Signature 1. Let L, L' be lists. $L \mathbin{+\!\!+} L'$ is a list.

Axiom 2. Let L be a list. Then $[] \mathbin{+\!\!+} L = L$.

Axiom 3. Let a be an object and L, L' be lists. Then $(a :: L) \mathbin{+\!\!+} L' = a :: (L \mathbin{+\!\!+} L')$.

Proposition 4. $L \mathbin{+\!\!+} [] = L$ for every list L.

Proof by induction on L. Let L be a list.

Case $L = []$. $\square$

Case $L = a :: L'$ for some object a and some list L'. Consider an object a and a list L' such that $L = a :: L'$. Then $L' \prec L$. Hence $L' \mathbin{+\!\!+} [] = L'$. Thus

$$\begin{aligned} L \mathbin{+\!\!+} [] &= (a :: L') \mathbin{+\!\!+} [] \\ &= a :: (L' \mathbin{+\!\!+} []) \\ &= a :: L' \\ &= L \end{aligned}$$

. $\square$

$\blacksquare$

Proposition 5. $L \mathbin{+\!\!+} (L' \mathbin{+\!\!+} L'') = (L \mathbin{+\!\!+} L') \mathbin{+\!\!+} L''$ for any lists L, L', L''.

Proof by induction on L. Let L be a list.

Case $L = []$. $\square$

Case $L = a :: L'''$ for some object a and some list L'''. Consider an object a and a list L''' such that $L = a :: L'''$. Then $L''' \prec L$. Let L', L'' be lists.

Then $L''' \mathbin{+\!\!+} (L' \mathbin{+\!\!+} L'') = (L''' \mathbin{+\!\!+} L') \mathbin{+\!\!+} L''$. Thus

$$\begin{aligned} L \mathbin{+\!\!+} (L' \mathbin{+\!\!+} L'') &= (a :: L''') \mathbin{+\!\!+} (L' \mathbin{+\!\!+} L'') \\ &= a :: (L''' \mathbin{+\!\!+} (L' \mathbin{+\!\!+} L'')) \\ &= a :: ((L''' \mathbin{+\!\!+} L') \mathbin{+\!\!+} L'') \\ &= (a :: (L''' \mathbin{+\!\!+} L')) \mathbin{+\!\!+} L'' \\ &= ((a :: L''') \mathbin{+\!\!+} L') \mathbin{+\!\!+} L'' \\ &= (L \mathbin{+\!\!+} L') \mathbin{+\!\!+} L'' \end{aligned}$$

. $\square$

$\blacksquare$