+M)$, respectively (Table 1) and independent of spatial frequency. For achromatic thresholds ($L+M$), the slopes were frequency-dependent and increased with spatial frequency (Table 1), consistent with Mustonen et al. (1993).
The transition from the DeVries-Rose to Weber behaviour was independent of spatial frequency for chromatic modulations (Figure 182); for achromatic stimuli, on the other hand, the inflection point shifted to higher retinal illuminances when spatial frequency was increased. Díez-Ajenjo and Capilla (2010) and Valero et al. (2004) reported a similar difference between chromatic and achromatic...
Table 1: Slopes of log threshold contrast vs log retinal illuminance (trolands) in linear range

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Spatial frequency (cpd)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>( \mathcal{L} + \mathcal{M} )</td>
<td>-0.31259</td>
</tr>
<tr>
<td>( \mathcal{L} - \mathcal{M} )</td>
<td>-0.43583</td>
</tr>
<tr>
<td>( \mathcal{S} - (\mathcal{L} + \mathcal{M}) )</td>
<td>-0.37897</td>
</tr>
</tbody>
</table>

For achromatic gratings, the transition from DeVries-Rose to Weber-law behavior was dependent on spatial frequency and occurred between 1 and 2 cd/m² for the lowest spatial frequency measured (0.5 cpd), consistent with our findings. For chromatic modulations, sensitivity decreased approximately linearly with background luminance in log-log space, without a clear transition point up to 100 cd/m². Valero et al. (2004) only investigated luminances up to 100 cd/m², which is well below our maximum luminance range (7000 cd/m²); in our experiments (Figure 7) the transition point occurred at around 200 cd/m² for chromatic stimuli.

The failure of Weber’s Law behavior for very high luminances may be due to incomplete adaptation to the display background for luminances greater than 200 cd/m². We investigate this possibility in Experiment 2, presented in the following section.

### Experiment 2: Control for Incomplete Adaptation

The purpose of Experiment 2 was to determine whether incomplete adaptation to the mean luminance level affected the contrast sensitivity measurements at high luminances (> 200 cd/m²). Though luminance adaptation is largely local and typically limited to a 0.5°-radius neighborhood (Vangorp, Myszkowski, Graf, & Mantiuk, 2015), the adaptation level can nonetheless be influenced by more distant parts of the visual field. As Experiment 1 was conducted in a dark room and the display subtended only a small portion of the visual field, we considered the possibility that the dark surroundings prevented observers from becoming fully adapted to the high luminance of the display.

Our hypothesis was that such incomplete adaptation was responsible for the drop in sensitivity that we observed at luminance levels above 200 cd/m². To test this hypothesis, we measured contrast sensitivities in bright surroundings. We kept the room light on and placed additional light sources around the display, in order to reduce the difference between the mean luminance of the display and of the region surrounding the display.

#### Methods

Contrast sensitivity was measured at 7000 cd/m². Four observers (3 female, 1 male, mean age = 29.0 ± 8.2) participated; two were authors. The stimuli and the apparatus were identical to those in Experiment 1.

In addition to the HDR display, we placed two photographer’s softboxes near the display, with the goal of increasing the luminance of the region surrounding the HDR display as uniformly as possible. Each softbox was fitted with five 5500K CFL bulbs and enclosed with a white fabric diffuser. From the observer’s perspective, one softbox was directly above the display and one was directly to the right. Due to space restrictions, we did not place any to the observer’s left. The softboxes added 1000 lux of light as measured from the observer’s viewing position with a handheld digital light meter.

#### Results

For the stimulus conditions tested, we did not find any systematic differences in contrast sensitivity when observers were in a dark room or in a bright room with high ambient light levels (Figure 8). This suggests that incomplete adaptation alone cannot explain the
Figure 8: Contrast sensitivity measures in dark (dark symbols) and bright (bright symbols) surroundings. In the DARK SURROUND condition, only the HDR display emitted light. No systematic differences were found between these two conditions.

Figure 9: Chromatic contrast sensitivity extended to lower spatial frequencies from 0.125 cpd to 6 cpd.

Experiment 3: Low Spatial Frequencies

In Experiments 1 and 2, contrast sensitivity for the red-green and lime-violet modulations was low-pass in shape, i.e., the peak sensitivity occurred at the lowest spatial frequency measured. In Experiment 3, we examined whether chromatic contrast sensitivity measurements at extremely low spatial frequencies would reveal a bandpass shape as observed for achromatic modulations. We therefore tested additional low frequencies ranging from 0.125 cpd to 6 cpd, at three luminance levels: 0.02, 200, and 7000 cd/m$^2$, for red-green and lime-violet stimuli.

Methods

Five observers (two male, three female, mean age = 27.2 ± 4.3) from Cambridge and Liverpool participated in this experiment. One observer was naïve; the rest were authors or had previously participated in Experiment 1 or 2. Two observers participated in the
full set of spatial frequency conditions; the remaining three participated only in the three lowest spatial frequency conditions.

