Definition 1. Let n, m be natural numbers such that $n \geq m$. $n - m$ is the natural number k such that $n = m + k$. Let the *difference of n and m* stand for $n - m$.

Proposition 2. Let n, m be natural numbers such that $n \geq m$. Then $n - m = 0$ iff $n = m$.

Proof.

Case $n - m = 0$. Then $n = (n - m) + m = 0 + m = m$. □

Case $n = m$. We have $(n - m) + m = n = m = 0 + m$. Hence $n - m = 0$. □

■

Corollary 3. Let n be a natural number. Then $n - n = 0$.

Proposition 4. Let n be a natural number. Then $n - 0 = n$.

Proof. We have $n = (n - 0) + 0 = n - 0$. ■

Proposition 5. Let n, m be natural numbers such that $n \geq m$. Then $n - m \leq n$.

Proof. We have $(n - m) + m = n$. Hence $n - m \leq n$. ■

Proposition 6. Let n, m be natural numbers such that $n \geq m$. If $0 \neq m < n$ then $n - m < n$.

Proof. Assume $0 \neq m < n$. Suppose $n - m \geq n$. We have $(n-m)+m = n$. Then $n+m = (n-m)+m = n = n+0$. Hence $m = 0$. Contradiction. ∎

Proposition 7. Let n, m, k be natural numbers such that $n \geq m$. Then $(n-m)+k = (n+k)-m$.

Proof. We have

$$\begin{aligned} &((n-m)+k)+m \\ &= ((n-m)+m)+k \\ &= n+k \\ &= ((n+k)-m)+m. \end{aligned}$$

Hence $(n-m)+k = (n+k)-m$. ∎

Corollary 8. Let n, m, k be natural numbers such that $n \geq m$. Then $k+(n-m) = (k+n)-m$.

Proposition 9. Let n, m, k be natural numbers such that $n \geq m+k$. Then $(n-m)-k = n-(m+k)$.

Proof. We have

$$\begin{aligned} &((n-m)-k)+(m+k) \\ &= (((n-m)-k)+k)+m \\ &= (n-m)+m \\ &= n \\ &= (n-(m+k))+(m+k). \end{aligned}$$

Hence $(n-m)-k = n-(m+k)$. ∎