Lemma 1. For every ordinal α there exists a cardinal greater than α.

Proof. Let α be an ordinal. Take $\kappa = |\mathcal{P}(\alpha)|$. Then $\kappa > |\alpha|$.

Let us show that $\kappa > \alpha$. Assume the contrary. Then $|\mathcal{P}(\alpha)| = \kappa \leq \alpha$. Hence $\kappa = |\mathcal{P}(\alpha)| = ||\mathcal{P}(\alpha)|| \leq |\alpha|$. Contradiction. End. ∎

Definition 2. Let κ be a cardinal. κ^+ is the cardinal such that $\kappa < \kappa^+$ and there is no cardinal ν such that $\kappa < \nu < \kappa^+$.

Definition 3. A *successor cardinal* is a cardinal number κ such that $\kappa = \nu^+$ for some cardinal number ν.

Proposition 4. Let κ be a cardinal. Then $|\alpha| \leq \kappa$ for every $\alpha \in \kappa^+$.