All stimulus parameters were as described in Experiment 1 but thresholds were only measured for the two chromatic directions.

For the 0.125 cpd, 0.25 cpd and 0.5 cpd conditions, observers were seated at 45.5 cm, such that the HDR display subtended 24.8° × 18.7° and could show up to four 9.0° × 9.0° Gabor patches at a time. Observers did not see a sharp boundary at the border of the 9° × 9° region, since the experiment was conducted near the observers’ contrast detection threshold.

**Results**

We did not find a systematic reduction in contrast sensitivity at the very low frequency (0.125 cpd) for the low and intermediate (0.02 and 20 cd/m²) luminance levels (Figure 9). For the highest luminances (7000 cd/m²), there was some evidence that the chromatic contrast sensitivity drops off as the achromatic sensitivity does. However, these differences are within measurement error and our experiments do not provide any strong evidence against the low-pass characteristics of the chromatic contrast sensitivity.

**Experiment 4: Effect of Stimulus Size**

The contrast sensitivity for periodic stimuli is known to depend on the number of cycles displayed (Hoekstra, Goot, Brink, & Bilsen, 1974). Gratings with fewer cycles result in higher contrast thresholds, suggesting summation across cycles and/or spatial extent (Howell & Hess, 1978) until a critical summation area has been reached (Piper, 1903). Effect of stimulus area and number of cycles has been studied both in the fovea and the periphery, primarily for achromatic gratings (Manahilov, Simpson, & McCulloch, 2001). Studies using chromatic stimuli reported subthreshold spatial summation to be similar for achromatic and red-green gratings (Sekiguchi et al., 1993), but show a different dependence on eccentricity (Mullen, 1991) and larger integration areas for S-cone isolating gratings (Vassilev, Zlatkova, Manahilov, Krumov, & Schaumberger, 2000). The purpose of this additional experiment was to enable us to predict contrast sensitivity for stimuli of different sizes from our fixed-cycles data.

![Figure 10](image-url)  
**Figure 10:** Results of Experiment 4: each line represent contrast sensitivity for a series of stimuli with varying in spatial frequency and fixed number of cycles. The size of the Gaussian envelope was fixed to 0.5, 1, and 2 times the wavelength (the inverse of spatial frequency).
Methods

In Experiment 1, the Gaussian envelope size was equal to half wavelength, where wavelength is the inverse of spatial frequency. For the current experiment, we introduced two more envelope sizes equivalent to 1 and 2 wavelengths respectively. This manipulation allowed us to investigate spatial summation for each spatial frequency since contrast sensitivity was measured for three different envelope sizes. This experiment was conducted at 20 cd/m$^2$ and only with a subset of the observers of experiment 1, namely eleven observers from Cambridge and Liverpool (4 male, 7 female, mean age = 30.7±11.9). The procedure and apparatus were identical to Experiment 1.

Results

Contrast sensitivity increased with stimulus size (Fig. 10). Due to display size restrictions, not all spatial frequencies could be measured at all three envelope sizes. However, the available data suggest that an increase in envelope size causes a fixed increase in sensitivity in log-log space. In Figure 11, contrast thresholds are replotted as a function of area (deg$^2$) or three different frequencies (2,4,6 cpd). The slopes of the linear fits in log-log space are shown for each condition.
Modeling

Our goal was to derive a spatio-chromatic contrast sensitivity function which could interpolate and extrapolate the collected data within an allowable range. We constructed a set of nested models, with each successive model being more restrictive and with fewer free parameters. In Model 1 (‘Spatio-chromatic contrast sensitivity function’), the CSF was fitted separately for each colour direction and each luminance level (each panel in Figure 12 is fitted separately). Model 2 (including ‘Luminance Intrusion’) restricts the fits by assuming that the CSF for chromatic stimuli is a mixture of a purely chromatic CSF and a luminance CSF for high spatial frequencies. In Model 3, a functional relationship between the model parameters and the adapting light level (‘CSF as a function of adapting light level’) was introduced.

Subsequently, contrast sensitivity measurements for different envelope sizes were used to generalize the model predictions from fixed-cycles stimuli to stimuli of arbitrary sizes (‘CSF as the function of the stimulus size’) and the extended model was used to predict previously published contrast sensitivity data (Mantiuk, Kim, Rempel, & Heidrich, 2011; K. J. Kim, Mantiuk, & Lee, 2013; Wuerger, Watson, & Ahumada, 2002).

