Part I

Countably Infinite Classes

Definition 1. Let X be a class. X is *countably infinite* iff X is equinumerous to $\mathbb{N}$.

Proposition 2. Let X, Y be classes. If X is countably infinite and Y is equinumerous to X then Y is countably infinite.

Proof. Assume that X is countably infinite and Y is equinumerous to X. Take a bijection f between $\mathbb{N}$ and X and a bijection g between X and Y. Then $g \circ f$ is a bijection between $\mathbb{N}$ and Y (by bijectivity of composition of bijections). Indeed X, Y are classes. Hence Y is countably infinite. ■

Part II

Countable Classes

Definition 3. Let X be a class. X is *countable* iff X is finite or X is countably infinite.

Proposition 4. Let X, Y be classes. If X is countable and Y is equinumerous to X then Y is countable.

Proof. Assume that X is countable and Y is equinumerous to X. If X is finite then Y is finite. If X is countably infinite then Y is countably infinite. Hence Y is countable. ■

Part III

Uncountable Classes

Definition 5. Let X be a class. X is *uncountable* iff X is not countable.

Proposition 6. Let X, Y be classes. If X is uncountable and Y is equinumerous to X then Y is uncountable.

Proof. Assume that Y is equinumerous to X. If Y is countable then X is countable. Hence if X is uncountable then Y is uncountable. $\blacksquare$