Definition 1. Let A be a class. A is *transitive* iff every element of A is a subset of A.

Proposition 2. Let X be a system of sets. Then X is transitive iff for every $x \in X$ and every $y \in x$ we have $y \in X$.

Definition 3. A *system of transitive sets* is a system of sets X such that every member of X is a transitive set.

Proposition 4. Every transitive class is a system of sets.

Proposition 5. Let X be a system of sets. Then X is transitive iff $\bigcup X \subset X$.

Proof.

Case X is transitive. Let $x \in \bigcup X$. Take a member y of X such that $x \in y$. Then y is a subset of X. Hence x is an element of X. □

Case $\bigcup X \subset X$. Let $x \in X$.

Let us show that $x \subset X$. Let $y \in x$. Then $y \in \bigcup X$. Hence $y \in X$. End. □

■

Proposition 6. Let A be a transitive class. Then $\bigcup A$ is transitive.

Proof. Let $x \in \bigcup A$.

Let us show that $x \subset \bigcup A$. Let $y \in x$. Take a member z of A such that $x \in z$. Then $z \subset A$. Hence $x \in A$. Thus y is an element of some member of A. Therefore $y \in \bigcup A$. End. ■

Proposition 7. Let X be a system of transitive sets. Then $\bigcup X$ is transitive.

Proof. Let $x \in \bigcup X$ and $y \in x$. Take $z \in X$ such that $x \in z$. Then z is transitive. Hence $x \subset z$. Thus $y \in z$. Therefore $y \in \bigcup X$. ■

Proposition 8. Let X be a system of transitive sets. Then $X \cup \bigcup X$ is transitive.

Proof. Let $x \in X \cup \bigcup X$.

Let us show that $x \subset X \cup \bigcup X$. Let $u \in x$. We have $x \in X$ or $x \in \bigcup X$. If $x \in X$then $u \in \bigcup X$. If $x \in \bigcup X$ then $u \in \bigcup X$. Indeed $\bigcup X$ is transitive. Hence $u \in \bigcup X$. Thus $u \in X \cup \bigcup X$. End. ■

Proposition 9. Let X be a system of sets. Then X is transitive iff $X \subset \mathcal{P}(X)$.

Proof.

Case X is transitive. Let $x \in X$. Then $x \subset X$. Hence $x \in \mathcal{P}(X)$. □

Case $X \subset \mathcal{P}(X)$. Let $x \in X$. Then $x \in \mathcal{P}(X)$. Hence $x \subset X$. □

■

Proposition 10. Let A be a transitive class. Then $\mathcal{P}(A)$ is transitive.

Proof. Let $x \in \mathcal{P}(A)$. Then $x \subset A$.

Let us show that $x \subset \mathcal{P}(A)$. Let $y \in x$. Then $y \in A$. Hence $y \subset A$. Thus $y \in \mathcal{P}(A)$. End. ■