Spatio-chromatic contrast sensitivity function

As a function of spatial frequency, the achromatic CSF is band-pass and the chromatic CSFs have a low-pass shape (Fig. 5, 9). We modelled this behavior using a truncated log-parabola (Ahumada Jr & Peterson, 1992; Rohaly & Owsley, 1993; Watson & Ahumada, 2005; Y. Kim et al., 2017):

\[
\log_{10} S(f; S_{\text{max}}, f_{\text{max}}, b) = \log_{10} S_{\text{max}} - \left(\frac{\log_{10} f - \log_{10} f_{\text{max}}}{0.5 - 2b}\right)^2
\]

\[
S'(f; S_{\text{max}}, f_{\text{max}}, b, t) = \begin{cases} 
S_{\text{max}} & \text{if } f < f_{\text{max}} \text{ and } S(f; S_{\text{max}}, f_{\text{max}}, b) < \frac{S_{\text{max}}}{t} \\
S(f) & \text{otherwise}
\end{cases}
\]

Equation 6 has four parameters: peak frequency \( f_{\text{max}} \), peak sensitivity \( S_{\text{max}} \), bandwidth \( b \), and an optional truncation parameter \( t \). \( t \) describes the low-pass behavior in sensitivity functions where the sensitivity saturates to a constant value for spatial frequencies below the peak frequency.

In our experiments, achromatic contrast sensitivity was bandpass (Fig. 5) and both chromatic contrast sensitivities were lowpass (Fig. 5, 9). Thus, we first model all CSFs as log-parabola without the truncation parameter and then model the chromatic CSFs as truncated log-parabolas. The three color channels and the seven luminance levels are modeled independent of each other. We fitted the average data for each of the 21 conditions (7 luminances and 3 color channels) with either three \( (f_{\text{max}}, S_{\text{max}}, b) \) or four \( (f_{\text{max}}, S_{\text{max}}, b, t) \) free parameters.

We made the implicit assumption that the contrast sensitivity of the chromatic stimulus modulations (‘red-green’, ‘lime-violet’) is determined by the sensitivity of two putative chromatic mechanisms \((L - M, S - (L + M))\). While chromatic mechanisms favor low temporal and low spatial frequencies, it is unlikely that chromatic contrast variations at medium to high frequencies (4 and 6 cpd) are only seen by chromatic mechanisms (due to luminance artifacts; see Introduction for details). Based on the data from Mullen (1985), we fitted the chromatic channels using only data from spatial frequencies \( \leq 2 \) cpd.

The results are in Figure 12 and Table 2. The log-parabola model fits the achromatic data well, but a truncated log-parabola model is needed to explain the chromatic data, especially at the lower frequencies, which were measured only at 20 cd/m\(^2\). The chromatic data...
shows a small dip in sensitivity at the extreme luminance levels of 0.02 cd/m² and 7000 cd/m². Because we are lacking data for low frequencies at these luminance levels, we cannot confirm whether the dip reflects a real effect or measurement error.

Table 2: Parameters for log-parabola fit with truncation parameter for chromatic channels

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.02</th>
<th>0.2</th>
<th>2</th>
<th>20</th>
<th>200</th>
<th>2000</th>
<th>7000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{\text{max}})</td>
<td>(\mathcal{L} + \mathcal{M})</td>
<td>0.6839</td>
<td>0.6371</td>
<td>1.023</td>
<td>1.372</td>
<td>1.624</td>
<td>1.689</td>
</tr>
<tr>
<td></td>
<td>(\mathcal{L} - \mathcal{M})</td>
<td>0.5704</td>
<td>0.2596</td>
<td>0.4536</td>
<td>0.3094</td>
<td>0.4422</td>
<td>0.5547</td>
</tr>
<tr>
<td></td>
<td>(S - (\mathcal{L} + \mathcal{M}))</td>
<td>0.2702</td>
<td>0.4407</td>
<td>0.3543</td>
<td>0.1679</td>
<td>0.3344</td>
<td>0.4783</td>
</tr>
<tr>
<td>(S_{\text{max}})</td>
<td>(\mathcal{L} + \mathcal{M})</td>
<td>7.825</td>
<td>17.63</td>
<td>37.45</td>
<td>46.46</td>
<td>50.89</td>
<td>36.44</td>
</tr>
<tr>
<td></td>
<td>(\mathcal{L} - \mathcal{M})</td>
<td>15.73</td>
<td>53.93</td>
<td>142.6</td>
<td>347.8</td>
<td>508.9</td>
<td>417.4</td>
</tr>
<tr>
<td></td>
<td>(S - (\mathcal{L} + \mathcal{M}))</td>
<td>3.845</td>
<td>5.536</td>
<td>17.16</td>
<td>54.57</td>
<td>64.42</td>
<td>53.69</td>
</tr>
<tr>
<td>(b)</td>
<td>(\mathcal{L} + \mathcal{M})</td>
<td>3.019</td>
<td>4.867</td>
<td>3.999</td>
<td>4.047</td>
<td>4.431</td>
<td>5.799</td>
</tr>
<tr>
<td></td>
<td>(\mathcal{L} - \mathcal{M})</td>
<td>3.516</td>
<td>7.113</td>
<td>4.072</td>
<td>7.389</td>
<td>6.631</td>
<td>5.177</td>
</tr>
<tr>
<td></td>
<td>(S - (\mathcal{L} + \mathcal{M}))</td>
<td>7.216</td>
<td>7.166</td>
<td>7.147</td>
<td>11.34</td>
<td>6.676</td>
<td>5.499</td>
</tr>
<tr>
<td>(t)</td>
<td>(\mathcal{L} - \mathcal{M})</td>
<td>0.0339</td>
<td>0.000</td>
<td>0.000</td>
<td>0.0132</td>
<td>0.000</td>
<td>0.0024</td>
</tr>
<tr>
<td></td>
<td>(S - (\mathcal{L} + \mathcal{M}))</td>
<td>0.0576</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Figure 12: The results of fitting parabolic CSF models to the data, individually for each luminance level (columns) and color direction (rows). Note that the frequencies below 0.5 cpd were measured only at 20 cd/m² and for the chromatic color channels.
Figure 13: Channel summation model with 11 free parameter; see Table 3 for fitted parameters. Including luminance intrusion improves the model prediction for chromatic channels at higher frequencies.

