Proposition 1. Let λ be a limit ordinal. Then

$$\omega \leq \lambda.$$

Proof. Assume the contrary. Then $\lambda < \omega$. Consequently $\lambda \in \omega$. Hence $\lambda = 0$ or $\lambda = \mathrm{succ}(n)$ for some $n \in \omega$. Thus λ is not a limit ordinal. Contradiction. ■