Proposition 1. $\mathbb{N}$ is a set.

Proof. Define $f(n) := n + 1$ for $n \in \mathbb{N}$. Then f is a map from $\mathbb{N}$ to $\mathbb{N}$. Hence we can take a subset X of $\mathbb{N}$ that is inductive regarding 0 and f. Then $0 \in X$ and for all $n \in X$ we have $n + 1 \in X$. Hence X contains every natural number. Thus we have $\mathbb{N} \subset X$ and $X \subset \mathbb{N}$. Therefore $\mathbb{N} = X$. Consequently $\mathbb{N}$ is a set. ■