**Luminance intrusion**

The CSF model in Fig. 12 predicted lower sensitivities for $L - M$ and $S - (L + M)$ at frequencies greater than 4 cpd than what we found in the experiments. We hypothesized that this was caused by the intrusion of a luminance mechanism at higher spatial frequencies (Flitcroft, 1989), possibly because we did not make the stimuli isoluminant for each observer using heterochromatic flicker photometry. We modeled this luminance intrusion by predicting chromatic sensitivity as the combination of responses of both luminance and chromatic mechanisms.

The probability that a stimulus defined by color contrast will be detected by achromatic or chromatic channels can be modelled as probability summation:

$$P_{Ach+Chr} = 1 - (1 - P(\alpha C S_{Ach})) (1 - P(C S_{Chr}))$$  \hspace{1cm} (7)

where $P_{Ach+Chr}$ is the probability of detecting stimulus of the contrast $C$, $S_{Ach}$ is the sensitivity of the achromatic channel and $S_{Chr}$ is the sensitivity of one of the chromatic channels (either red-green or yellow-violet). $\alpha$ is the portion of the original contrast that is detected by the luminance mechanism. Note that the product $C S_{Ach}$ gives the perceptually "normalized" contrast that is equal to 1 at the detection threshold. The function $P(c)$ is the psychometric function that can be expressed as:

$$P(c) = 1 - \exp(\tau c^\beta),$$  \hspace{1cm} (8)

where $\beta$ controls the slope of the psychometric function and $\tau$ controls the probability at the detection threshold. Since the thresholds were estimated from the 4AFC data for $P = 0.81$, we set $\tau$ to ln(0.81). If we introduce the psychometric function to Equation 7, we
Table 3: Parameters for channel summation fit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Channel</th>
<th>Luminance (cd/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>$f_{\text{max}}$</td>
<td>$L + M$</td>
<td>0.5052</td>
</tr>
<tr>
<td></td>
<td>$L - M$</td>
<td>0.4735</td>
</tr>
<tr>
<td></td>
<td>$S - (L + M)$</td>
<td>0.2463</td>
</tr>
<tr>
<td>$S_{\text{max}}$</td>
<td>$L + M$</td>
<td>7.138</td>
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<tr>
<td></td>
<td>$L - M$</td>
<td>14.44</td>
</tr>
<tr>
<td></td>
<td>$S - (L + M)$</td>
<td>3.595</td>
</tr>
<tr>
<td>$b$</td>
<td>$L - M$</td>
<td>4.802</td>
</tr>
<tr>
<td></td>
<td>$S - (L + M)$</td>
<td>5.680</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$L - M$</td>
<td>2.858</td>
</tr>
<tr>
<td></td>
<td>$S - (L + M)$</td>
<td>0.3480</td>
</tr>
</tbody>
</table>

get:

$$P_{\text{Ach+Chr}} = 1 - \exp\left(\tau \left(\alpha C S_{\text{Ach}}\right)^\beta\right) \exp\left(\tau \left(C S_{\text{Chr}}\right)^\beta\right)$$

$$= 1 - \exp\left(\tau C^\beta \left(\alpha^\beta S_{\text{Ach}}^\beta + S_{\text{Chr}}^\beta\right)\right)$$

If we introduce the psychometric function on the left side of the equation, we get:

$$1 - \exp(\tau C^\beta S_{\text{Ach+Chr}}^\beta) = 1 - \exp\left(\tau C^\beta \left(\alpha^\beta S_{\text{Ach}}^\beta + S_{\text{Chr}}^\beta\right)\right)$$

$$S_{\text{Ach+Chr}} = \left(\alpha^\beta S_{\text{Ach}}^\beta + S_{\text{Chr}}^\beta\right)^{1/\beta}$$

Therefore, the sensitivity for the combined response of the chromatic and achromatic channels can be modeled as a weighted Minkowski summation of the sensitivities of the individual mechanisms.

