Convention 1. Let $\mathsf{lim} x$ stand for $\bigcup x$.

Proposition 2. Let x be a subset of $\mathbb{O}$. Then $\mathsf{lim} x$ is an ordinal.

Proof. (1) $\mathsf{lim} x$ is transitive.

Proof. Let $y \in \mathsf{lim} x$ and $z \in y$. Take $w \in x$ such that $y \in w$. Hence w is transitive. Thus $z \in w$. Therefore $z \in \mathsf{lim} x$. $\square$

(2) Every element of $\mathsf{lim} x$ is transitive.

Proof. Let $y \in \mathsf{lim} x$. Let $z \in y$ and $v \in z$. Take $w \in x$ such that $y \in w$. Hence w is an ordinal. Thus y is an ordinal. Therefore y is transitive. Consequently $v \in y$. $\square$ $\blacksquare$

Definition 3. A *limit ordinal* is an ordinal λ such that neither λ is a successor ordinal nor $\lambda = 0$.