

# Russell's Paradox in Naproche

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2025

*Russell's Paradox* is a set-theoretical paradox discovered by Bertrand Russell around 1902 [1, chapter XV] which shows that there exist statements  $\varphi$  whose extension  $\{x \mid \varphi(x)\}$  cannot constitute a set – or in other words: Not every class is a set.

**Theorem 1 (Russell's Paradox).** There exists a class that is not a set.

*Proof.* Assume the contrary. Define  $R := \{x \mid x \text{ is a set such that } x \notin x\}$ . Then  $R$  is a set. Hence  $R \in R$  iff  $R \notin R$ . Contradiction. ■

## References

- [1] Gottlob Frege. *The Philosophical and Mathematical Correspondence*. Ed. by Gottfried Gabriel et al. Basil Blackwell, 1980.

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