The achromatic sensitivity is modeled using the log-parabola model from Equation 6:

$$S_{\text{Ach}} = S\left(f; f_{\text{max}}^{(\text{Ach})}, S_{\text{max}}^{(\text{Ach})}, b^{(\text{Ach})}\right)$$

where $f_{\text{max}}^{(\text{Ach})}, S_{\text{max}}^{(\text{Ach})}, b^{(\text{Ach})}$ are the peak frequency, peak sensitivity, and bandwidth of the achromatic channel, at a given luminance level. The sensitivity to the two chromatic directions is modelled as the Minkowski summation of both chromatic and achromatic sensitivity:

$$S_{\text{Ach+RG}} = \left(\alpha_{RG}^\beta S_{\text{RG}}^\beta \left(f; f_{\text{max}}^{(\text{RG})}, S_{\text{max}}^{(\text{RG})}, b^{(\text{RG})}, t^{(\text{RG})}\right) + S_{\text{Ach}}^\beta \left(f; f_{\text{max}}^{(\text{Ach})}, S_{\text{max}}^{(\text{Ach})}, b^{(\text{Ach})}\right)\right)^{1/\beta}$$

$$S_{\text{Ach+YY}} = \left(\alpha_{YY}^\beta S_{\text{YY}}^\beta \left(f; f_{\text{max}}^{(\text{YY})}, S_{\text{max}}^{(\text{YY})}, b^{(\text{YY})}, t^{(\text{YY})}\right) + S_{\text{Ach}}^\beta \left(f; f_{\text{max}}^{(\text{Ach})}, S_{\text{max}}^{(\text{Ach})}, b^{(\text{Ach})}\right)\right)^{1/\beta}$$

where $f_{\text{max}}^{(\text{RG})}, b^{(\text{RG})}, t^{(\text{RG})}, f_{\text{max}}^{(\text{YY})}, b^{(\text{YY})}, t^{(\text{YY})}$ are the parameters of the two chromatic mechanisms, fitted independently for each luminance level. The parameters $\alpha_{RG}$ and $\alpha_{YY}$ control the amount of luminance intrusion. At each luminance level, we fit all
three sensitivity functions, 13 parameters in total (3 peak frequencies, 3 peak sensitivities, 3 bandwidths, 2 summation coefficients, 2 achromatic channel gains). The optimization was performed for the data of all 20 observers individually as well as the average CSF for all the observers. The fitting results for the average CSF data are presented in Fig. 13. The log-parabola fits (truncated in cases of chromatic channels) are shown as dotted lines in Figure 13. The model assumes that the achromatic stimuli are picked up solely by a luminance channel (upper row) and can completely specified by Eq. 13. For chromatic stimuli, we assumed that a luminance channel also contributes to the overall contrast sensitivity. In the second and third rows in Figure 13, the dotted lines represent the contributing luminance channel which adds to the chromatic sensitivity via probability summation (Eq. 7) and determines the response at higher spatial frequencies. The effect is more evident for the lime-violet stimuli.

The fitted parameters for the model are listed in Table 3. The values for $\alpha_{RG}$ are much higher than for $\alpha_{YV}$, which is due to the sensitivity values for $L - M$ are being higher than for $S - (L + M)$ or $L + M$ channels. This difference in sensitivity is partly due to the contrast is defined (Eq. 5). A quick investigation of the table reveals that many of the parameters are related to the logarithmic value of luminance. In the next section we model, such a functional relationship so that the model can be generalized to any luminance level within the measured range.

![Figure 14: The relationship between the fitted CSF parameters and luminance. The orange dots indicate parameters fitted for individual observers and the black dots the parameters fitted for the average observer. The dashed lines show the functions we fitted to the parameters from average observer data to build a luminance-dependent CSF. The adjusted $R^2$ values of the fits to the average observer are reported. $b$ for all channels and $f_{max}$ for the lime-violet channel did not fit well to a simple function and were thus fixed to the median value across luminance levels. Left: Log-parabola parameters; peak frequency $f_{max}$, peak sensitivity $S_{max}$, and bandwidth $b$. Right: Achromatic channel gain $\alpha$ used in Minkowski summation.](image-url)
### Contrast sensitivity as the function of mean luminance

