

**Proposition 1.**  $\text{length}(\text{rev}(L)) = \text{length}(L)$  for any list  $L$ .

*Proof by induction on  $L$ .* Let  $L$  be a list.

*Case  $L = []$ .*  $\square$

*Case  $L = a :: L'$  for some object  $a$  and some list  $L'$ .* Take an object  $a$  and a list  $L'$  such that  $L = a :: L'$ . Then  $L' \prec L$ . Hence  $\text{length}(\text{rev}(L')) = \text{length}(L')$ . Thus

$$\begin{aligned}\text{length}(\text{rev}(L)) &= \text{length}(\text{rev}(a :: L')) \\ &= \text{length}(\text{rev}(L') \uplus [a]) \\ &= \text{length}([a]) + \text{length}(\text{rev}(L')) \\ &= 1 + \text{length}(\text{rev}(L')) \\ &= 1 + \text{length}(L') \\ &= \text{length}(L)\end{aligned}$$

.  $\square$

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