Fig. 14 shows the relationship between the fitted CSF parameters and the logarithmic luminance. The plots clearly show that some parameters, such as $f_{\text{max}}$, $S_{\text{max}}$, and the inverse of $\alpha$, are strongly related to log-luminance, while the relation of $b$ is less clear given our data. To be able to generalize our model to different luminance levels (between 0.02 cd/m$^2$ and 7.000 cd/m$^2$), we fit functions for the CSF parameters that show strong relationship with luminance and find constant values for the parameter $b$, as listed in the equations below:

$$f_{\text{max}} = \begin{cases} 1.663\phi(\log l; 3.045, 4.008), & L + M \\ 0.06069 \log l + 0.3394, & L - M \\ 0.4095 & S - (L + M) \end{cases}$$

$$\log_{10} S_{\text{max}} = \begin{cases} 1.705\phi(\log l; 1.867, 4.443), & L + M \\ 2.715\phi(\log l; 2.663, 4.757), & L - M \\ 1.843\phi(\log l; 2.696, 3.688), & S - (L + M) \end{cases}$$

$$b = \begin{cases} 5.434, & L + M \\ 6.086, & L - M \\ 6.254 & S - (L + M) \end{cases}$$

$$1 = \frac{0.9323\phi(\log l; 0.6986, 2.826),}{\alpha} = \frac{4.099\phi(\log l; 0.3328, 3.303),}{S - (L + M)}$$

where $\phi$ is a Gaussian function: $\phi(x; \mu, \sigma) = \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$. The summation coefficient $\beta$ was fixed to 3.5. Fig. 15 shows model predictions for the achromatic (Eq. 13) and two chromatic (Eq. 14 and 15) components of the model when the parameters are predicted by the functions and constants from Eq. 16 above. Despite the approximations made to predict luminance-dependent parameters, the model provides good fit to the data.

The three models and their root-mean-squared-error (RMSE) are compared in Table 4. Model 1 was fitted individually for each measured luminance level and color direction. Model 2 was fitted for each luminance level, but jointly for all color directions. Model 3 was fitted for seven luminance-dependent parameters, and can generalize predictions to any arbitrary luminance level at the cost of higher RMSE.

### Table 4: Summary of nested models

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Model description</th>
<th>Summary</th>
<th>Equations</th>
<th>Mean RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Log-parabola</td>
<td>Optimization with 3 free parameters for Ach; $f_{\text{max}}^{(\text{Ach})}$, $S_{\text{max}}^{(\text{Ach})}$, $b^{(\text{Ach})}$, 4 free parameters for RG; $f_{\text{max}}^{(\text{RG})}$, $S_{\text{max}}^{(\text{RG})}$, $b^{(\text{RG})}$, $f^{(\text{RG})}$, and 4 free parameters for YV; $f_{\text{max}}^{(YV)}$, $S_{\text{max}}^{(YV)}$, $b^{(YV)}$, $f^{(YV)}$</td>
<td>Eq. 6 fitted separately for each color and luminance</td>
<td>$L + M$ 0.0463</td>
</tr>
<tr>
<td>2</td>
<td>Model 1 + Luminance intrusion</td>
<td>Optimization with 13 free parameters; $f_{\text{max}}^{(\text{Ach})}$, $S_{\text{max}}^{(\text{Ach})}$, $b^{(\text{Ach})}$, $f_{\text{max}}^{(\text{RG})}$, $S_{\text{max}}^{(\text{RG})}$, $b^{(\text{RG})}$, $f_{\text{max}}^{(YV)}$, $S_{\text{max}}^{(YV)}$, $b^{(YV)}$, $\alpha^{\text{ RG}}$, $\alpha^{\text{ YV}}$, $\beta^{\text{ RG}}$, $\beta^{\text{ YV}}$, and 2 fixed parameters: $f^{(\text{RG})}$, $f^{(\text{YV})}$</td>
<td>Eqs. 13 - 15 fitted simultaneously for all colors, independently for each luminance</td>
<td>$L + M$ 0.0701</td>
</tr>
<tr>
<td>3</td>
<td>Model 1 + 2 + Luminance dependence</td>
<td>Coefficients in Eqs. 16 optimized with 3 free parameters (Gaussian), and 2 free parameters (linear)</td>
<td>Eqs. 13 - 15 with parameters from Eq. 16</td>
<td>$L + M$ 0.1458</td>
</tr>
</tbody>
</table>
Contrast sensitivity as the function of stimulus size

When measuring stimuli of different frequencies we fixed the number of cycles. This made the stimulus size become smaller as frequency increased. We had decided upon this approach in order to collect more applicable data — in most applications, it is more important to know the exact threshold of a small pattern of high frequency rather than a large field of a high-frequency sine grating. But this choice also made our data harder to compare with other measurements, which were mostly done for stimuli of fixed size. In this section we describe a model that can generalize our predictions to stimuli of arbitrary size and frequency so that model predictions can be compared with other datasets.

Rovamo et al. (1993) modeled spatial integration as a function that increases with the stimulus area and saturates after reaching a critical area. The key observation they made was that the increase in sensitivity is proportional to the square root of the product of grating area and the squared frequency. We follow their model, but use the log-parabola sensitivity function rather than the OTF used in the original paper:

$$S_A(f, a; S_{\text{max}}, f_{\text{max}}; b, a_0, f_0) = S(f; S_{\text{max}}, f_{\text{max}}; b) \cdot \sqrt{\frac{a f^2}{a_0 + a f_0 + a f^2}},$$  

(17)

where $S(f)$ is the log-parabola model from Equation 6, $f$ is the spatial frequency in cycles per degree and $a$ is the area in deg$^2$. For our stimuli, which were smoothly modulated by Gaussian envelopes, we approximate $a$ with $\pi \cdot \sigma^2$; the area of a disk of the same radius as the standard deviation of the Gaussian envelope. $a_c$ and $f_0$ are the two parameters of the stimulus size model. We used the same equation but with different parameters for each color direction. We modeled the sensitivity using the OTF model from Rovamo et al. (1993) (Eq. 25) but found that it does not account for the drop in sensitivity at low frequencies and in our data.

Ideally, we would like to fit all 5 parameters of the model, but we found our data to be insufficient for that. Therefore, instead, we use the spatial integration parameters from the original paper for achromatic sensitivity: $a_0 = 114$ and $f_0 = 0.65$. For the two chromatic
sensitivities, we set $a_0$ to 40 and $f_0$ was kept the same as for the achromatic sensitivity. More data for large-size chromatic gratings would need to be collected to fully establish the values of these coefficients. As before, the data was fitted to the average observer data, but only for chromatic frequencies up to 2 cpd. The model was fitted to the 20 cd/m$^2$ data, which contained the variation in stimulus size (Experiment 4). The parameters of the model are presented in Table 5.

Table 5: Area dependent parameters of log-parabola at 20 cd/m$^2$

<table>
<thead>
<tr>
<th>Channel</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{\text{max}}$</td>
</tr>
<tr>
<td>$L + M$</td>
<td>447.5</td>
</tr>
<tr>
<td>$L - M$</td>
<td>2780</td>
</tr>
<tr>
<td>$S - (L + M)$</td>
<td>555.7</td>
</tr>
</tbody>
</table>

The fits to the data from Experiment 4 are shown in Figures 16 and 17. The model from Equation 17 can well account for the size of both achromatic and chromatic stimuli. However, the predictions are less accurate at higher frequencies for the two chromatic channels. This is to be expected as we did not intend to fit these data points, which would require modeling luminance intrusion.

Figure 16: Contrast sensitivity predictions for fixed-cycles stimuli, compared to the results of Experiment 4. Each column represents a separate color direction. Each row is plotted for a different stimulus size, determined as a fraction of the wavelength.
Figure 17: Contrast sensitivity predictions as the function of stimulus size (σ of the Gaussian envelope), compared with the results of Experiment 4. Each row shows predictions for a separate color direction. Each column is plotted for a different spatial frequency.

Figure 18: Contrast sensitivity at different luminance levels, as a function of stimulus size. From Mantiuk et al. (2011).
Figure 19: Comparison of our model with Wuerger et al. ColorFest dataset. The data is well explained by the continuous lines, showing the predictions for fixed size stimuli, which was used in the original experiment.

To use our model to predict datasets measured at different luminance levels, we extend the model to include the previously derived light-level dependency. Figure 18 shows the data from (Mantiuk et al., 2011), where contrast sensitivity was measured at different luminance levels for stimuli of different size. For a fixed spatial frequency, the sensitivity curve is simply shifted upwards in log-log space, suggesting that there is little interaction between the effect of light level and the effect of stimulus size. Therefore, contrast sensitivity can be simply modelled as:

\[ S_{AL}(f, l, a) = S_A(f, a) \cdot \frac{S_L(f, l)}{S_L(f, 20)} \]  

where \( S_L \) is luminance-dependent chromatic/achromatic CSF from the previous section (Eqs. 13-15) and \( S_A \) is the area-dependent CSF from Equation 17. The \( S_L(f, 20) \) in denominator accounts for the fact that \( S_A \) was fitted to the data measured at 20 cd/m\(^2\).

**Comparison with other datasets**

In the previous sections we showed that a relatively simple model can predict contrast sensitivity variation due to frequency, stimulus size and adapting luminance level, both for chromatic and achromatic gratings, as measured in our experiments. In this section we demonstrate that the same model can generalize and predict data from other experiments. We selected datasets that contained variability in luminance levels and/or included both chromatic and achromatic stimuli.

First we use the model from Equation 18 to predict the data from the ColorFest study (Wuerger et al., 2002). It should be noted that the ColorFest study used stimuli of fixed size and stimuli were temporally modulated (Gaussian modulation with a standard deviation of 0.125 sec). The sensitivity in the ColorFest data is uniformly, across all three colour directions, higher by a factor of by \( 0.3 \log_{10} \) units. To obtain comparable sensitivity values, we reduced the sensitivity of the original data by this amount, which resulted in reasonable good fits (Figure 19). The difference in overall sensitivity could be explained by the differences in experimental procedures: while ColorFest data were collected sequentially for each stimulus variation so that the same pattern was presented in consecutive 2AFC trials, in our 4AFC procedure we randomly selected a stimulus of a different frequency, color direction or orientation in each trial.

Figure 19 shows the original data together with the model predictions. Predictions for that data are shown as solid lines (labelled 'fixed size'). In addition to that, we show as dashed lines the predictions for the stimuli with the fixed number of cycles (and varying size), similar to the stimuli used in our experiments (labelled 'fixed cycles'). The model from Equation 18 was used for both curves.
Finally, we use the model to predict the data from the measurements of achromatic and chromatic gratings at luminance levels varying from 0.002 cd/m$^2$ to 200 cd/m$^2$ from (K. J. Kim et al., 2013). Since the experimental procedure was the same as in (Wuerger et al., 2002) and different than in our experiments, we reduced the contrast sensitivity of the data by the same amount of $0.3 \log_{10}$ units. The predictions for achromatic gratings are shown in Figure 20 and for chromatic gratings in Figure 21. We use the same notation as before: solid lines for fixed size stimuli used in (K. J. Kim et al., 2013) experiments, and dashed line for the fixed-cycles stimuli used in our experiment. The predictions of the model (solid lines) for achromatic gratings are close to the data except for the two lowest frequencies. This could be both due to the limitation of the simple log-parabola model we use and the lack of data for low-frequencies and achromatic gratings. The predictions for chromatic gratings (Figure 21) are reasonably accurate for the $L-M$ color direction, but slightly higher than the measurements for the $S-(L+M)$ color direction. We could not determine what was the cause of that difference.

![Figure 20](image-url)  
Figure 20: Comparison of our model predictions with the achromatic contrast sensitivity measurements from Mantiuk et al. (2011). Solid lines represent the same stimuli as used for the measurements.

![Figure 21](image-url)  
Figure 21: Comparison of our model predictions with chromatic contrast sensitivity measurements from K. J. Kim et al. (2013). Solid lines represent the same stimuli as used for the measurements.
Summary and Conclusions

Spatial contrast sensitivity measurements are commonly used to characterise the sensitivity of the human visual system at different spatial scales. We have extended existing measurements of contrast sensitivity to cover light levels ranging from low mesopic (0.02 cd/m$^2$) to high photopic (7000 cd/m$^2$) levels, and, crucially, measured sensitivity as a function of light level in all three directions of color space, an achromatic direction and two chromatic ones (red-green, lime-violet).

All our measurements were performed under steady-state adaptation to a particular light level. A notable feature of these extended contrast sensitivity measurements is that the adapting light level has a differential effect on the chromatic and achromatic contrast sensitivity in several important aspects: (1) We extended the contrast sensitivity measurements by Van Nes, Koenderink, Nas, and Bouman (1967) and demonstrated that the achromatic contrast sensitivity does not saturate at 200 cd/m$^2$, but it decreases again at higher light levels (Figure 22) (2) The light level at which Weber-law behaviour was observed was frequency-dependent for achromatic stimuli (2 cd/m$^2$ for 0.5 cpd; 200 cd/m$^2$ for 6 cpd), whereas for chromatic sensitivity we observed the transition to Weber’s law to occur at about 200 cd/m$^2$ at all spatial frequencies (Figure 7). (3) We extended the chromatic contrast sensitivity measurements of Mullen (1985) to very low and high light levels and showed that chromatic sensitivity saturates at about 200 cd/m$^2$ for spatial frequencies above 1 cpd.

We used these contrast sensitivity measurements, in conjunction with supplementary measurements on spatial summation in both the chromatic and achromatic domain, to derive a computational CSF model that predicts spatial contrast sensitivity for a wide range of ambient light levels in the mesopic and photopic range.

Our measurements were obtained under conditions of unlimited viewing time which precluded investigations of the interactions of spatio-chromatic sensitivity with temporal factors. Further extensions of the model need to take into account temporal factors as well as light-level-dependent spatial summation. A general model applicable to natural spatio-chromatic images will also need to characterise the summation across the chromatic and luminance channel at detection threshold.

Finally, our contrast sensitivity measurements will be important to develop computational appearance models for supra-threshold stimuli (Kulikowski, 1976) that allow to predict how appearance is affected under mesopic and photopic viewing conditions.

Figure 22: Summary of our model for spatio-chromatic contrast sensitivity at multiple luminance levels.
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The Matlab code used to calibrate the displays and the conversion from DKL to RGB space will be made publicly available. The link to the code with the fitted functions and the original data will also be provided upon acceptance.

References


The contrast sensitivity of human colour vision to red-green and blue-yellow chromatic gratings. 


